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SUMMARY TECHNICAL REPORT
OF THE
NATIONAL DEFENSE RESEARCH COMMITTEE

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SUMMARY TECHNICAL REPORT OF DIVISION 7, NDRC

VOLUME 3

AIRBORNE FIRE CONTROL

OFFICE OF SCIENTIFIC RESEARCH AND DEVELOPMENT
VANNEVAR BUSH, DIRECTOR

NATIONAL DEFENSE RESEARCH COMMITTEE
JAMES B. CONANT, CHAIRMAN

DIVISION 7
H. L. HAZEN, CHIEF

WASHINGTON, D. C., 1946

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NOTES ON THE ORGANIZATION OF NDRC

The duties of the National Defense Research Committee were (1) to recommend to the Director of OSRD suitable projects and research programs on the instrumentalities of warfare, together with contract facilities for carrying out these projects and programs, and (2) to administer the technical and scientific work of the contracts. More specifically, NDRC functioned by initiating research projects on requests from the Army or the Navy, or on requests from an allied government transmitted through the Liaison Office of OSRD, or on its own considered initiative as a result of the experience of its members. Proposals prepared by the Division, Panel, or Committee for research contracts for performance of the work involved in such projects were first reviewed by NDRC, and if approved, recommended to the Director of OSRD. Upon approval of a proposal by the Director, a contract permitting maximum flexibility of scientific effort was arranged. The business aspects of the contract, including such matters as materials, clearances, vouchers, patents, priorities, legal matters, and administration of patent matters were handled by the Executive Secretary of OSRD.

Originally NDRC administered its work through five divisions, each headed by one of the NDRC members. These were:

Division A—Armor and Ordnance
Division B—Bombs, Fuels, Gases, & Chemical Problems
Division C—Communication and Transportation
Division D—Detection, Controls, and Instruments
Division E—Patents and Inventions

In a reorganization in the fall of 1942, twenty-three administrative divisions, panels, or committees were created, each with a chief selected on the basis of his outstanding work in the particular field. The NDRC members then became a reviewing and advisory group to the Director of OSRD. The final organization was as follows:

Division 1—Ballistic Research
Division 2—Effects of Impact and Explosion
Division 3—Rocket Ordnance
Division 4—Ordnance Accessories
Division 5—New Missiles
Division 6—Sub-Surface Warfare
Division 7—Fire Control
Division 8—Explosives
Division 9—Chemistry
Division 10—Absorbents and Aerosols
Division 11—Chemical Engineering
Division 12—Transportation
Division 13—Electrical Communication
Division 14—Radar
Division 15—Radio Coordination
Division 16—Optics and Camouflage
Division 17—Physics
Division 18—War Metallurgy
Division 19—Miscellaneous
Applied Mathematics Panel
Applied Psychology Panel
Committee on Propagation
Tropical Deterioration Administrative Committee

NDRC FOREWORD

As events of the years preceding 1940 revealed more and more clearly the seriousness of the world situation, many scientists in this country came to realize the need of organizing scientific research for service in a national emergency. Recommendations which they made to the White House were given careful and sympathetic attention, and as a result the National Defense Research Committee [NDRC] was formed by Executive Order of the President in the summer of 1940. The members of NDRC, appointed by the President, were instructed to supplement the work of the Army and the Navy in the development of the instrumentalities of war. A year later, upon the establishment of the Office of Scientific Research and Development [OSRD], NDRC became one of its units.

The Summary Technical Report of NDRC is a conscientious effort on the part of NDRC to summarize and evaluate its work and to present it in a useful and permanent form. It comprises some seventy volumes broken into groups corresponding to the NDRC Divisions, Panels, and Committees.

The Summary Technical Report of each Division, Panel, or Committee is an integral survey of the work of that group. The first volume of each group's report contains a summary of the report, stating the problems presented and the philosophy of attacking them, and summarizing the results of the research, development, and training activities undertaken. Some volumes may be "state of the art" treatises covering subjects to which various research groups have contributed information. Others may contain descriptions of devices developed in the laboratories. A master index of all these divisional, panel, and committee reports which together constitute the Summary Technical Report of NDRC is contained in a separate volume, which also includes the index of a microfilm record of pertinent technical laboratory reports and reference material.

Some of the NDRC-sponsored researches which had been declassified by the end of 1945 were of sufficient popular interest that it was found desirable to report them in the form of monographs, such as the series on radar by Division 14 and the monograph on sampling inspection by the Applied Mathematics Panel. Since the material treated in them is not duplicated in the Summary Technical Report of NDRC, the monographs are an important part of the story of these aspects of NDRC research.

In contrast to the information on radar, which is of widespread interest and much of which is released to the public, the research on subsurface warfare is largely classified and is of general interest to a more restricted group. As a consequence, the report of Division 6 is found almost entirely in its Summary Technical Report, which runs to over twenty volumes. The extent of the work of a Division cannot therefore be judged solely by the number of volumes devoted to it in the Summary Technical Report of NDRC: account must be taken of the monographs and available reports published elsewhere.

The Fire Control Division, initially Section D2 under the leadership of Warren Weaver and later Division 7 under Harold L. Hazen, made a significant contribution to an already highly developed art. It marked the entrance of the civilian scientist into what had hitherto been regarded as a military specialty.

It was one of the tasks of the Division to explore and solve the intricate problems of control of fire against the modern military aircraft. Gunnery against high-speed aircraft involves fire control in three dimensions. The need for lightning action and superlatively accurate results makes mere human skills hopelessly inadequate. The Division's answer was the development of the electronic M-9 director which, controlling the fire of the Army's heavy AA guns, proved its worth in the defense of the Anzio Beachhead and in the protection of London and Antwerp against the Nazi V-weapons. In addition to producing mechanisms such as the M-9, the Division made less tangible but equally significant contributions through the application of research methods which had a profound, even revolutionary, influence on fire control theory and practice.

The results of the work of Division 7, formerly Section D2, are told in its Summary Technical Report, which has been prepared at the direction of the Division Chief and has been authorized by him for publication. It is a record of creativeness and devotion on the part of men to whom their country will always be grateful.

VANNEVAR BUSH, Director
Office of Scientific Research and Development

J. B. CONANT, Chairman
National Defense Research Committee

FOREWORD

VOLUME 3 of Division 7, the Summary Technical Report of Section 7.2, NDRC, contains three parts. In Part I on aiming controls in aerial ordnance, Mr. G. A. Philbrick discusses the work of the Section in all fields except that of gunnery and the assessment of gunnery devices. Mr. A. L. Ruiz has contributed Part II, in which developments in aerial torpedo directors subsequent to those in which Mr. Philbrick took part are discussed. The third principal part of the report is that on aerial gunnery and assessment, written by Professor J. B. Russell.

It is indeed fortunate that such a large part of this work could be written by one individual, who could thus provide a unity of treatment which otherwise would be very difficult to obtain. In assuming responsibility for his part of the Summary Technical Report, Mr. Philbrick took on a heavy task—and discharged it with zeal. Under the circumstances a more conventional report would have fulfilled all the requirements, but Mr. Philbrick has served his reader a tasty dish of skilled technical exposition. We can ask no more.

In general the part contributed by Professor Russell stresses the instrumental features of aerial gunnery, and relies upon the writing of Dr. Paxson, in Volume 2 of the Summary Technical Report of the Applied Mathematics Panel, for the basic mathematical substance of the subject. Professor Russell brings to his treatment continuous experience in the field dating from before Pearl Harbor. He has participated in all of its growth, first as a Technical Aide in Section 7.2, and during the closing months of the war as an Expert Consultant to the Secretary of War.

H. L. HAZEN
Chief, Division 7

S. H. CALDWELL
Chief, Section 7.2

CONTENTS

PART I

AIMING CONTROLS IN AERIAL ORDNANCE

By *G. A. Philbrick*

PREFATORY COMMENTS

GENERAL

During the past three years and a half the present writer has served as technical aide to S. H. Caldwell, Chief of Section 7.2, whose section has been charged with airborne developments within the more general fire control framework of Division 7 of National Defense Research Committee [NDRC]. By delegation from the section chief, the writer has shared, with other technical aides and section members, several domains of responsibility in the conduct of research and development on airborne fire controls. The initiation for such work in typical cases occurs through a request by Army or Navy to Office of Scientific Research and Development [OSRD] for a particular study of development; following acceptance by the latter organization, which is made through agreements of the relevant section and division of NDRC, a program is laid out and presented as specification for a project to the appropriate contractor. Guidance of this project through the stages of theory or experiment, design or test, and the maintenance of liaison with the same agencies concerned, constitute functions of the NDRC section. Such duties have in turn become those of such operatives as the present writer, who enter personally into the technical phases of the development of the project as deeply as appears necessary or appropriate, and who answer to the section for the conduct and outcome of the work.

SUBJECT MATTER

By reasons of familiarity and plausibility, the material treated in the following pages is naturally restricted to those branches of airborne fire control with which the writer has been concerned at first hand. Although rather generously distributed over the field, these branches have not been all inclusive. They are considered sufficiently pervasive, however, to warrant the title given to the present report. In particular, the subject matter revolves about the development of computors and computing sights for aerial torpedoing, bombing, and aerial rocketry, with a final attempt to combine all of these functions with that of fixed gunnery for the fighter airplane. The most impressive omission is that of flexible gunnery, for while the writer has been exposed to and has gained familiarity with the equipment and developmental procedures in this branch, he has had little or no tangible responsibility there. Another characterization of the present material refers to the character, or technical position, of the researches which are reported upon, and reflects the corresponding nature of the activities of the writer. The latter has been substantially a creature of the laboratories, operating for the most part between theory and design. His contacts with the using Services have usually been with respect to a particular equipment under development, and his involvement with proving organizations and with aerial firing tests has been for the specific purpose of gaining knowledge of the properties of one device rather than of a larger category. That is to say, the morphology of localized apparatus, from the instrumental standpoint, is here stressed rather than a broader survey of available ordnance or a presentation of assessment techniques, in spite of the recognized status of the latter. The flavor is thus dynamical rather than statistical, constructional rather than evaluational, instrumental rather than logistic, and physical rather than administrative. This is not to indicate any absolute preference or desirability, but straightforwardly to identify the aspect of the material to be treated, and to admit that this aspect stems, quite naturally, out of the writer's own predilections and propensities, quite aside from whether these are important.

In addition to the description, in successive developmental forms, of a number of specific computing devices and controls, considerable space is here allotted to certain tools of research themselves, where it is felt that these constitute advances in the techniques of instrument development or design. It is believed, for example, that the use of increasingly compre-

hensive electronic model structures can bridge enormous gaps which classically have intervened between theory and concrete facilities. Such model structures, owing to their ability to "cooperate" with the real human operator and to incorporate discontinuities against which analysis is almost completely impotent, can bring realism to the laboratory and can shorten the interval over which optimum dynamics are attained. With completely automatic assemblages the need for such models is as great or greater.

As to weapons, we are concerned here with the airplane, the projectile, and the man at the firing-key, the whole group operating as a unit. The airplane types include principally the fighter, fighter-bomber, and bomber. The projectiles are bomb, guided bomb, rocket, torpedo, and bullet or shell, in approximately that order of concern. A typical aiming control, or computing sight, involves a group of input variables, which may be either manually or automatically introduced, a computer proper, and a presentation-component or *sight* whereby the aiming process is reduced to some sort of null between an index and the target. Automatic firing may be involved, either permanently or at the choice of the operator.

It is difficult confidently to predict the shape of fire controls for the future, at least from the present standpoint. The trend toward more thoroughgoing automaticity is certainly evident, with the task of the operator becoming increasingly supervisory and eclectic. At this level the distinctions between offense and defense, and even between strategy and tactics, are so interwoven with other relationships as to be almost nonexistent, so that we shall not here presume to speak of these. While the rocket and guided bomb, together with their logical combinations, are outstandingly weapons of great future significance, the bomb, bullet, and shell are simple structures which should not be ignored. As to superexplosives, with potentially a million times greater payload ratio, the need for fire control should not decrease any more than as between flintlock and main battery. While the appearance and character of weapons may suffer revolutionary changes, in whatever era, with the science even of orbital underwater trajectories being replaced, for example, by that of underground trajectories, it will still be essential to direct and deliver fire in the vicinity of the target. Of the homing and automatic interception missiles on whose protection survival may depend, and which ultimately may truly battle among themselves, those with the more recondite controls will triumph along with their masters.

COLLEAGUES

In view of the apparent virtuosity of this report, the framework of personnel within which the writer has operated deserves particular comment. Both categorical and special, the people among whom the progress herein reported has taken place have made it what it is.

The staff of NDRC Section 7.2, with chief, members, and fellow technical aides, has been closest of all to these operations, and knows most about them. Thus are listed: S. H. Caldwell (Chief), J. B. Russell (since transferred to the Office of the Secretary of War), A. L. Ruiz, H. C. Wolfe, C. G. Holschuh, W. A. MacNair, E. G. Pickels, E. W. Paxson, and (latterly) A. F. Sise; in whose collective company it has been an honor for the writer to work. Knowledge of the branches of airborne fire control which are not herein treated, and more complete discernment of those which are, must be sought among these gentlemen. Also within Section 7.2 as technical aide, R. M. Peters is to be mentioned by the writer as his staunch mathematical assistant. She has helped materially in every phase of progress, and is to be credited with whatever significant mathematics adheres to the present report.

The remainder of Division 7, with H. L. Hazen as chief and K. L. Wildes as divisional technical aide, and with the adjacent section chiefs, D. J. Stewart, E. J. Poitras, and I. A. Getting, has served in all connections as guidepost and beacon. Considerable and appreciated interchange has been enjoyed with such members and technical aides of adjacent sections as G. R. Stibitz, L. M. McKenzie, and J. F. Taplin.

Of other divisions of NDRC with which developmental business has been conducted,

Division 5 should be mentioned especially. Control apparatus for new missiles has been dealt with in this relationship, both through an arranged collaboration between the two divisions, and by the writer as an officially appointed consultant to Division 5 itself. Constructive and informative intercourse has taken place with H. H. Spencer, Chief, Division 5, as well as with J. C. Boyce, L. O. Grondahl, P. Mertz, and E. W. Phelan, among others. Further relations have been important with Division 3 (F. L. Hovde) on rockets, with Division 4 (A. Ellett) on toss bombing, with Division 6 on antisubmarine warfare, with Division 14 on radar and with Section 16.1 (T. Dunham, Jr.) on optical instruments. Valuable personal contact on professional matters has been possible also with W. Weaver, T. C. Fry, J. D. Williams, M. S. Rees, and others of the Applied Mathematics Panel. Among other groups which cannot be mentioned in entirety are the administrative and engineering staffs of OSRD and NDRC, both central and local.

A detailed accounting of the Service agencies and personnel which have been directly concerned with these efforts would be all out of order in the present location. Such agencies and personnel will appear, however, in reference to the project chronologies in the main body of the report. Relations with the Services have, on the whole, been extremely pleasant and positive. The writer's dealings have been predominately with the Navy, and within Navy with the Bureau of Ordnance, and within BuOrd with the aircraft fire control sections: Re4 (Re4d), Re8 (Re8c). Further extensive affairs have been enjoyed with NAS Norfolk, NAF Philadelphia, NAS Squantum, NAS Quonset, NOTS Inyokern, NOP Indianapolis, and the various special devices depots, to mention only several branches. The Navy projects on which the writer has been engaged, and which are herein reported upon, include NO-106 (torpedo director: now TD Mark 32), NO-129 (antisubmarine bombsight: now BS Mark 20), NO-180 (maneuverable bombing target), NO-190 (blimp bombsight: now Mark 24), NO-191 (bombsight presetting computer), NO-216 (rocket sights: now RS Mark 2 and RS Mark 3, and computers Mark 35 and Mark 36), AN-4 (low altitude bombsight; BARB: now BS Mark 23), NA-168 (slant range computer), NO-242 (range-type torpedo director), NA-232 (Razon attachment for TA3 trainer), and NO-265 (pilot's universal sighting systems: now AFCS Mark 3). With the Army, we have dealt most directly with the armament laboratory (ATSC) at Wright Field, as well as with Langley Field, Foster and Matagorda Fighter Fields, Dover Air Base, and other installations. The Army projects have been AN-4 (low altitude bombsights), AC-36 (CRAB guided bombsight), and AC-121 (rocket sights). In the case of both Services, a good deal of work, both concrete and of an advisory nature, has been done under no official project whatsoever. Thus, in connection with projects which were related to those for which control numbers had been assigned to us, the writer was requested by BuOrd of Navy to advise directly on certain projects placed with Navy contractors. Instances include Specialties, Inc., at Syosset, New York, and Polaroid Corporation in Cambridge, Massachusetts.

CONTRACTORS

First, with regard at least to groups under contract to NDRC Section 7.2, it is well known that in general all organized technical pursuits under NDRC direction are set up through the facilities of such contractors. The present writer has had most to do with the contract (OEMsr-330) at The Franklin Institute in Philadelphia, where a major portion of the section's airborne fire control developments have been conducted. The research group here has been built, with considerable assistance and molding by the section, from a nucleus of four engineers to a staff of several score technical personnel, augmented somewhat by the staffs and facilities of several subcontractors. Laboratory, office, and drafting space have been prepared and occupied gradually as needed. Sharing such facilities as experimental, computing, drafting and model shop, the project staff blossomed horizontally into groups devoted to torpedoing, bombing, gunnery, rocketry and integrated equipments. The heads of these groups reported to the coordinator for

the contractor, who has been R. H. McClarren. Guidance in technical policy and project planning has been given by the section staff through the agency of a steering committee which was formed rather early in the history of this contract. McClarren served this committee as nonvoting secretary, and the remainder of the members was constituted of S. H. Caldwell (ex officio), A. L. Ruiz, H. C. Wolfe, E. G. Pickels, and until recently J. B. Russell, with the present writer as chairman. Aside from his existence in the latter capacity, however, the writer was also held responsible for a specific number of developmental projects as such. Although in connection with such projects he owes much to specific personnel at The Franklin Institute, mention of the numerous men thus concerned will be reserved for the more detailed and relevant portions of the report.

In connection with a contract undertaken for Section 7.2 by the Stanolind Corporation in Tulsa, Oklahoma, the writer was given surveillance of the development of a mechanical pursuit-collision course plotter, proposed by M. Alkan of Specialties, Inc., for assistance in the design of a Navy dive bombsight. Principal contact at Stanolind was with D. Silverman.

Through a contract with Columbia University, at the Marcellus Hartley Laboratory, the section conducted electronic projects under J. A. Balmford and J. R. Ragazzini there. As was the intent, such work largely served and supplemented that in progress within the larger group at The Franklin Institute, and covered certain servomechanism developments as well as simulative endeavors. In addition, however, electronic simulative studies were conducted on steered projectiles for projects being pursued by Section 7.2 in collaboration with Division 5. This branch of the contract was taken under the direct sponsorship of Division 5 in the summer of 1945 and was placed within a contract with Specialties, Inc. The reader is referred to Chapters 4 and 8.

Under contract to the section, The Bristol Co. has also contributed to researches at The Franklin Institute, and through the services of C. A. Mabey, A. W. Jacobson, and G. M. Thynell has aided in the preparation of special mechanical linkages for components of computing sights under development.

The writer has had considerably less to do with the others of the section's contracts, which include those at General Electric on B-29 computers, at Northwestern University on assessment methods, at University of Texas on gunnery evaluation, and at Jam Handy Corporation on vector sights, etc.

With several of the other contractors of Division 7, highly beneficial cooperation has been indulged in on projects of mutual interest. Through the provision, by Section 7.3 for example, of certain facilities of Lawrance Aeronautical Corporation, Linden, New Jersey, the design of pneumatic components has been greatly furthered on our Navy projects. Facilities also of Eastman Laboratories in Rochester have similarly been made available, under the stewardship and incentive of E. J. Poitras and J. F. Taplin of Section 7.3. An earlier example of such collaborative effort on pneumatic instrumentation, which resulted ultimately in bombsight Mark 23, involved the McMath-Hulburt Observatory at Lake Angelus, Michigan.

Far too numerous for exhaustive tabulation are the contractors of NDRC divisions other than Division 7. However, the Radiation Laboratory at the Massachusetts Institute of Technology [MIT] must be mentioned as being involved in several connections, in connection particularly with the provision of automatic plane-to-plane and plane-to-ground ranging equipment. Tangible help on a variety of problems has been received from many of the staff there. We mention also the California Institute of Technology, under Division 3, in connection with rockets, where C. C. Lauritsen, W. A. Fowler, C. G. Anderson, and others have rendered assistance. With that portion of the Bureau of Standards under Division 4, we have dealt profitably with W. B. McLean on toss-bombing studies. The contractors of Division 5 have been concerned, both through interdivisional collaborative arrangement, and by the writer directly as consultant to that division; these dealings are reported jointly since separation is not feasible. Such contractors include:

Gulf Research and Development Co. in Pittsburgh, where we have cooperated on guided bomb controls with E. A. Eckhardt, R. D. Wyckoff, and J. P. Molnar among others; RCA Laboratories (Zworykin) at Princeton in connection with television for homing bombs; Douglas Aircraft in Santa Monica, where W. B. Klemperer, E. W. Wheaton, and many others were extensively cultivated with regard to the ROKH projectile in its several phases; L. N. Schwien Engineering Co. in Los Angeles, with L. N. Schwien and H. A. VanDyke on stabilization and other control techniques; and Bendix Pacific Division in Hollywood, with W. S. Leitch on radio links.

Special mention must be made of the Applied Mathematics Group at Columbia, under contract to the Applied Mathematics Panel [AMP] of NDRC. Incalculable aid has been received from this group, with which we have worked quite closely on several fire-control projects. The notables include S. MacLane (Director), H. Whitney, H. Pollard (the two latter having had local office headquarters in the writer's office in Cambridge), I. Kaplansky, L. C. Hutchinson, and D. P. Ling. Particular operations thus valiantly served were rocket sights, toss-bombing equipment, and pilot's universal sighting systems; relating to Service projects NO-216 and NO-265. Certain other groups under AMP were similarly useful, though as it happened in less major connections.

In connection with researches on controls for guided bombs the differential analyzer at MIT has been made available for an extensive study of controlled trajectories in two and three dimensions. Engaged at first through the contractual machinery of Division 7, and later through that of Division 5, this facility made possible an articulate numerical treatment of a problem which could not otherwise have been handled without years of computation by a large staff expertly led at every point. The staff of this analyzer, as of the Center of Analysis itself at MIT, contributed mightily in this work.

Wherever, in the present report, the writer refers to "our laboratories," this impropriety should be interpreted as a manner of speaking, which has arisen out of habit, rather than in a precisely literal sense. The laboratories of The Franklin Institute contract are most likely thus to be connoted, and perhaps, even those at Columbia. The possessive pronoun is a consequence of the part the section has played in building up such facilities and in organizing the research which has been pursued.

FORM AND REFERENCES

The monographic form for this report, with its set of sub-monographs on the separate techniques and fields of endeavor, was chosen by the writer out of his personal preference for a unified literary entity. It should be possible, however, to separate, discard, or reassemble the various parts hereof in whatever manner appears desirable to suit a larger need. No details are here bodily reproduced which are available elsewhere, although brief outlines of such fuller material may be categorically included in reference to related topics.

Certain of the writer's own contributions, in either theoretical branches or concrete mechanism, may here be found discussed with an apparently unwarranted emphasis. In such cases the reason for such treatment is that documentation of these items is not likely to be found at other sources, or may there be but baldly referred to.

A serious attempt is made to give proper credit and to make equitable references to original enunciations and reductions to practice. If this is imperfectly achieved, it is without malice. Contractor's reports are referred to liberally, but more thorough search of these is indicated for the details of any given aspect which may later become significant. Writings of such collaborating mathematicians as H. Whitney and H. Pollard (of AMG-C) are voluminous and frequently contain definitive documentation of methods and components which we have been too eager to build and try, in contrast to the compulsions of our manifest duty to disseminate.

Chapter 1

GENERAL THEORY OF AIMING PROCESSES

1.1 SCOPE AND LIMITATIONS

To the present chapter we have relegated as much as possible of the general and theoretical background which is relevant to the rest of the report. There is introduced here not only an account of the mathematical scaffolding which is typically to be employed, but also a number of the physical and dynamical principles on which the numerous aiming devices rest. The presentation is elementary, as indeed are the principles themselves, but it is considered that a service will hereby be afforded in characterizing both the mode of approach to problems and the sort of description and explanation which is employed throughout.

The various branches of fire control are based to a remarkable degree on a common technical foundation. Although this is not surprising on the face of it, nevertheless it is found impressive to the worker in tangible apparatus as his attention shifts from one field to another, and this is true no matter how extensive his experience and however sophisticated his philosophy. The realm of statistics and probability is not strictly included in these remarks, for it is of broader significance yet. This realm, although inseparable from the theory and instrumental techniques of the subject of this report and serving it in all phases when applied in the economy of effort and the concentration of effectiveness, is entered into here only to a minor degree. Being treated far more completely and competently elsewhere, and applying more indispensably, for example, in the larger senses of assessment, the methods and accomplishments of statistics are substantially omitted from these pages. Thus, even in the development of automatic mechanism as such, what is included must be considered only a factor in the whole assemblage; but it is this factor on which we have specialized, and such emphasis is properly descriptive of the activity to be documented.

Another reason for the inclusion of the present chapter is to record, in close association with the individual treatments of apparatus and of its development, a self-consistent, albeit somewhat diluted, compendium of definitions for the several terms and phrases which are peculiar to this field and to which allusion is frequently made. This is considered advisable owing to the rather large and widespread disparity which is evident among the current users of such nomenclature. It will be apparent to the reader that certain departures are herein made from usages and conventions which are orthodox in this field. In such cases the divergence is commonly for the purposes of increased generality, to which end the writer perpetually strives; and an attempt is made to identify such innovations, locally, with the more familiar and restricted concepts.

1.2 THE POSITION VECTOR

Among the symbolic languages which are available to the dealer in fire controls, that of the free vector is one of the most direct and articulate. We build here the structure of such vectors which describes a system of points in motion in a medium. While falling short of the more general rigid-body dynamics, such systems are everlastingly serviceable for the embodiment of a great fraction of the fundamentals hereto relevant.

Consider first a point which is stationary in a given medium, the latter being unaccelerated in space and for the present purposes uniform. This point, which may be called a, is typical of and may identify the medium. It may have been quite arbitrarily chosen.

A second point, moving in or with respect to this medium, may be symbolically identified as b. The *position* of the point in the medium, or with respect to point a, is to be described completely by the vector $\mathbf{R}_{ab}$, which may be thought of alternatively as the "directed

range" from point a to point b, in that order. The vector so expressed denotes both distance and direction, in accord with the normal capability of vectors. When this vector is constant,

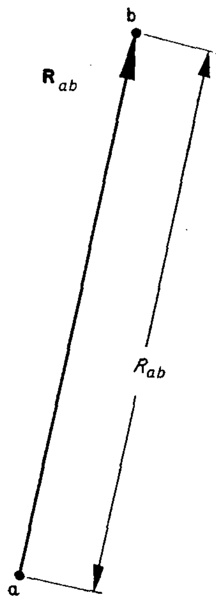


FIGURE 1. Standard position vector.

the second point is stationary with respect to the first point, and hence in the medium thereby identified. When this vector is variable, being for example a function of time, then if its origin is located in point a, the tip of the vector traces out the space-path of the moving point b in the medium.

When point b approaches point a, the vector $\mathbf{R}_{ab}$ approaches $\mathbf{R}_{aa}$, which is the null vector although not necessarily the number zero. The scalar or inner product of $\mathbf{R}_{ab}$ with itself, that is $\mathbf{R}_{ab} \cdot \mathbf{R}_{ab}$, gives the square of the absolute scalar magnitude of the vector $\mathbf{R}_{ab}$, and in this case is also the square of the undirected distance or true range of point b from point a. Of the many notations which are current for the scalar magnitude itself, we mention $|\mathbf{R}_{ab}|$. We shall denote this quantity here by the scalar symbol R_{ab}, as in

$$R_{ab}^2 = \mathbf{R}_{ab} \cdot \mathbf{R}_{ab}. \tag{1}$$

It is intended that this quantity contain, or carry with it, the physical dimensions, if any, of the vector $\mathbf{R}_{ab}$. The quantity R_{ab}, then, is the distance (in feet, say) from the point a to the point b.

Consider next the vector $k\mathbf{R}_{ab}$, defined as the position vector, with respect to point a, of a point collinear with a and b, and which is k times as far from a as b is. Its scalar magnitude is certainly kR_{ab}. The scalar multiplier k may be purely numerical, in which case it produces merely a stretched version of the original vector, or it may have physical dimensions and thus alter the character of the original vector. If we arrange that $k = R_{ab}^{-1}$, then the so-called unit vector, having a scalar magnitude of unity, is produced. This unit vector indicates only the direction of the point b from the point a and may be given the symbol $\mathbf{r}_{ab}$; thus:

$$\mathbf{r}_{ab} = R_{ab}^{-1}\mathbf{R}_{ab} \quad \text{or} \quad \mathbf{R}_{ab} = R_{ab}\,\mathbf{r}_{ab}. \tag{2}$$

The unit vector is undefined when the scalar R_{ab} is zero. It is further evident that the unit vector does not have, in this sense at least, any dimensions.

If the position of point b with respect to

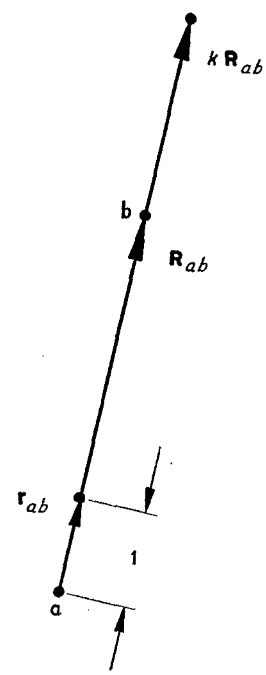


FIGURE 2. Unit vector and its multiples.

point a depends in either a preassigned or a determinable way on time, then the vector $\mathbf{R}_{ab}$ is said to be a function of time. That is

$$\mathbf{R}_{ab} = \mathbf{R}_{ab}(t).$$

In many circumstances, particularly in signifi-

cant physical cases, this variable vector will be differentiable. For example, if it is considered that $\mathbf{R}_{ab}$ is the triple of three rectangular coordinates, then in such circumstances these coordinates are also differentiable in the scalar sense. On this assumption we define the vector

$$\mathbf{V}_{ab} = \dot{\mathbf{R}}_{ab} = \frac{d}{dt}\mathbf{R}_{ab} \tag{3}$$

as the vector velocity of the point b with respect to the point a, or in the medium thereby identified. This vector, as a function of time,

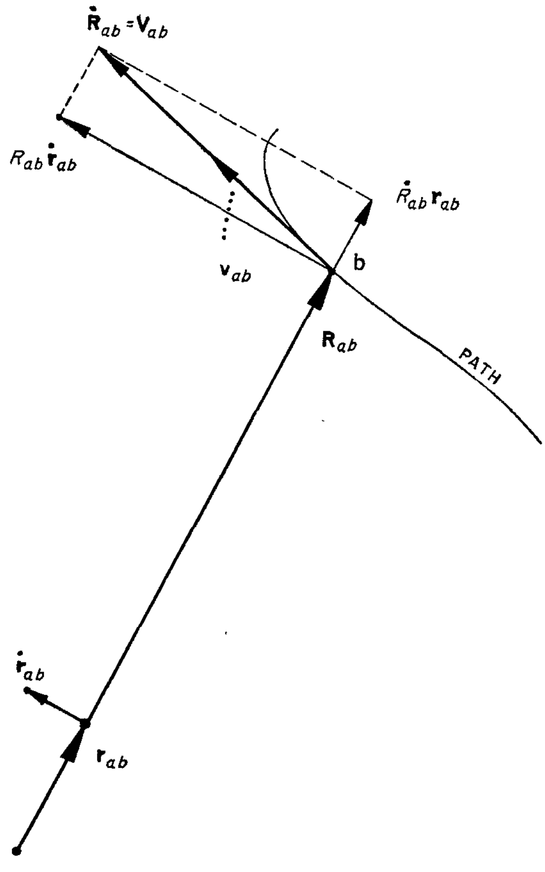


FIGURE 3. The vector derivative.

is parallel to the tangent to the space-path of point b in this medium and in length is proportional to the velocity of b along this path, both being instantaneously evaluated at the time t. The scalar value or magnitude of $\mathbf{V}_{ab}$ is similarly defined as $|\mathbf{V}_{ab}|$ or V_{ab}, where

$$V_{ab}^2 = \mathbf{V}_{ab} \cdot \mathbf{V}_{ab}, \tag{4}$$

and the unit vector, giving purely the direction of motion of the point b in the medium and with respect to a, is symbolized by $\mathbf{v}_{ab}$, where

$$\mathbf{V}_{ab} = V_{ab}\,\mathbf{v}_{ab}. \tag{5}$$

Individual constancy of the various quantities in equation (5) is significant as follows. If the unit vector $\mathbf{v}_{ab}$ is constant, motion of the point b in the medium is rectilinear; if the scalar V_{ab} is constant, then this motion involves an unchanging speed along the path; and finally the constancy of the vector velocity $\mathbf{V}_{ab}$, which implies both the above, means unaccelerated motion of the point b in the medium identified by the point a.

1.3 ANGULAR RATES

Still with regard to the two points a and b, and to the fundamental vector $\mathbf{R}_{ab}$, we may show how rates of rotation enter the scheme as the time derivatives of unit vectors. First differentiate the second expression given above as (2) for the vector $\mathbf{R}_{ab}$, obtaining thus

$$\dot{\mathbf{R}}_{ab} = \mathbf{V}_{ab} = R_{ab}\,\dot{\mathbf{r}}_{ab} + \dot{R}_{ab}\,\mathbf{r}_{ab}. \tag{6}$$

Noting that $\dot{\mathbf{r}}_{ab}$ is in magnitude proportional to the angular rate of point b about point a, and is normal to $\mathbf{R}_{ab}$ and coplanar with $\mathbf{R}_{ab}$ and $\mathbf{V}_{ab}$, and noting also that $\dot{R}_{ab}$ (which is different from V_{ab}) is merely the range rate between a and b, we can identify the terms on the right-hand side of the vector equation (6) as the components of $\mathbf{V}_{ab}$ which are normal to and along the direction $\mathbf{r}_{ab}$.

Pursuing further the derivative $\dot{\mathbf{r}}_{ab}$ of the unit vector as a measure of rotation in space, we may construct an even more articulate such vector by taking the outer or vector product of $\mathbf{r}_{ab}$ into $\dot{\mathbf{r}}_{ab}$, which may be symbolized as $\mathbf{r}_{ab} \times \dot{\mathbf{r}}_{ab}$. Thus we define

$$\Omega_{ab} = \mathbf{r}_{ab} \times \dot{\mathbf{r}}_{ab}, \tag{7}$$

and it will be seen that this vector is a measure of the time rate of angular rotation in space of the point b about a which is normal to the plane of rotation and which is directed in the right-handed, or screw-thread, sense.

When the vector velocity of the point b in the medium is invariant, that is when $\dot{\mathbf{R}}_{ab} =$

[redacted]

$\mathbf{V}_{ab}$ is constant, then the area of the parallelogram formed, in the invariant plane of relative motion, by the vectors $\mathbf{R}_{ab}$ and $\mathbf{V}_{ab}$ is evidently constant. Thus by definition of the vector product we have, in such a case,

$$\mathbf{R}_{ab} \times \mathbf{V}_{ab} = \text{Constant.}$$

But now $\mathbf{R}_{ab} = R_{ab}\,\mathbf{r}_{ab}$, and by performing vector multiplication on both sides of (6) with this latter product we find also, referring to (7),

$$R_{ab}^2\,[\mathbf{r}_{ab} \times \dot{\mathbf{r}}_{ab}] = R_{ab}^2\,\Omega_{ab} = \text{Constant.} \qquad (8)$$

This relation is seen to embody a vectorial expression of the well-known invariant (range-squared into angular rate) for unaccelerated straight line courses.

1.4 SEVERAL POSITION VECTORS

Keeping the initial point a as identifying symbol for the single medium, consider now two points, b and c, having general motion therein. With the same convention for a relative position vector, we can express the position of the point c in the medium as

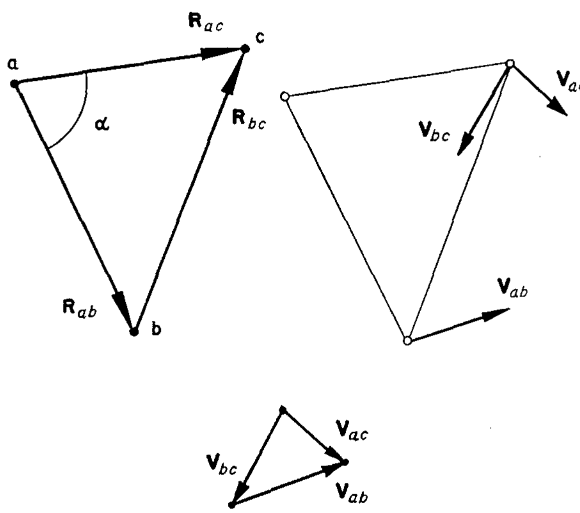


FIGURE 4. Relative position and relative velocity as vectors.

$$\mathbf{R}_{ac} = \mathbf{R}_{ab} + \mathbf{R}_{bc}, \qquad (9)$$

in recognition of the triangle formed in the medium by the three points a, b, and c. The notation employed provides a convenient criterion for cancellation of indices on summing.

Note that scalar multiplication, of each of the two sides of (9) into themselves, yields the familiar law of cosines.

Differentiation of (9), giving

$$\mathbf{V}_{ac} = \mathbf{V}_{ab} + \mathbf{V}_{bc}, \qquad (10)$$

shows that the vector velocities, one relative and two with respect to the medium, add vectorially and form a closed figure as do the position vectors themselves. A similar relation holds no matter how many points are involved and no matter how many derivatives are taken.

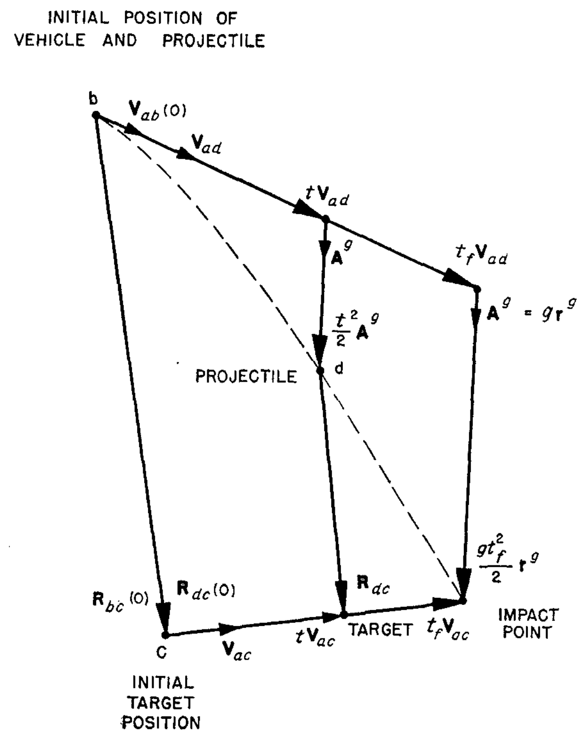


FIGURE 5. Hitting criterion as vector sum.

If point c is moving in an arbitrary manner in the medium, and if the instantaneous motion of point b, with respect to the medium, is directly toward point c, then

$$\mathbf{V}_{ab} = k\mathbf{R}_{bc} \qquad (11)$$

where k is some positive scalar quantity. But since we may write

$$V_{ab}\,\mathbf{v}_{ab} = kR_{bc}\,\mathbf{r}_{bc}$$

it follows that

$$k = R_{bc}^{-1}\,V_{ab}$$

and

$$\mathbf{V}_{ab} = \mathbf{r}_{bc}, \qquad (12)$$

since the scalar and the directional equations

must be satisfied. The latter equation may hold either instantaneously or identically and forms the definition of a *pursuit course* with reference to a path in the medium. Where b is a point in a body, of which some axis may be pointed at c, the pursuit course may be differently defined. We shall prefer here, however, to use the term as characterized by (12). Note that the derived criteria

$$\mathbf{V}_{ab} \times \mathbf{R}_{bc} = 0\,,$$

and

$$\mathbf{V}_{ab} \times \mathbf{r}_{bc} = 0 \tag{13}$$

may describe either a pursuit course, in the above sense, or its negative: the one point moving *away from* the other.

When the two points b and c are moving uniformly in the medium, so that $\mathbf{V}_{ab}$ and $\mathbf{V}_{ac}$ and hence also $\mathbf{V}_{bc}$ are constant, a simple criterion may be exhibited for the coming together of these two points. Such coming together is of course reflected in the vanishing of both the relative position vector $\mathbf{R}_{bc}$ and of its scalar magnitude R_{bc}, and may be a desired event if, say, b is a bullet and c its target. Since now $\mathbf{V}_{bc}$ is a constant, and since consequently

$$\mathbf{R}_{bc}(t) = \mathbf{R}_{bc}(0) + t\,\mathbf{V}_{bc}\,, \tag{14}$$

which must vanish at some value of t, in the future for example, we have, somewhat as before,

$$\mathbf{r}_{bc} + \mathbf{v}_{bc} = 0 \tag{15}$$

as the criterion for the *collision course* in the sense indicated. Since absolute motion has not latterly been employed, in the medium that is, this criterion implies collision even when $\mathbf{V}_{ab}$ and $\mathbf{V}_{ac}$ are variable, provided only that their difference $\mathbf{V}_{bc}$ is constant. The oppositely pointing unit vectors of (15) insure the "closing of the range." A hypothetical collision in the past, with the range opening, is of little comfort in those forms of fire control which require a coincidence; such a "recessive" collision course, however, may be quite useful in some aiming processes to be described.

A type of *generalized* collision course is defined by the simple criterion

$$\dot{\mathbf{r}}_{bc} = 0, \tag{16}$$

which implies equation (15) when the motions are uniform. In general, however, neither condition insures a collision since the range may increase or approach a limit. Equation (16) is the vectorial version of the *constant true bearing* criterion which is familiar in homing operations. Dynamical relationships which subside to this condition, or which are able to approximate it identically, are of extreme interest in such operations.

Of course the necessary and sufficient condition (we repeat) for ultimate coincidence of points b and c is that, at some time, $R_{bc} \to 0$, however this is brought about.

When equation (16) is taken together with the constant-range criterion $\dot{R}_{bc} = 0$, then they together imply $\dot{\mathbf{R}}_{bc} = 0$. Points b and c are then translating similarly in the medium or are "flying formation" in a very ideal sense.

1.5 REMARK ON ANGLE

Although there appear to be several natural forms of notation for angular rates, such as that shown above, which come out of the straightforward vector constructions, there seems to be no supremely convenient notation for the angle between two vectors.

The vector product is proportional to the sine of such an angle, as well as to the product of the magnitudes of the vectors themselves, and shows the sense in which the angle is swept out. If the vectors are first normalized, or reduced to unit vectors, and the vector product then evaluated, the resulting vector shows the sine of the angle, the plane which contains it, and the sense of rotation.

Further as to unit vectors, the difference between two of them is demonstrative of the angle (and its sense) between the two directions thereby indicated. The scalar magnitude of this vector difference is twice the sine of half the angle.

A further measure of the angle, in terms of vector concepts, was considered.[a] Referring to any two vectors, a vectorial measure of the tangent of the angle between them, properly signed, is given by the ratio of their vector product to their scalar product. The resulting

[a] In conversation with A. L. Ruiz.

vector is normal to the plane in which the angle may occur. With a simple notation worked out, this measure of angle should be extremely useful, owing partly to its compactness and to the remarkable usefulness of the tangent in many classes of problems.

1.6 A PROJECTILE

Retaining the terminology of vectors, we shall next discuss a problem in the aiming of a projectile, and while giving the problem a certain generality we shall assume certain ideal circumstances which are not always satisfactorily descriptive of real conditions. Frequently, however, under real circumstances, transformations may be made in a problem so that an equivalent problem in the more idealized state is obtained; and even more commonly the simpler but less realistic study has high educational value, and may provide a starting point from which to expand toward a more complex reality.

Following a presentation of the rationalized problem, with its significance properly qualified as above, we shall discuss several special cases, having fewer space dimensions and involving more specialized physical phenomena, as being introductory to some of the aiming stratagems which are described elsewhere in this report.

In a medium as above, identified by the point a (not shown in Figure 5), let there be a vehicle b, that is to say which is represented by the point b, and an enemy target c. Let it be required to launch a projectile from the vehicle so as to coincide at some time with the target; and let the projectile be identified by point d. Assume first that the target c is moving uniformly in the medium, which may be the air mass. Assume further that the projectile, when launched or released or projected or fired, proceeds initially in the same direction in the medium as the vehicle at that instant is proceeding, except k times as fast. Its initial velocity, with respect to the vehicle, is then $(k-1)$ times the velocity of the latter. Suppose also that, although the medium offers no net resistance to the projectile, gravity acts upon it in a normal manner. With time measured from an origin at the instance of firing, the position of the projectile in the medium is

$$\mathbf{R}_{ad}(t) = \mathbf{R}_{ab}(0) + t\mathbf{V}_{ab}(0) + (k-1)t\mathbf{V}_{ab}(0) + \tfrac{1}{2}t^2\mathbf{A}\,, \quad (17)$$

where the vector $\mathbf{A}^g$ is the acceleration of gravity, acting downward. This vector may be considered invariant, and may further be written $g\mathbf{r}^g$ where g is the scalar acceleration of gravity, and $\mathbf{r}^g$ is a unit vector, downward pointing.

The target position, however, is

$$\mathbf{R}_{ac}(t) = \mathbf{R}_{ac}(0) + t\,\mathbf{V}_{ac}(0), \quad (18)$$

to which, of course, acceleration terms may be added in the more general case of nonuniform motion. Equating (17) and (18) for a hit, and letting the value of t which satisfies the resulting equation be t_f, the time of flight, we have

$$\mathbf{R}_{bc}(0) = -t_f\mathbf{V}_{bc}(0) + (k-1)t_f\mathbf{V}_{ab}(0) + \tfrac{1}{2}gt_f^2\mathbf{r}^g. \quad (19)$$

The relative position vector $\mathbf{R}_{bc}(0)$ is the vectorial present range, while $\mathbf{V}_{bc}(0)$ is the present relative velocity of the target. We note that if these quantities are continuously available, together with the vehicle velocity $\mathbf{V}_{ab}$ and the "down" direction $\mathbf{r}^g$, then the solution to the aiming problem is given completely by equation (19). It is unnecessary to know the scalar t_f since this equation determines $\mathbf{V}_{ab}$, for example, in terms of the other three vectors, and a mechanism could be built to steer the vehicle along the vector $\mathbf{V}_{ab}$ which would make the equation hold, for whatever t_f were thus required. The magnitude of the latter variable would be an outcome, incidentally, of this automatic computation.

Although vectorial mechanism, in three dimensions, would be thoroughly practical for computation, the commoner types of mechanism deal in scalars and in fewer dimensions at a time.

For the moment let us omit the consideration of gravity, and restrict attention to a particularly simple situation in two space dimensions (Figure 6). The relative position vector $\mathbf{R}_{bc}$ becomes a line segment in the plane connecting the vehicle and the target. The velocity vectors of vehicle and target may then be attached at the appropriate ends of this line segment. It

is evident that if the vehicle velocity $\mathbf{V}_{ab}$ lies along the relative position vector $\mathbf{R}_{bc}$, a pursuit course is signified. If, on the other hand, the projections, normal to the position vector $\mathbf{R}_{bc}$ of the two velocities $\mathbf{V}_{ab}$ and $\mathbf{V}_{ac}$ are equal,

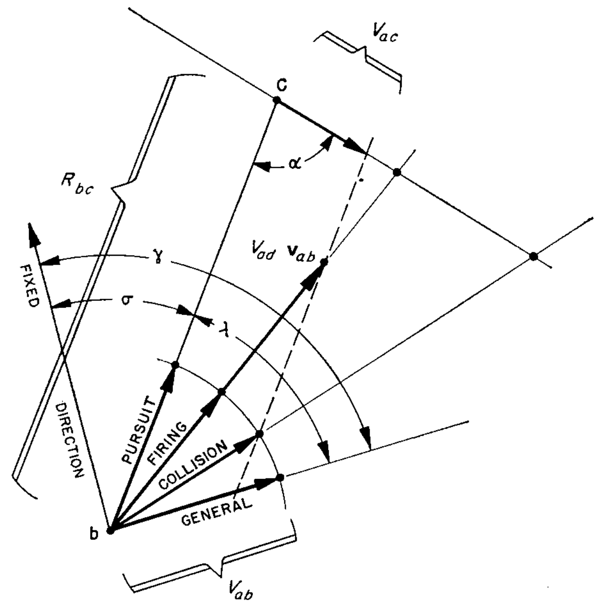


FIGURE 6. The approach in the plane.

and if further these are constant, then we have one condition for a collision course: in the restricted sense. The other condition is a uniform decrease of the scalar range R_{bc}, which criterion may also be expressed in terms of the projections of the two vector velocities along the connecting line segment or vector. Similarly, a collision course for a projectile is describable by replacing the vehicle velocity by its appropriate multiple

$$\mathbf{V}_{ad} = k\mathbf{V}_{ab} = \mathbf{V}_{ab} + (k-1)\mathbf{V}_{ab}\,.$$

Referring to the scalar magnitudes only of the figure last indicated, we may reiterate, as follows, the several remarks already made. If the "lead" angle λ, as shown in this figure, is zero, the pursuit course is in effect. If the lead λ is such that

$$V_{ab} \sin \lambda = V_{ac} \sin \alpha, \tag{20}$$

then a collision course results. Further, if

$$V_{ad} \sin \lambda = (V_{ab} + V_{bd}) \sin \lambda = V_{ac} \sin \alpha\,, \tag{21}$$

a projectile launched under these conditions will hit the target.

A general fact which relates to the circumstances here pictured is contained in the identity (cf. Figure 6)

$$\dot{\sigma} \equiv V_{ac}R_{bc}^{-1} \sin \alpha - V_{ab}R_{bc}^{-1} \sin \lambda\,. \tag{22}$$

Thus by relation (21) it is seen that a criterion for hitting is given by

$$\sin \lambda = R_{bc}V_{bd}^{-1}\,\dot{\sigma}\,, \tag{23}$$

which requires no measurement of the target velocity. It should be noted that all these scalar expressions may be given as concisely, if somewhat less familiarly, in vector notation.

To return to equations (20) and (21), we shall illustrate methods for finding aiming criteria without explicit use either of the target velocity or of the range. We restrict ourselves, naturally, to the assumptions already enforced. Suppose that a collision course be momentarily obtained, for example, through use of the criterion

$$\dot{\sigma} = 0. \tag{24}$$

Under these circumstances, as indicated both by equations (20) and (22), we know that

$$V_{ac} \sin \alpha = V_{ab} \sin \lambda^*\,, \tag{25}$$

where λ^* indicates the (measurable) lead on a collision approach. Thus from equation (21) we find that the angle λ for a hit is given in terms of quantities locally available, by

$$\sin \lambda = (1 + V_{ab}^{-1}\,V_{bd})^{-1} \sin \lambda^*\,. \tag{26}$$

The flight path of the vehicle need only be rotated through the angle $\lambda^* - \lambda$ toward the target, to pass from a collision course to a firing position.

Instead of the initial collision course, suppose that a pursuit course is initially employed. Since then $\lambda = 0$, we have

$$\dot{\sigma}^* = V_{ac}R_{bc}^{-1} \sin \alpha\,, \tag{27}$$

where $\dot{\sigma}^*$ is the angular rate in space of the line to the target during the initial pursuit approach. Then assuming the range does not change substantially during the transition, we alter the course so that, by equations (22), (23), and (27)

$$\dot{\sigma} = \dot{\sigma}^* - V_{ab}V_{bd}^{-1}\,\dot{\sigma}\,,$$

or so that

$$\dot{\sigma} = (1 + V_{ab}V_{bd}^{-1})^{-1}\,\dot{\sigma}^*\,. \tag{28}$$

Again a firing criterion, in terms of an initial

maneuver, is made available without employment either of the target velocity or of the range. The evident similarity in form between equations (26) and (28) is rather remarkable. With regard to the latter criterion it should be noted that the inevitable change in range during the transition may be roughly approximated, and that this must be done if high accuracy is required. In the writer's opinion, both of these fundamental methods for kinematic lead computing are worthy of note, for example when the range becomes difficult to compute, and it seems to him that there may exist practical composite methods which lie somewhat between the two. The principal purpose, however, of their present introduction is illustrative rather than developmental. Along practical lines, the stability of the aiming process, in each case, would require much closer attention than has here been indicated.

1.7 SPACE-TIME GEOMETRY

We wish here to exemplify, in connection with some of the aiming criteria discussed above, a method of visualization, or rather of symbolic representation, which may always be substituted for the vectorial one, and which in many cases, particularly to the geometrically inclined thinker, may be preferable thereto.

Again suppose that we are dealing with two space dimensions, but that in addition time is taken to be represented as a dimension normal to the plane of "space." In the resulting three-dimensional volume then, a point represents an *event*, and the coincidence or intersection of two paths generated by such points corresponds to a simultaneous coincidence of real points. This is in contrast to the properties of vectors in the normal portrayal where time is not a dimension.

In the adjacent figure, the projection of all points on to the space-plane reduces the whole affair to the conceptual scheme which has been employed above. For the rest, the interpretation is remarkably simple, although admittedly this is true, in a conceptual sense, only because we are temporarily restricted to two dimensions of space. It is evident, for example, that unaccelerated motion of a point results in a straight line in the space-time volume shown; a stationary point forming a line parallel to the time axis, and an infinitely fast point lying altogether in one space-plane. It is further seen that the velocity of any point is given by the cotangent of the angle between the space-plane

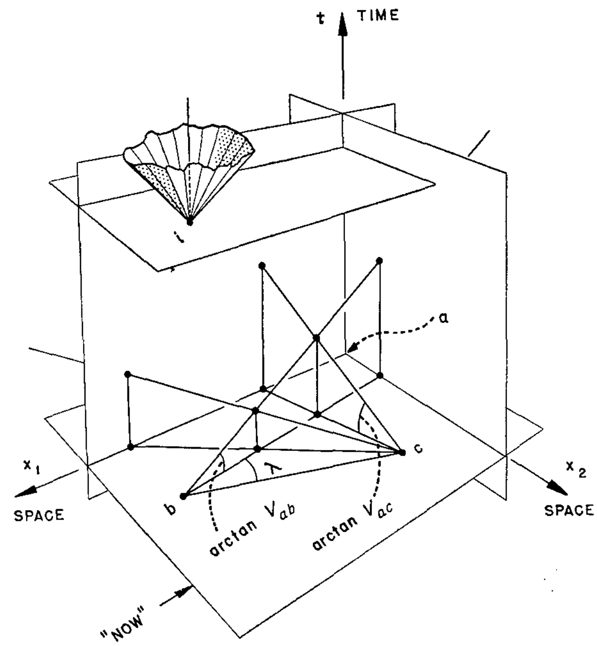


FIGURE 7. Space-time geometry.

and the tangent line drawn at the given point to the three-dimensional path traced out by this point.

Consider two points in motion in a medium which is stationary in the space-plane, the origin of the latter being, for example, the identifying point of the medium in the manner already explained. At a particular time, the positions of these points in the particular space-plane corresponding to this time may be taken as initial positions for the two points. Assuming then that the two points have a given constant velocity and are to be unaccelerated in the future, we see that their possible space-time paths lie along the elements of cones which have axes parallel to the time axis. The tangent of half the vertex angle of each cone corresponds directly to the preassigned velocity of each point. If for one point a given direction in space be chosen, and hence just one element of its cone of motion is singled out, then the one element of the other cone

[redacted]

which intersects that of the first cone is likewise singled out, and consequently the direction of motion for the second point is determined which will result in collision. Such a solution, which may readily be reduced to straightforward mechanical form, will apply whether it is desired that the vehicle itself collide, or that an aimed projectile coincide at some future point with the target.

The generalizations of the above conceptual scheme to include such effects as that of gravity, accelerated motions, and so on, may well be imagined. Mathematically at least, there is no need to perpetuate the restriction to two space dimensions, although indeed only the sections and projections of the resulting hypercones can conveniently be visualized. The present writer has found the space-time mode of thinking to be extremely useful, even when limited to special cases, for the tangible illustration of the more drastically complex situations of typical and practical problems.

1.8 SYNCHRONOUS OPERATIONS

As an example of the useful employment of an operation of the synchronizing variety, so-called, suppose that it is required that the absolute velocity vector of a target be accurately determined from a moving vehicle, and that only the direction and distance to the target, that is to say its relative position, are available to some approximation. Note first that the relative position vector at any time t may be expressed as follows

$$\mathbf{R}_{bc}(t) = \mathbf{R}_{bc}(0) + \int_0^t \mathbf{V}_{bc}(t)dt\,;$$

or further as

$$\mathbf{R}_{bc}(t) = \mathbf{R}_{bc}(0) + \int_0^t [\mathbf{V}_{ac}(t) - \mathbf{V}_{ab}(t)]dt\,. \quad (29)$$

If now $\mathbf{V}_{ab}$ be continuously measured, as a local variable, and if the relative position $\mathbf{R}_{bc}(0)$ of the target be once determined at a given instant, then by generating an artificial version $\overline{\mathbf{V}}_{ac}(t)$ of target velocity, an artificial version of the relative position may be produced as

$$\overline{\mathbf{R}}_{bc}(t) = \mathbf{R}_{bc}(0) + \int_0^t [\overline{\mathbf{V}}_{ac}(t) - \mathbf{V}_{ab}(t)]dt\,. \quad (30)$$

If adjustment of $\overline{\mathbf{V}}_{ac}(t)$ is made in continuous dependence on the observed vector error $\overline{\mathbf{R}}_{bc}(t) - \mathbf{R}_{bc}(t)$ and is made so as to reduce the latter stably and accurately to small magnitude, and if such stability and accuracy is continuously maintained, the artificially generated $\overline{\mathbf{V}}_{ac}(t)$ can be a faithful version of the true target velocity $\mathbf{V}_{ac}(t)$. Errors in the initial target position $\mathbf{R}_{bc}(0)$ are integrated out in this process, but errors in the determination of $\mathbf{V}_{ab}(t)$ contribute to errors in the target speed in direct proportion to the numerical speed advantage.

In tracing out the causal sequence involved here, we note that the operator (whether human or automatic) observes the difference between the generated vector $\overline{\mathbf{R}}_{bc}$ and the vector $\mathbf{R}_{bc}$ describing the relative target position, and on the basis of this difference manipulates the artificial target-speed vector $\overline{\mathbf{V}}_{ac}$. Through the agency of apparatus which mechanizes equation (30), this artificial target speed influences the generated position vector $\overline{\mathbf{R}}_{bc}$, which in turn affects the observed difference, and so on in continuous fashion around the loop. If the operator manipulates the vector $\mathbf{R}_{bc}(0)$ as well as $\overline{\mathbf{V}}_{ac}$, this may result in a closer following of $\mathbf{R}_{bc}$ by $\overline{\mathbf{R}}_{bc}$, or in "better tracking," but such manipulation if irregularly and continually employed results in serious errors in the approximation of $\overline{\mathbf{V}}_{ac}$ to $\mathbf{V}_{ac}$. Such combined operation is a vectorial generalization of "aided tracking." If, however, the two adjustments are appropriately interconnected, easier and better tracking as well as faster subsidence to an accurate version of the target speed will result. Thus notice that for a given error in the latter, in $\overline{\mathbf{V}}_{ac} - \mathbf{V}_{ac}$ that is, the requisite adjustment of $\mathbf{R}_{bc}(0)$ to make $\overline{\mathbf{R}}_{bc}$ momentarily equal to $\mathbf{R}_{bc}$ increases in proportion to the time. Hence if each adjustment in $\mathbf{R}_{bc}(0)$ is accompanied by a simultaneous and proportionate change in $\overline{\mathbf{V}}_{ac}$, where this change is also proportional inversely to the time intervening since the last previous adjustment, then subsequent readjustment need only deal with errors of higher order.

We note that this whole process can be generalized as far as may be desired. In equation (30), $\overline{\mathbf{V}}_{ac}$ may be replaced by

$$\overline{\mathbf{V}}_{ac}(0) + \int_0^t \overline{\mathbf{A}}_{ac}(t)dt\,,$$

[redacted]

where $\overline{\mathbf{A}}_{ac}$ is the artificial target acceleration vector (in space). Three adjustables now replace the above two, and these may be similarly interconnected, with the intercoupling between $\overline{\mathbf{A}}_{ac}$ and $\overline{\mathbf{V}}_{ac}(0)$ resembling that between $\overline{\mathbf{V}}_{ac}$ and $\mathbf{R}_{bc}(0)$. This procedure may improve the tracking and the representation of target speed, and may furthermore provide a "synchronous" measure of target acceleration for higher order solution to fire-control problems.

1.9 MORE ON THE APPROACH

We have discussed above, both in vectorial and in scalar form, the properties and criteria pertaining to pursuit and collision courses for the approach by a vehicle to a target. Among the infinite variety of such modes of approach, of which these are but special cases, we shall outline one here which may begin as a course of pursuit and then subside with arbitrary rapidity toward a course of collision. In this process we shall show means whereby such an approach may be controlled.

Suppose initially, in the plane of Figure 6, for example, that the vehicle b is in direct pursuit of the target c, or that

$$\lambda(0) = 0. \tag{31}$$

We propose subsequently to change the direction of motion of the vehicle, measured by the angular sum $\sigma + \lambda$, at a rate proportional to that at which the direction to the target, measured by the angle σ itself, is changing. We propose further to make the changes in the direction of motion of the vehicle occur k times as fast as those in the direction of motion of the target, or to make

$$\lambda + \sigma - \sigma(0) = k[\sigma - \sigma(0)].$$

The derivative of this criterion provides a more succinct description:

$$\dot{\lambda} = (k - 1)\,\dot{\sigma}. \tag{32}$$

Thus a paraphrased criterion is to make the rate of change of the lead angle $k-1$ times as large as the rate of change of the absolute direction of the target. This mode of approach, for which it is evident that several other symbolic descriptions may be given, is sometimes referred to as *proportional navigation.* The technique will reappear at several points within the present report.

In order simply to illustrate the consequences which follow the imposition for this criterion of approach, the following assumptions shall be made: We assume the scalar speeds of vehicle and target, V_{ab} and V_{ac}, are constant, and further that the target has motion only normal to the line connecting it with the vehicle. We assume also that the lead angle λ is sufficiently small that the angle is an adequate approximation for its sine, and so that the rate of change of the range is contributed to only by the vehicle speed.

It is evident, under the above assumptions, and by reference to Figure 6, that for a collision course

$$\lambda = \overline{\lambda} = V_{ab}^{-1} V_{ac}, \tag{33}$$

where the new symbol $\overline{\lambda}$ identifies the lead for a collision course. Equation (22), under the same assumptions, becomes

$$R_{bc}\,\dot{\sigma} = V_{ac} - V_{ab}\lambda \tag{34}$$

while also

$$\dot{R}_{bc} = -V_{ab}$$

Now combining (32), (33), and (34) we find

$$\frac{d\lambda}{\overline{\lambda} - \lambda} + (k - 1)\,\frac{dR_{bc}}{R_{bc}} = 0; \tag{35}$$

and upon integration

$$\lambda(t) - \overline{\lambda} = \left(\frac{R_{bc}\,(t)}{R_{bc}\,(0)}\right)^{k-1}: \tag{36}$$

which means that the lead λ approaches the value $\overline{\lambda}$ for a collison course in proportion to the $(k - 1)$ power of the fractional closure of the range. It is evident that for large values of the ratio k the collision condition is attained very rapidly. For $k = 2$, notice that $\dot{\lambda}$, $\dot{\sigma}$, and consequently $\dot{\lambda} + \sigma$ are approximately constant. This implies a constant rate of turn of the vehicle and corresponds to the so-called *circular-interception approach.*

We repeat that the above analysis is approximate, but within reasonable limits it shows substantially the same phenomena as does the considerably more difficult exact treat-

ment (for example in three space dimensions). A number of generalized applications will be found in this report.

1.10 FEEDBACK IN GENERAL

Operations which form, in the causal sense, a closed sequence or chain, and which thus occur in a completed ring or loop, form a special branch of dynamics of great importance in many aiming controls, as well as in controlling and regulatory devices generally. Such causal loops may be entirely automatic in nature, or may contain one or more human elements as an essential connecting link. A variety of terms, such as feedback, retroaction, regeneration, and degeneration, have come to be applied in the identification of systems which are so constructed; a servomechanism is basically of this character.

The most important property of such a physical arrangement is the stability with which it operates. In cases where the components and the interconnections involved are linear, that is to say where the principles of additivity or superposition for cause and effect apply, the criteria and the instrumental stratagems for stability are now quite well understood, and a good body of literature has accumulated on this subject. The power of feedback processes, thus restricted, is considerable. They are applied for a variety of computational purposes, as where a characteristic is to be reciprocated or inverted, and, for example, in certain complicated problems of smoothing and prediction.

On the other hand when nonlinear components are involved, and where consequently the additivity principle is violated, as in general with a human operator forming a connecting link in the loop, a considerably more recondite situation is attained. Beyond the inferences which are available from linear approximations, and which must be very carefully drawn, the major recourse for such questions must be to experimental methods and to model studies. We refer for example to the methods of simulation treated elsewhere.

For purposes of broad reference, and to set down concretely a rather purified example of the feedback phenomenon, we give here briefly what may be considered the simplest and hence the most fundamental system in which feedback is involved. There is intended to be nothing new in this.

In the accompanying figure, the operator Φ, being the characteristic in the "box," is of arbitrary nature and is merely assumed to connect dynamically the incoming and outgoing variables. Thus the operator Φ is a physically realizable functional which determines the local output as a function of the time t when the local input is predicated as a function of time. The input variable $q = q(t)$ is arbitrary. The output of the box is the variable r, which is also the "response" to the operation Φ when performed on the input to the box. Further, the variable u is the "unbalance" or difference between q and r. Thus

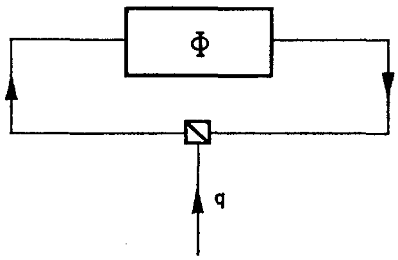


FIGURE 8. Elementary feedback loop or following system.

$$u = q - r, \tag{37}$$

$$r = \Phi \cdot u. \tag{38}$$

Consequently

$$u = q - \Phi \cdot u, \tag{39}$$

and

$$r = \Phi \cdot q - \Phi \cdot r. \tag{40}$$

When the operator Φ is linear, expressible, for example, as a rational function of the derivative operator p then under appropriate restrictions we may write

$$u = \frac{1}{1 + \Phi(p)} \cdot q, \tag{41}$$

and

$$r = \frac{\Phi(p)}{1 + \Phi(p)} \cdot q. \tag{42}$$

In equations (41) and (42) appear the familiar operators of linear feedback and servo theory. The roots of the rationalized denomina-

tors of these operators are significant to stability, while the merits of "performance" are obtainable in terms of the results of the entire operation when $q(t)$ is specified. Stability may also be studied, still under the linear restriction, through the exploration of $\Phi(p)$ itself, either experimentally, or analytically in the complex plane by the methods of Nyquist, Black, et al. If linearity is not clearly indicated, the expressions (39) and (40) are to be preferred.

A meaningful interpretation of the closed causal system of Figure 8 may be given as follows. Suppose it is required to duplicate q, which represents, for example, a variable weight in the pan of a balance. If then the response r is proposed as such a "duplicating" variable, the unbalance u indicates continually its failure in this regard. The operator Φ, which may now be considered a "follower," adjusts r through interpretation of the unbalance u. Thus the response r is a measure, better or worse, of the arbitrarily variable weight q. In more general cases the follower may have to operate through an additional chain of components, and q may contain irrelevant signals, and so on. An example of such a "feedback" following operation was given in Section 1.8. We remark that all cases of feedback, by proper identification of the input quantity q and of the following operator Φ, reduce to this simple example.

1.11 INSTRUMENTAL CONSIDERATIONS

We wish to include, while on the theoretical plane of the present chapter, some remarks on the general instrumental character of a fire-control system such as has been developed in some quantity for airborne application.

It should be pointed out immediately that such a system is typically only part of a more inclusive system which may encompass also the vehicle in which the apparatus is borne and the man or men who manipulate both.

A fire-control system, or a connected group of instruments forming means for the control of aim in an airplane, may be divided into three parts. One subdivision will be constituted of apparatus which feeds in all primary data. Such data include continuously measured variables, which may be supplied either directly or through power-boosting servos. Feedback systems may be involved in the measurement of the input variables, where, for example, null methods are essential to attain a dynamically faithful replica of the physical quantity which is sought. Other input data may be supplied intermittently or periodically, and these data may arise either automatically or by the manual settings of a principal operative or by those of one of his confederates. The precision with which the primary inputs must be supplied, for a given aiming accuracy, is a prime property of the system as a whole, or rather of the principle on which it functions.

Another subdivision of the typical fire control is that which performs the computation which is required, and in which the various input data are correlated, collated, operated upon, and interpreted with regard to their significance to the problem at hand. The components of this subdivision must constitute a mechanization, or physical embodiment, of the equation or equations which describe the method employed for the control of aiming. The computation involved here may be algebraic, geometric, or may require equipment versed in the calculus. In general the mathematical operations performed are nonlinear, although in a good many cases linear approximations suffice and are employed. Errors in computation arise from a number of sources, and their allowable limits is one of the important specifications for this subdivision of the system. Such variables as temperature and acceleration may be inadvertently "measured" by the computer, and it is important to predict such effects and thoroughly to test their final magnitudes. The outputs of the computing subdivision are principally the presentation variables, which are handed on to the next subdivision. We note that some of these variables may return to the initial subdivision and may in turn become input data for the computing subdivision itself: this occurs where it is to make use of a feedback stratagem in the realization of certain dynamic characteristics, or again simply where servo methods are required for the effective delivery of a significant variable for the presentation

subdivision. Further outputs, or results of the computation, may be locally displayed at the computer for auxiliary purposes such as checking and calibrating, or for warning indicators of various sorts. These, however, in a liberal sense, may also be thought of as presentations.

The final subdivision, for presenting the appropriate variables to the one or more operators, accepts such variables from the computer

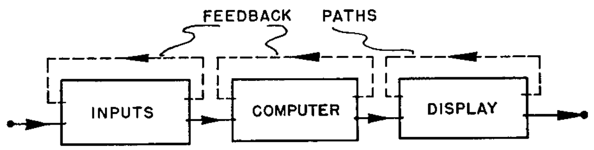


FIGURE 9. Major subdivisions of fire-control system.

and converts them into a manifestation which is perceptible to those individuals or mechanisms. This equipment may consist in a simple reflecting sight, an oscilloscope screen, or may involve complex follow-up or feedback components whereby null methods are applied to put into effect the final consummation intended. In many cases an alignment is to be carried out between a moving index and a target (or alternatively this operation be involved in the primary data subdivision). All variety of combinations are possible and have been in evidence. In manipulating the vehicle to obtain a match between his optical presentation and the target, the pilot of a fighter supplied with an aiming-control system will produce angular rates which are introduced into the computer and affect the presentation, and hence the pilot's manipulations, and so on around the major causal loop which is then involved. The stability of tracking which results, and which in turn may affect the accuracy of fire, is of primary concern; and a central problem in the art is that of so constructing the dynamics of the aiming-control system that this process will be rapid, stable, and precise.

To return to the initial subdivision, which is responsible for supplying the data to the computer, we should point out that certain remote controls are there to be included which might not normally be considered as related to the input variables as such. Examples are: adjustments of the computer which depend on the type of airplane involved; settings made by the pilot in indication of his choice of projectile; and the firing operation itself, which may require resetting of the computing components for the next attack.

Note further that although the excellence of a control or of a computing system depends pivotally (aside from considerations of size, weight, permanence, and so on) on the partial derivatives of the firing errors committed, taken with respect to the errors in the input data, it depends as well on the inherent accuracy with which such input data may be supplied. Thus if range, for example, may be measured with great precision, relatively speaking, it is less important that a given system or principle of operation be insensitive to this particular input variable. In such considerations, statistical methods are supremely useful, although great attention is warranted to the assumptions made in any given case; to pass beyond the region of validity of these assumptions deserves and should receive severe criticism.

A few remarks are here warranted on some of the commoner variables which are continuously measured in an airplane for the purpose of aiming controls. Briefly these comprise: distances, velocities, and accelerations; and angles, angular rates, and angular acceleration. The passage of time itself should be added. Such quantities may be measured directly, or obtained by inference from related quantities. For example a velocity may be measured as the integral of the corresponding acceleration. We mention as follows, however, only the primary measurables themselves. Distances are frequently obtainable, to appropriate objects, by radar-ranging methods. Altitude as ground clearance is included, although the barometric pressure may contribute with other measurements to a knowledge of the altitude over an appropriately defined point. The so-called indicated airspeed may be measured in terms of the difference between dynamic and static pressure, and may be converted to true airspeed through auxiliary measurement and computation. Accelerometers, in a variety of forms and excellences, will provide measurements of the acceleration of the airplane in a coordinate system fixed therein or, by appro-

priate stabilization (which may be considered to involve measurements of angle), with respect to an unaccelerated coordinate system. Angle is usually measured with respect to fixed directions, these being manifested by free — or approximately free — gyros. Absolute directions are provided by North and Down, and these represent long-term standards. Angular rates, of the vehicle or of independent platforms therein, may be given extremely quickly and accurately by captured gyros, which are discussed elsewhere (particularly in Chapter 3). Finally, angular accelerations are provided through the measurement of torques on a body possessing rotational inertia.

We have referred above, and purposely so, to a relatively complete fire-control system. The opposite extreme is represented by the warrior who fires "by eye," and who by dint of training and experience has developed aiming controls in his judgment and in his reflexes. This procedure, which can lead to remarkable precision, has attained however an unfortunate aspect of glamour, and has very definite limitations for modern operations. Most aiming controls extant, and these are far from perfect, lie somewhere between these two extremes. Thus, there are numerous cases where automatic apparatus combines with visual estimates, as of the speed of a warship, the duration of a brief time interval, or the angular depression of a target below the horizon.

Chapter 2

ON CERTAIN ASPECTS OF TRACKING

2.1 TRACKING AND OUR CONCERN THEREWITH

By "tracking" is here meant a continuous following or alignment procedure which is pursued through cognizance of its error or unbalance, as in the basic feedback or following operation described in Section 1.10. A good example, although somewhat abstractly disclosed, is given in Section 1.8, where an arbitrarily varying position vector is to be followed by an artificially generated position vector which is influenced by manipulation of an integration constant and a velocity.

Tracking pervades all branches of fire control, but it arises principally in aiming operations as such, whether these be manual or automatic. With regard to the latter, automatic tracking is essentially indistinguishable from other types of automatic control and will here be considered in that light. However, our major interest in the present chapter is in tracking by manual means, a human operator being involved. While the removal of human limitations gives the greater promise to automatic tracking devices and techniques, still there is a good bit to be learned, even for such unlimited future trends, from tracking of the human variety. Besides this, the human element can hardly be altogether eliminated, even ultimately, and until we give to our machines the final powers of judgment and choice we shall find it necessary personally to direct their efforts. It may also be doubted that the adaptability or educability of the brain, whereby it adjusts to altered circumstances, can soon be imparted to the automaton in any but the most trivial of cases. Or rather, if this is to be done, it will probably be through imitation of the processes by which such adaptations take place in the animal mechanism.

It is typical of the writer that he considers the entire subject of tracking to come within the broader boundaries of regulatory controls, which to him include all corrective apparatus having a closed causal loop. In this connection, and although the region has not been very far explored, there will be traced out briefly in what follows an analogy between the past development of automatic regulators and that of tracking aids. From this analogy a deduction will be made as to the future of the latter field of endeavor. Aside from this conjectural activity, we have been intimately concerned with tracking in a number of development programs where fire-control apparatus was being devised. Tracking in range, and we speak now of manual tracking, whether by stadiametric means or otherwise, has not entered particularly into our immediate sphere, and it is further evident that this brand of tracking has been dealt with very thoroughly by other groups. Thus it may be said that tracking in *angle,* or more precisely in *direction,* is under discussion. The problems and technology of such tracking have appeared in gunnery systems, and in those for bombing, guided bombing, and for airborne rockets.

For flexible gunnery, an elaborate instrumental, statistical, and psychological study of the man-machine interactions which tracking comprises was undertaken and pursued by Section 7.2 at The Franklin Institute. This project (NO-268) dealt with standard types of computing dynamics and with known kinds of aiding controls. Many significant results were obtained; some unexpected. Other members of the section have been more directly involved, however, and their writings should be referred to rather than the present material. (See Part III, for example.) While allusions may be made here to this work, it will not be further discussed or described.

2.2 THE HUMAN FOLLOWER

People "track" during every conscious moment, unless their eyes be closed, their hands

tightly tied, and their tongues clamped solidly between their teeth. Alignment processes, in which the alignment error serves as datum for its own annihilation, are forever being carried out in the familiar operations of living. We may thus expect that there is a rudimentary sort of tracking circumstance in which the human operator will be at home, and in which he will be found to be greatly and innately skilled. Pointing at a moving object with a pencil, or with a rifle under beneficial conditions of support and inertia, is a tracking operation which may be carried out relatively well. Provision of a reflecting sight, or of a nonmagnifying telescope with cross hairs, does not improve the operation extensively and has even been seen to impair it. Of course it is a different matter with magnification, since then visibility and visual resolution may be vastly increased. This is not to say that this "natural" type of tracking cannot be improved upon, for in tracking targets which are changing very slowly and uniformly an adjustable-rate device is superior, this being only one example, but on the whole for typical cases, and where scale factors in the presentation are reasonably adjusted, such natural tracking would suffice admirably. It is assumed here that closeness of tracking as such is desired, or simply smallness of tracking error. It is evident that for the larger purposes of aiming contol this is not the only index of excellence. The needs and nature of the interpretative and computing equipment cannot finally be separated from those of the tracking controls. We are thus discussing a subsidiary problem, but more on this again.

We repeat, those tracking arrangements would suffice in which the dynamic relationship between the immediate manual manipulation and the direction index were of the same character as in the arrangements referred to as natural. If this is an overstatement of the case, then we should like to assume that there might be discovered some even more ideal dynamic connection between the manual manipulation and, say, the visual index, and that the dynamic nature hereof might be expressible in quantitative form, even though the ideal differed from one individual to the next.[2,4]

2.3 THE CAUSAL LOOP

It is frequently pointed out that the elements in a tracking system form a chain, and that consequently each element must perform as perfectly as the whole assemblage is intended to perform. But it has not been sufficiently reiterated that these elements form a *closed* chain, or a *complete loop,* together with the human operator, where one is present. This fact brings to the tracking process all the special characteristics, including the unique conditions for stability and periodicity, which are peculiar to that type of system. The formation of such a loop, in the causal sense, is particularly significant when there is included the dynamic characteristic of a "disturbed" or lead-computing sight, so-called. For in spite of the fact that the little understood dynamics of the human operator are included, many of the properties of those loops which are entirely automatic, and which are familiar in automatic regulatory devices and in many automatic controls, are seen to be present in the tracking sequence. It seems evident that an approach to the tracking problem, thinking of this problem as requiring the provision of improved aiding equipment, would best be made a basis of the recognition that the operations involved are of this cyclic character. Operations in a closed loop are fundamentally different from those in an "open" or "straight-through" system. Both advantages and disadvantages follow from such an arrangement, and this will define our principal topic. We wish first to map out the causal circuit involved in a tracking system in operation.

Suppose a directional index is to be made to coincide, as nearly as possible, with the direction of a target which has motion only partly predictable. We shall assume at first that such motion may also be contributed to by motion of a vehicle from which the tracking is taking place; that is, tracking is to be in vehicle coordinates. At any rate there is a dynamic connection to the directional index (or *sight index*) from some sort of handle or control which is under direct manipulation. If a coordinate μ is assigned to the displacement of the handle, and $\bar{\sigma}$ (for reasons later to become evident) to that of the sight index, then the tracking dy-

namics are expressible as a relationship between these two variables coordinates, or as

$$\bar{\sigma} = T \cdot \mu, \tag{1}$$

where T is what may be called the tracking operator. Note that in general T may be a nonlinear operator, whereas in the analytically manageable cases it is linear; at worst, linear approximations serve admirably as rationalized ideals. When thus rationalized, $T = T(p)$.

We should state that the coordinates such as μ and $\bar{\sigma}$, which are used here, may be considered as multiple-valued quantities, as vectors that is, or again simply as one of the components of the problem, in circumstances in which such a one is representative and in which a significant and symmetrical such separation into components is possible.

Now the sight index $\bar{\sigma}$ is to be compared with the true sight direction or direction to the target, which may be given the symbol σ. Thus the difference between these two, or the error, which is visible to the human operator, is an important variable. We define the error ϵ as

$$\epsilon = \sigma - \bar{\sigma}. \tag{2}$$

While under ideal tracking this difference will remain identically zero, still in real operation its value must be continually observed and interpreted to approximate this ideal.

Perception, we assume by visual means, of the tracking error ϵ, and manipulation of the handle μ on the basis of such perception, is the office of the human operator. Symbolically,

$$\mu = H \cdot \epsilon$$

where the operational symbol H presumes to embody what takes place functionally between the eye and the hand. This operator must thus include not only the delicate reflexes and inhibitions of the human nervous system, sensory, motor, and as much of the central system as may be involved, but also the random excitations and "nervousness" which are characteristic of the organism. It must embody further, for complete representation, the ability to learn,[7,8] which implies a fundamental nonlinearity.

The causal loop should now be quite distinct, from eye to hand to sight index and around again, continually and cyclically. Figure 1 shows this circuit in symbolic form. It will readily be seen, by comparison with Figure 8, that the operators H and T, in series, correspond directly to the follower-operator Φ of the latter figure. Before passing on to a more detailed consideration of the components which may, in relevant cases, be involved in the causal loop of tracking, a few words should be spent toward clarifying what is to be gained by recognizing the existence of the loop as an essential feature of the whole phenomenon under discussion. In dealing perpetually with equipment which operates in this sort of closed

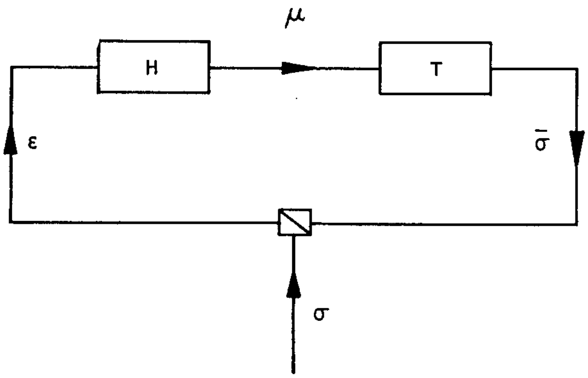


FIGURE 1. Causal loop for tracking.

causal connection, as one does with servomechanisms and with regulatory mechanisms generally, a number of special characteristics come to be familiar which are peculiar thereto, and which are not typical of all arrangements. One such characteristic, having to do with the decay of transients, is that such decay occurs quite uniformly among all the variables which are directly included in the loop. This is evident either by analytic considerations or from observation of the operation which is typical of such systems in practice. That is, if the parameters of the various components of the loop are so adjustable in their mutual relationship that any one of the variables of the system subsides stably and rapidly to an equilibrium condition following the imposition of given initial conditions, or following a transient disturbance of any type, then all of the variables directly included around the loop will subside stably and rapidly for the same adjustments. This is not true for open chains of components, for example, and is but one

of the properties unique to causally closed arrangements. One consequence of this property which may be worth citing is as follows. While the precision itself of tracking, in a director system where the sight index is not derived from the final aiming operation, may not assure accuracy in the latter operation owing to the response of an intermediate computer to other characteristics of the tracking, a very different circumstance holds true for the disturbed-sight type of equipment. Here the closed system, as explained above, keeps the dynamic performance of gun and sight index, as variables of the loop, much closer together, and an adjustment which makes for better tracking more nearly succeeds also in giving a corresponding improvement in aiming than is the case with a director. The present argument is admittedly qualitative, but the facts are borne out by the quantitative data which have been obtained. We are indicating only a mode of thinking; it remains to be put to more articulate use. It is recognized further that these arguments may by now have become trite in fields of which the writer has little knowledge. His own interpretations of need are based, naturally, on the work he has seen in progress.

2.4 FURTHER BREAKDOWN OF THE LOOP

Speaking particularly of the lead-computing or disturbed sight, so-called, we wish to illustrate, through elaboration of the operational tracking circuit of Figure 1, the principal difference between the uses of such computing components and of those which occur in director systems. Thus in Figure 2 the two general classes of systems are given, in the one case the tracking function T of Figure 1 breaking into the series components A and S, and in the other the sight index $\bar{\sigma}$ being accepted by an independent channel. In each case the symbol γ stands for the coordinate of the gun, say, or for that of whatever direct aiming agent governs the initial direction of the projectile. The component C, in the lower portion of Figure 2, besides computing the "kinematic lead" for the gun on the basis of the dynamic behavior of $\bar{\sigma}$ and of the range to the target, applies whatever additional ballistic corrections are essential, including parallax, for example. Owing to the rich literature which is available for director systems, beyond anything we can include here, and to the rather different nature of the smoothing problem (in which transients

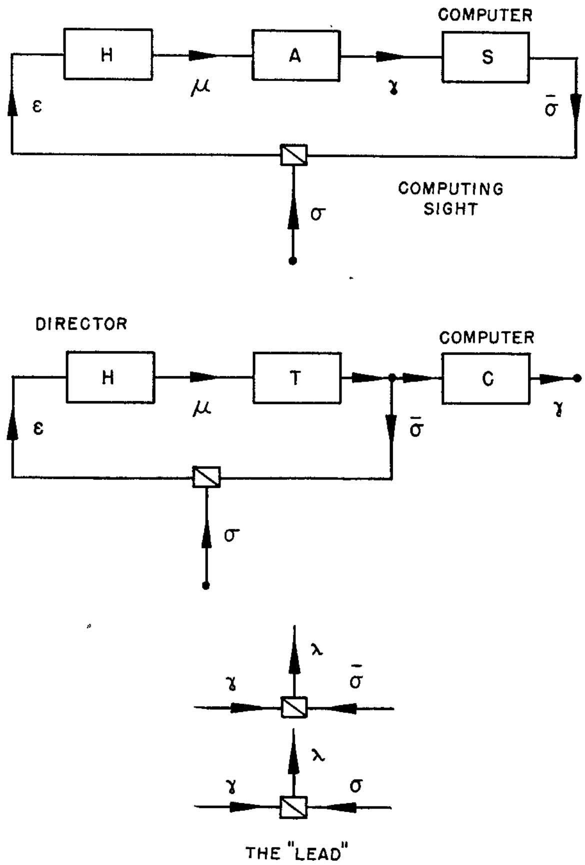


FIGURE 2. Positions of computer with respect to tracking loop.

in $\bar{\sigma}$ are to be kept from harmfully affecting γ), we shall deal henceforth with the system in the upper portion of Figure 2. Further references, however, may be made to director systems in the present report, but these will be conjectural only and will show possibilities for future development rather than being descriptive of work we have followed.

Speaking basically, it is as legitimate to achieve a given dynamic relation between the gun coordinate and that of the line of sight to the target by means of the upper system of Figure 2 as by the lower one, the most manifest difference being the inclusion of the dynamic gun-to-sight computing function, in the former case, as a component of the tracking

loop. Whereas, in that case, the power requirements and the inertias of the gun itself may restrict the mobility of the tracking loop, it is evidently possible to replace the variable γ in this loop by a lightweight and lower-power mock gun or index $\bar{\gamma}$, and then separately to reproduce $\bar{\gamma}$, in the real gun coordinate γ, as effectively as the external high-power controls will permit. As in the director, such corrections as trail and parallax may be additively included in this final transmission, which is not then part of the loop.

We now consider only the upper system in Figure 2, where the variable γ may be manipulated without the restrictions which a heavy turret (or an airplane) would impose. What is the best characteristic for the "aiding" operation A? First assume that there is a satisfactory form for the tracking function T, and that this form T' is linear, so that $T' = T'(p)$. This is either the "natural" form spoken of above or some "idealized" version thereof. Under the relevant exterior circumstances, suppose that this form $T'(p)$ of the tracking function or operation will produce a very close alignment of $\bar{\sigma}$ with σ. It is then evident that if the computing function S is such as to give rapid and accurate performance when $\bar{\sigma} = \sigma$ (nearly), that it is only necessary to make $A(p)S(p) = T'(p)$, where it is assumed that A and S are linear, although their parameters may vary with the time. Thus an ideal, or nearly ideal, form for the aiding function is given as $T'(p)S^{-1}(p)$. Since T' is certainly not critical, one would suspect that this new value for A need not be critical. Reasonable approximations should suffice.

2.5 AUTOMATIC REGULATORS – AN ANALOGY

There is a remarkable parallelism, historically, between the development of tracking aids and the progressive steps which have been made in the types of automatic regulatory equipment for commercial processes. This is not altogether surprising, since both involve the gradual perfection of components which are added to apparatus operating in a (causally) closed loop, and since both have as purpose to improve the stability and performance, in a special task, of the systems to which these components are added. In discussing tracking aids we are referring for example to the component A in Figure 2.

It is not a far cry from the tracking loops shown above to the regulatory loop of Figure

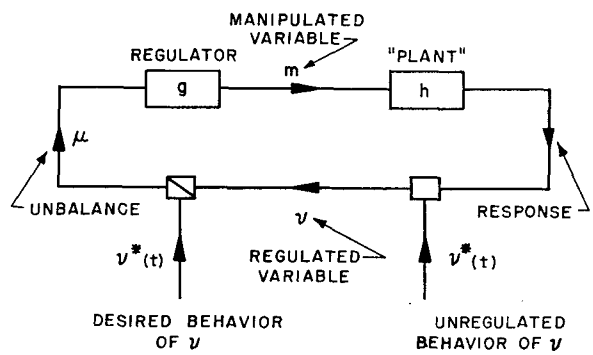


FIGURE 3. Symbolic representations for regulatory control loop.

3, where the components are given in symbolism recently recommended to the ASME by a Committee on Symbolism for Industrial Regulators and Controls. Here the *regulator g* may be considered to correspond approximately to the A component of Figure 2, whereas the *plant h* may correspond to the components H and S, in series, of the latter figure. Both H and S, in typical cases, involve *lags* or characteristics responses which fall off in amplitude at the higher frequencies. The typical plant also involves lag, which generally makes for difficulty in regulation, and requires advanced forms for the dynamics of the regulator. But we proceed with our analogy.

The dynamic characteristics of velocity tracking, displacement tracking, and aided tracking, as these names have come to be used, may be illustrated by the responses, for example, in the gun coordinate γ when a step input is applied to the handle μ. Thus, in Figure 4, the responses shown by curves a, b, and c as functions of time are characteristic of the classical tracking dynamics in the order named above. Now these same responses happen also to characterize the dynamics, such as would occur between the measured unbalance and the manipulated variable of Figure 3, of the classical regulator types as they occurred in his-

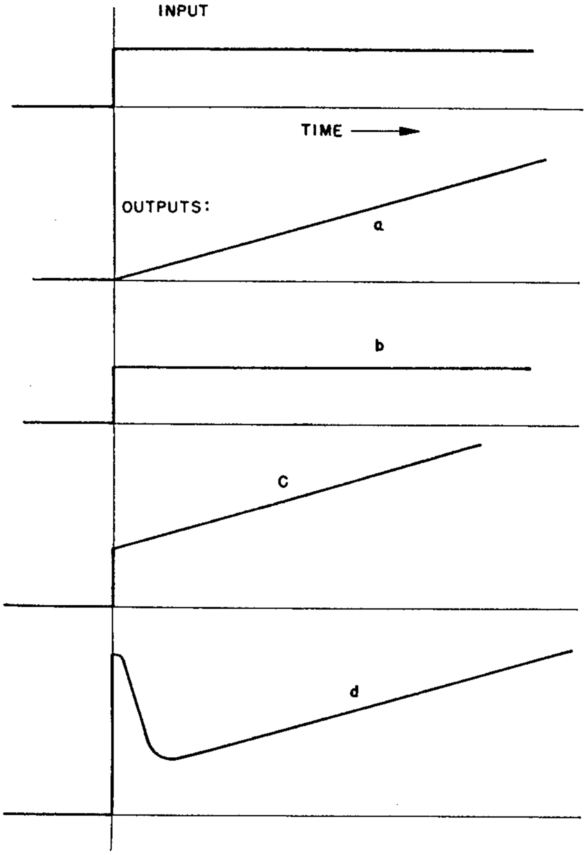


FIGURE 4. Tracking and regulatory responses.

torical order. These are called, respectively, the floating (or integrating) regulator (or controller), the proportional regulator, and the proportional-plus-floating regulator. The latter, like aided tracking, proved to constitute a big advance when it became generally available. It is now interesting to conjecture whether a more universally potent regulator characteristic, such as the newer one which leads to the response d of Figure 4, would correspond to an improved tracking function. To the writer it seems that the evidence of the analogy points unmistakably in that direction.

A word more: In the cases both of tracking and of the regulator problem, one deals not with systems in which the remainder of the loop is of well-known and identifiable dynamical character, but with systems in which that portion of the loop is rather vague and changeable. Thus the human operator is hard to describe, not only by virtue of the intricate nature of its response at any given time, but also because this nature changes in time in dependence on conditioning and on fortuitous causes. So in the regulatory case, the regulator must cope with an ill-defined mechanism, and must be flexible for initial adjustment and noncritical for continued effectiveness. It is tempting to suspect that both fields are now only in rudimentary stages of development.

2.6 CHARACTERISTICS OF HIGHER ORDER

We have a circumstance in tracking in which it is likely that more elaborate dynamics will lead to greater effectiveness when the human operator and the kinematic computing equipment are included together in the tracking loop. In Figure 4, the response d implies a differential equation of higher order than do the preceding responses (a, b, and c). In the techniques of automatic regulation, it is a familiar experience to find that the extra degree of freedom which is permitted by passing to a higher-order regulatory law enables an adjustment of the parameters which offers results, in stability and performance, far beyond those obtained with the simpler arrangement. Along with such an increase in complexity, for its management to be possible, must come a knowledge of how to handle the new complex of adjustments, and how consequently to exploit the generalized dynamic adaptability which is at hand. In the field of regulation such knowledge was acquired, but the analogy with tracking aids is as yet somewhat less distinct than that traced out in the last section.

With regard to the familiar classical dynamics of the lead-computing sight, there are several reasons to suspect that a characteristic of higher order would serve more effectively. This is true on the one hand through the straightforward argument that linear prediction, here in angle, should properly be generalized to a prediction of higher order, since the typical engagement involves acceleration of relative target motion. We enter here into a basic controversy which is familiar in ground and seaborne equipment. Further, however, in the coupling constant of the lead-computing

sight, which, in addition to the thus far unavoidable adjustment for time of flight, is the only adjustable parameter of the computation, an extremely crude compromise has been necessary, especially in the presence of the available tracking dynamics. Adjustment of the coupling constant in one sense gives an improved stability of tracking, and in the other sense gives a more rapidly decaying transient error in the computation. The region of overlap does not lead to great satisfaction on either score, and a very shallow optimum, in terms of gun error, is shown on exploration of this single adjustment. This is precisely the sort of situation which is typical of regulatory circuits, and for which very frequently a more articulate dynamic characteristic has stepped in to put the whole affair on a new basis. For example, in many regulatory cases where a regulator of first- or second-order dynamics has been used, and in the adjustment of which unsatisfactory compromises must be accepted, the insertion of a higher derivative term in the regulatory dynamics can sever the interdependence of parameters which necessitates the compromise and can allow a readjustment giving the results which are in an entirely new class.

Now it may be noted that the characteristics of the aiding component and those of the kinematic computer are intimately related in the tracking loop. There is thus, in view of the possibilities for improvement indicated above for each component, an even more striking possibility that a mutually beneficial restyling might be carried out jointly between the two together. A proposal involving this sort of innovation was made[a] for application to the case of a lead-computing sight as used in a velocity tracking turret. This proposal was only for a minor change, and was premised on the possibility that a physical alteration of simple form might improve circumstances, but it did result in a conversion both of tracking function and of the lead-computing dynamics, and to higher order. While never actually applied to full-scale trial installations, it was reduced to simulative form, and enough work was done to indicate a possible improvement. It is evident that this is not an unpromising history in view of the embryonic status of such modifications.[b]

2.7 MULTIPLE SIGHT INDICES

There have been several examples, which to the writer's knowledge have first appeared over the past three years, of the employment, in aiming controls, of sights or directional sighting means in which multiple indices occur. It is considered that an important new technique is embodied in these, and that it is worth while briefly to sketch out what they involved. For simplicity we restrict the discussion to visual aiming equipment; the developments referred to, as it happens, have been similarly restricted, although the underlying principle is certainly more general.

The examples referred to above shall now be mentioned. First, in the British method for low-altitude angular rate bombing, as embodied in their LLBS Mark III, a row of collimated luminous lines, parallel and equally spaced, was rotated downward with respect to space. The lines were oriented horizontally, and the operator merely observed the target through the optical grid thus formed. When the target, which at first appeared to be progressing "upward" across the lines, became momentarily stationary with respect to the grid, the operator acted on this instantaneous synchronism as his signal to release a bomb.

Second, in the proposal[c] for the so-called *Texas sight*, of which models were built and tested on the ground, a constellation of luminous points was substituted for the single collimated reticle of the classical lead-computing sight. The points of the constellation, which formed an extended two-dimensional pattern in angle, moved with the single reticle and converged exponentially toward it as focus. Ideally, it should be added, the points of the pattern should not rotate about the central one; not with respect to space, that is. In use, the operator merely manipulates his tracking

[a]By H. Whitney.

[b]We refer, however, to the writings of H. Whitney on tracking, and to the more detailed accounts of simulative tracking studies which were carried out under Section 7.2 at Columbia University and at The Franklin Institute.

[c]By L. LaCoste.

controls, or the gun itself, so as to achieve equality of angular velocity between the target and the *local* points of the pattern. Two results ensue; for one thing it is far easier to track, since an integration is omitted from the tracking loop, the operator observing rate rather than position; furthermore the opportunities to fire are multiplied tremendously, since as may easily be demonstrated the sufficient conditions as well as the necessary ones are given for effective fire. The trouble has been that a visual stadia for manual ranging was difficult to incorporate along with the optical pattern, but this circumstance might be altered in view of modern ranging facilities or of the use of methods in which range is not explicitly accounted for.

A third instance of the need or desirability for many sighting indices has been involved in the writer's proposals (Section 10.3) for techniques to be used in the pilot's universal sighting systems [PUSS] (Project NO-265) and in certain optical systems for the control of guided bombs from a static position at ground level (Chapter 8). In both cases the tracking display would be similar to that in the two instances already mentioned. In neither case has the method been finally reduced to practice. It will suffice to mention here a simple means for arriving at a collision course.

It is well known that if the axis of a free gyro in a vehicle, for example, be initially pointed at a target, and that if subsequently the vehicle be so steered that the axis continues to point at the target, then a collision course will result subject to the conditions described already in Chapter 1. Now this criterion for steering is not an easy one, and to achieve stability it is evident that coupling must be introduced between the heading of the vehicle and the line of sight. Instead of this, an improved stability would also result if uncoupled lines of sight were displayed as a pattern in every direction, through one of the several available means for such stabilization and presentation. Transient errors would not need to be corrected all the way back to the starting point, it being essential only to keep those points of the pattern moving with the target which are already in proximity therewith. The stars themselves, if sufficiently dense, would serve admirably as such a pattern. It is not known by the writer if they have ever been so used.

2.8 TRACKING BY THE PILOT

In fixed gunnery, aerial torpedoing, bombing, and rocketry a pilot may be expected to carry out an aiming process which involves keeping an artificial line of sight, either fixed or moving with respect to the airplane, in approximate coincidence with the line of sight to the target. The pilot employs the normal flying controls in this operation, and in so doing deals with a system of tracking dynamics which differs radically from, and at the same time is considerably more complex than, the controls which are common, say, to a turret. For typical airplanes, and for the modes of approach which have been in demand, the pilot must continue to perform a number of special operations which are essential to safe and efficient flight, and which although they may become semi-automatic nevertheless provide distractions from the efficiency of his tracking, as such. Thus he must watch his indicated airspeed, keeping this quantity within specified bounds, he must trim all his control surfaces to maintain any sort of symmetry in his individual manipulations, he must so operate the rudder that skid is maintained below an allowable amount, and he must beware the possibility of flying underwater or underground.

It is clear that under these circumstances additional tribulations such as manual adjustments to a computing system or attention to complicated warning indicia are unwelcome and must be kept to a minimum or eliminated. In spite of the stringency of such requirements, the single-pilot airplane, principally the fighter and the fighter-bomber, has evolved into the most useful weapon in the air, and thus has deserved attention as a vehicle for which the development of aiming controls is a profitable pursuit.

The most striking feature of the tracking dynamics available to the pilot is the asymmetry in the up-down and sideways directions. For pulling up the nose of the airplane, or for

nosing downward directly, a motion of the stick forward or backward is all that is required. However, and especially if the pilot wishes to fly without skid, his operations in manipulating the airplane toward a goal which originally appeared to one side are very much more intricate. This is partly true since as he progresses in the operation the target does not maintain its direction with respect to airplane coordinates, but rotates approximately about a longitudinal axis therein, first one way as the maneuver begins and then the other way as the maneuver comes to completion. The pilot must operate both stick and rudder, and each in a different manner, during such a maneuver. Under these asymmetrical conditions, it is obviously even more complex to point the airplane, not to mention a sight index dynamically related to the airplane, in a direction which is neither directly above or below nor to one side of the general direction of flight. For some of the problems connected with tracking through the controls of an airplane, other chapters of Part I may be consulted. Thus in Chapter 4 a laboratory apparatus is discussed in which the phenomena of such tracking are reproduced electronically in the laboratory. Again in instruments for aiming rockets and for multiweapon aiming instruments, the special requirements which arise from the nature of tracking by the pilot are dealt with.

Although no apparatus has yet been prepared for this purpose, it is of interest to discuss a possible means for use by the pilot in tracking which has been seriously proposed[d] and which may be prophetic for future systems. There are analogous arrangements which have been proposed and tried out for bombers and for automatic missiles.

The idea is for the pilot to track the target through an independently stabilized system which is carried in the airplane. For such tracking he would manipulate conveniently mounted controls which would rotate in space a sight index having no direct dynamic coupling to the airplane. This tracking component would be expressly designed to assure minimum error and maximum smoothness in the continuous representation of the line of sight to the target. From the motion of the sight index thus developed, with reference to the stabilized system, all data on angular motion of the line of sight are extracted automatically and continuously. These data, together with the remaining significant variables (speed, range, acceleration) are submitted continuously to a computing component which determines the proper instantaneous heading for the airplane in which it can effectively launch a projectile at the target. A high-performance automatic pilot, given this information, flies the airplane in such a manner that the firing condition is satisfied. The pilot merely tracks the target and chooses when to fire, in which latter choice he may be assisted by instrumental means which show him the transient condition of the computing and piloting operations. In normal flight some simple adaptation might be found useful for navigating, and would serve to lighten the pilot's responsibilities in any case. On retiring from any given engagement it might seem appropriate for the pilot to take over the controls in the traditional manner; on the other hand, he might simply turn his tracking index suddenly away in the desired direction for retirement. It would be straightforward to incorporate an optionally usable automatic evasion. Safety interlocks would come into play at minimum "times-to-ground" and at maximum "gees."

The remaining steps to complete automaticity are not unthinkable, even when the vehicle becomes a projectile itself. Automatic detecting components can lock on the target, and computers and automatic pilots can work out the attack. With no human pilot, for example, it is no longer necessary to bank. But here we enter the field of guided missiles.

2.9 MORE ON THE HUMAN TRACKING OPERATOR

It is plausible that for a given state of conditioning or "learning," and within certain limits, the human operator in the following or tracking operation may be considered approximately linear. This belief in itself, however, is not to be accepted without question; there are a number of queries which are not easy to an-

[d]For example, in 1943 by Lt. Comdr. E. S. Gwathmey.

swer in connection with its justification. Thus if the operator is a linear one, why does it not react the same each time to a given stimulus? How account for random "dithering"? What linear mechanism is there which will exhibit these varying states of muscular preparedness at the onset of an experiment? Many such questions may be parried by pointing out that such irregularities may be provided in an automatic model by virtue of an artificial source of random signals which add directly to the output, or elsewhere in the circuit. For example they may be thought to arise as the generated response to random excitations in the internal feedback loops which one includes under the name *kinesthetic.* Or these questions may be parried rather differently by indicating that the fluctuating components of the human response are unimportant, and need not be represented in a linear model since they are rapidly attenuated in the remaining components of the tracking loop. This seems to be a particularly dangerous assumption. In any case the closest approximation to a model of the human following operator appears to be attained[e] by a linear function, say $H(p)$, together with an added random fluctuation, as illustrated symbolically in Figure 5. We shall consider what may be the nature of $H(p)$ which approximates most closely to the human example.

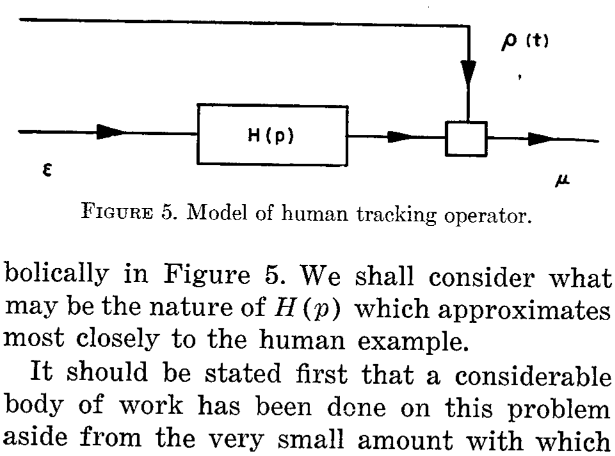


FIGURE 5. Model of human tracking operator.

It should be stated first that a considerable body of work has been done on this problem aside from the very small amount with which the writer has been associated. There are accounts available of experimental and theoretical work on this topic, both in Great Britain and within Division 9 at Radiation Laboratory at MIT. Those titles[1,2,4] which the writer can obtain are listed in the bibliography for this chapter, but it is certain that the interested investigator can find these records through other channels. In the various linear operators which have been proposed there is some uniformity, whereas wide disparity seems evident in the methods of approach to the problem, and in the significance, for example, which the different researchers have attached to the method of frequency analysis. Our own view on the latter question has been that any simple study based on the response to simple harmonic inputs, or even to the sums of a few such inputs having irrationally related frequencies and different amplitudes, will admit the possibility of learning by the operative under test, with a consequent fundamental alteration in his characteristic. The more complex of the composite harmonic inputs lead to tremendous difficulties in separation and interpretation.

The primary indication of linearity is that generally speaking the observed responses double when the stimulus is doubled, provided that a limit of apparent "saturation" is avoided. Such general proportionality of stimulus to response is encouraging for the devotees of the linear approximation.

There appear to be only two characteristics of the human tracking operator, beyond the limited linearity already mentioned and the larger fact that over longer periods a learning process occurs, which have been at all well established. One of these is the inclusion of lags, owing perhaps to the retardations of nerve conduction and perhaps again to some more mysterious process in the reflex itself. The other is the integrating response. Good tracking is never *deliberated,* although it may be *deliberate.* First, as to lags: It has frequently been proposed that these be described by a direct displacement in time of τ seconds, or by the effect of the "true" time-lag operator $\exp(-\tau p)$. This seems unnecessary on the one hand, since nerve conduction is probably not distortionless, and it leads to analytic difficulties on the other hand, owing to the peculiarities of the time-lag operator. On the basis of the known convergence

$$\lim_{n\to\infty}\left(1+\frac{\tau}{n}p\right)^{-n} = e^{-\tau p}$$

[e] In work done at Columbia[1] on the possibility of such a synthesis, the human operator as such came to be known as "Big Henry," or as H, whereas its model $h(p)$ came to be known as "Little Henry."

it has been proposed[f] we could use

$$\left(1 + \frac{\tau}{3} p\right)^{-3}$$

as an approximation to the lag involved in the human response. The writer has advocated larger values of n as being easily representable in linear model, and as probably more nearly representative of the situation in nature even than exp $(-\tau p)$. There is a curious situation here. Psychological data give reaction times as a single number, but this should not be interpreted as a direct time lag. For example the response to a step input of the above operator for $n = 15$ is a transient such as shown in Figure 6. If nerve conduction were represented by such a lag operator, then different absolute thresholds in the indication of

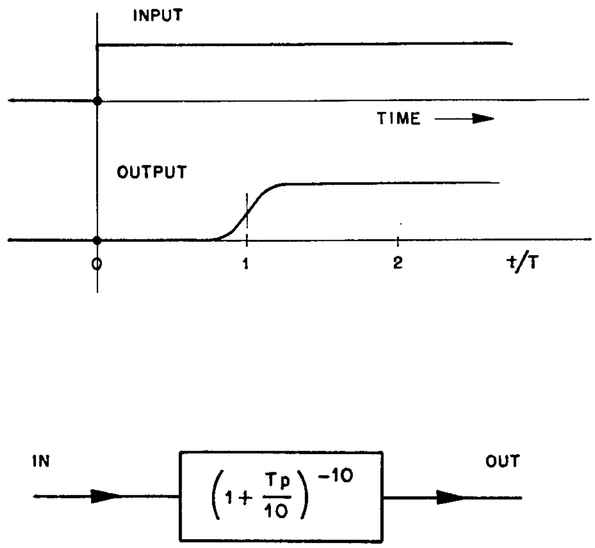


FIGURE 6. Response by approximate time lag.

the response and in the measuring apparatus would result in a different apparent reaction time for different amplitudes of stimulus, as is observed. Notice that the hypothesis of a velocity of conduction varying in dependence on the strength of stimulus would then be unnecessary. To the writer, this explanation would be eminently more satisfactory.

Aside from the lagging response, which is certainly a property of the human tracking operator, it is evident that there is another type of response, which, in addition, may be adapted gradually to the environing circumstances. In terms of the coordinate μ of the tracking handle as output, and of the observed error ϵ as input, and omitting the assumedly multiplicative lag-operator, there are reasons to believe that an integrating effect, perhaps alone and perhaps in combination with other effects, is present.

These conjectures are on the basis of a relatively simple system around the loop, as when the human operator replaces the follower Φ of Figure 8 of Chapter 1. Admittedly we are discussing a human operator which has been conditioned by the experience of operating in such a loop. However, if we knew the character of the operator thus conditioned, then, since this is a natural circumstance, it would be significant to include compensatory features within the loop on the basis of this conditioned character if the operation thus modified resembled the natural state of affairs. This is not an easy viewpoint to express.

An experiment may be arranged in which the operator "follows" a variable by a direct manipulation, seeing only the error of his following. If under these circumstances, during the performance and unknown to the operator, the error is frozen, the operator simply feels that he is exactly compensating for contemporary variations in the input variable but that he must correct the accumulated error. The very striking manipulation under these conditions is a continued steady motion in the direction which would normally produce such a correction. This corresponds substantially to an integrating response.

Numerous experiments, mainly generalizations on this simple one, may be carried out to determine more precisely what is the operational form of the intervening human dynamics. The difficulties of frequency methods have already been mentioned, but in all cases the problems of conditioning warrant extreme care in experimentation.

This general problem has been worked on[g] and the possibility suggested of a purposely established higher-frequency exploratory loop which the operator may superimpose on the regulatory

[f]By Sobczyk at MIT Radiation Laboratory.

[g]See reference 6 of Chapter 1, also later works by Wiener, as yet unpublished.

loop with which he must deal. This is for the purpose of determining the moment-to-moment nature of a system which may be changing, or for an invariant system (present writer's suggestion) merely to refresh his memory. Such a possibility seems not at variance with the results of one British writer who has examined numerous oscillographs of human tracking. He finds, after subtracting out certain lower frequency components, that there is a large "remnant" at higher frequencies which has special properties. He concludes that the human tracking operator, or human servomechanism, appears always to operate near instability.

Chapter 3

TECHNOLOGY OF ROTATION IN SPACE

3.1 THE IMPORTANCE OF ANGULAR RATES

THE MEASUREMENT and the production of angular rates form a fundamental branch of instrumental technique in airborne aiming controls. We distinguish immediately between two categories of angular rate: relative and absolute. In making this distinction it is convenient to call up the concept of a fixed direction or a fixed body, and to save trouble it is simplest to define such a direction or body as having no measurable angular motion with respect to the fixed stars. We are thus resting firmly on an empirical foundation and can expect no arguments from the philosophers. Absolute rotation is now merely relative rotation assessed with respect to a fixed direction or a fixed body. With few exceptions absolute rotation, or rather the *rate* of absolute rotation, is the subject dealt with here.

The rate of absolute rotation of a line, considered either as the total quantity taken about an axis normal to the plane of rotation, or as one of the components of the total about some other specified axis, appears as an essential variable in lead-computing gunsights, synchronous bombsights, angular rate bombsights, rocketsights, stabilizing systems, guided missile controls, and so on. As discussed in Chapter 1, a prime criterion for the interception approach is expressible in terms of this variable. One of the more spectacular applications of the absolute rate of rotation is in the principle of low-altitude bombing which is discussed more fully in Chapter 7. We wish here to signalize the general value of absolute angular rate by giving a brief geometrical description of this principle. The range solution only is demonstrated.

Consider a bomber flying horizontally over a target, at constant speed, and making a straight ground track in which the target is contained. The direction from the target to the bomber is increasing in elevation, whereas that from the bomber to the target is dropping in angle below the horizon in a precisely corresponding manner. In terms of the altitude h of the bomber, the remaining horizontal distance d on the ground track, and the horizontal speed v of the bomber in target coordinates, we may obtain an expression for the absolute rate of rotation ω of the sight line from bomber to target, as seen from the bomber. We find easily

$$\omega = \frac{vh}{h^2 + d^2}. \tag{1}$$

Now if a given value of ω, say ω_0, is considered, we see that the locus of points in which an airplane, flying with relative speed v in a vertical plane through the target, sees the target moving with angular rate ω_0, is given by

$$h^2 + d^2 = \frac{vh}{\omega_0}. \tag{2}$$

This is evidently a circle lying in the given vertical plane, containing the target as a point of its circumference, and having its center precisely $\frac{1}{2}v/\omega_0$ above the target. As v and ω_0 change, we see that the circular locus merely swells or shrinks but still passes through the target and is still symmetrically disposed above it.

The locus of release points for a bomb on the other hand, for negligible air resistance, is merely an inverted trajectory with apex in the target. Such a locus is a vertical parabola containing the target, and may be written, using g for the acceleration of gravity, as

$$h = \frac{g}{2v^2} d^2. \tag{3}$$

We now choose ω_0 in equation (2) so that the circle osculates the parabola, noting that the center of the circle, and the apex and focus of the parabola, are collinear. Placing the center of the circle at the center of curvature of the

parabola, referring to the nature of the curve at the origin, we have

$$\frac{v}{2\omega_0} = \frac{v^2}{g}.$$

It may readily be shown that for typical speeds the closeness of the mutually osculating curves is remarkably good for altitudes below 500 feet, and thus that they can replace one another there. Thus a bomb may be released when, approximately,

$$\omega = \frac{g}{2v}. \tag{4}$$

Several refinements must enter before a practical sight results, but the inputs other than those of equation (4) are of second order importance. The method is relatively independent of altitude at low altitude, for example, and as a slightly extended analysis will show is remarkably insensitive to a departure from horizontal flight. A number of bombing applications result from this principle, which differ among themselves by virtue of the manner in which the angular rate criterion is incorporated.

3.2 ALTERNATIVE METHODS OF MEASUREMENT

The phrases "rate of rotation in space," "absolute rate of rotation," and "absolute angular rate," are being used interchangeably here. When the context leaves no ambiguity, the term *angular rate* itself will frequently be employed as equivalent to the above phrases in order to simplify the construction of sentences.

When it is desired to determine, from a vehicle, the absolute rate of rotation of the direction from the vehicle to an external object, one has the choice of various instrumental procedures. It must first be well understood, in any given case, what axis or axes it is desired to employ as references, or about which the angular rate is to be measured. Suppose that a gyro,[2] in appropriate gimbals, is subjected to torques about axes normal to its polar axis, and that these torques are so regulated that the polar axis of the gyro is made to point directly and continuously toward the given external object, say the target. If the two axes, about which the applied torques occur, are mutually perpendicular, the torque applied about each of these axes measures the component of absolute angular rate, of the given direction, about the other axis, and the total angular rate in space of the given direction may be compounded vectorially from these two orthogonal components. The sensitivity of the measurement is proportional to the moment of momentum of the gyro, and consequently to the speed and inertia of the spinning wheel, both quantities being assumed to be so large that the torques corresponding to angular acceleration are relatively small. If this is not the case, as it usually is, the two angular rates may still be extracted, assuming always that the target is being perfectly tracked, by a dynamic compensation and a mixture of the torques corresponding to the two components. Of course when the applied torques are imposed about axes which are *not* normal to the polar axis, as is frequently the case in practice, then the appropriate resolution must be made on to equivalent axes which *are* so located.

Suppose on the other hand that a rigid framework, mounted in similar gimbals, is so manipulated by the control of torques with respect to the vehicle that a definable direction fixed in the framework is also pointed directly and continually at the target. This is a rather pure problem in servomechanism, with an inertia load, aside from the detection itself of the target direction. If now (absolute) angular rate meters, of which several types are to be described below, are mounted solidly with appropriate orientations in the rigid framework which "follows" the target, then the indications of these meters provide measurements of the absolute rate of rotation of the direction to the target taken about axes which are determined by the orientation of the meters in the framework, of the framework in the vehicle, and of the vehicle in space.

Again, assume that the vehicle carries a completely stabilized body, possessing no appreciable absolute angular motion. Assume further that means are available for the continuous determination of relative angular rate. Such means may, for high precision, consist in a feedback arrangement whereby angular motion is created under the control of a positional

null, or of positional error detection. If then an index is pointed continuously at the target, either through the application of torques, to the index member, with respect to the vehicle, or with respect to the stabilized body, and the above means is employed to measure the angular rate components with respect to the stabilized body, these measurements are also valid with respect to space. We mention some practical points:

Usually the direction to a target is capable of much slower instantaneous rotation than is the vehicle. Thus, if an articulated massive system is forced to point toward the target, the torque required will be largely used in overcoming the friction in the drive between the vehicle and the pointing system. This is not true, however, with regard to rotation of the pointing system *about* the direction to the target. The precession of a gyro, with spin axis as pointing index, to follow a target has several advantages. First it is self-stabilizing, opposing inherently those inadvertent torques which arise from angular motion of the vehicle. Furthermore it provides a more naturally stable dynamic system, for a given excellence of servo design, leading thus to an easier problem in servomechanism.

Finally there should be mentioned the method in which (absolute) angular rate meters are used to measure the (absolute) rates of rotation of the vehicle itself about axes which are fixed therein. It is evident that the angular velocity of the line in space from the vehicle to the target may be compounded from the angular rates of that line with respect to the vehicle, together with the absolute angular rates of (the coordinate system in) the vehicle. The geometry involved is not trivial, but is completely delineated — for example — in Euler's transformations for a rigid body. We note that the complete angular velocity of the vehicle in space may be expressed as the vector sum of its absolute angular rates about any three mutually orthogonal axes which may be chosen in the vehicle. Thus the measurements afforded by three rigidly mounted absolute angular rate meters, disposed in the vehicle so that the axis about which each is sensitive lies perpendicularly to those of the other two, such measurements that is, are completely definitive of the angular velocity of the vehicle in space. If the case is contemplated in which the target is seen more or less directly ahead of the vehicle, or in which the lead angles between target direction and vehicle heading are always less than, say, 45 degrees, then a geometrically simpler situation results. Here the system which measures the angular rates of the target direction with respect to the vehicle, the most significant of which are typically the time rates of change of the lead angles in two coordinates, combines naturally with the system for measuring the absolute angular rates of the vehicle. Thus for example the axes chosen for measurement of the latter may be the same as for the former, both being stationary in the vehicle. Quite valid approximations may be employed to simplify substantially the dynamic system which is necessary. Apparatus of this type is described in Section 10.5, where its application to the developments in the pilot's universal sighting systems [PUSS] project is described more fully.

The choice in general among the available instrumental methods outlined above depends on numerous circumstances. (A number of permutations are obviously possible among the systems referred to.) These include the immediateness of the basic components, in terms of procurement or developmental status, the requirements of precision, the life expectancy to be obtained, the size and weight, and the flexibility for subsequent alteration and adaptation which it may be desirable to incorporate. This latter item, in the opinion of the writer, is one of the most important and yet appears to be the easiest to overlook under the stress of emergency research. With regard to absolute angular rate meters, only those having small internal angular displacements are considered; certain unique advantages may be claimed for such components. One is the instrumental flexibility which follows from the technique of measuring accurately and permanently a fundamentally important quantity, involving, thus, a component which need not change form in dependence on the dynamics or the geometry of the particular problem at hand. Another advantage lies in the absence of angular discontinuities, or limits of predictable motion, which haunt the designer

of gimbals and bails for certain other angular rate components, and which lead to problems in locking and in protection against wear during idle intervals. A further advantage lies in the avoidance of friction in the measuring assembly, since, owing to the use of small angular displacement, or even of a null system with regard to such displacement, traditional bearings may be replaced by almost completely frictionless structures which require no care or maintenance.

3.3 PRINCIPLE OF THE CAPTIVE GYRO

By *captive gyro* is meant a component comprising a balanced rotor, spinning at substantially constant speed, of which the axis is constrained to remain very close to a fixed index, in one or two dimensions, within a supporting framework. The operation of constraint has been referred[a] to as *capturing.*

With a captive rotor of this sort, having sufficiently large moment of momentum, and presupposing a small angular departure between the spin axis and an arbitrarily varying index, the total effective torque which is applied to the rotor about all axes normal to the spin axis of the rotor is a direct measure of that angular rate in space of the latter axis, and hence very nearly of the varying index. More generally, the applied torque is the vector product of the moment of momentum into the precession rate, all three physical quantities being treated as vectors. Thus approximately

$$L\mathbf{r}^l = I\,\Omega\,[\mathbf{r}^s \times \dot{\mathbf{r}}^s]\,, \tag{5}$$

where the unit vectors $\mathbf{r}^l$ and $\mathbf{r}^s$ point in the direction of the torque and spin axes respectively, and where it is to be noted that $\dot{\mathbf{r}}^s$ is simply the vectorial precession rate in space. The amplitude of $\dot{\mathbf{r}}^s$ is the scalar precession rate ω, which is thus measured by the scalar torque L under the circumstances above named, since further the rotor speed Ω and its spin inertia I may be held constant.

Equation (1) describes the relation between precession and applied torque for a gyro whether or not the technique of capturing is to be used, and of course only holds when the acceleration torques are assumed negligible. In the captive gyro $\mathbf{r}^s$ is to be held close to some direction fixed in a supporting framework, and $\mathbf{r}^l$ is so arranged as to be restricted to directions normal to the first direction. In what we may call a *captive gyro of the first kind,* the spin axis $\mathbf{r}^s$ is allowed to rotate, however little, in one plane only in the framework; this plane ordinarily contains the spin axis and is normal to a second axis of rotation fixed in the framework, imposed, for example, by a single gimbal or by bearings between the framework and the nonspinning case of the rotor. Rotation of the spin axis in the "free" plane within the framework is opposed by a torque, applied about the above second axis, which depends on the relative displacement in one dimension of the spin axis from the fixed or index direction in the framework, and which operates to keep this relative displacement to a minimum. The applied torque thus measures the absolute angular rate of the spin axis, and consequently to a high approximation of the fixed index, about an axis fixed in the framework and normal both to the fixed index and the above second axis. The approximation improves in proportion to the closeness with which the displacement referred to is held near zero. In order that the total applied torque may be principally contributed by a measurable or reproducible agency, it is essential that the friction involved in the rotation of the gyro about the second axis, the *precession* axis that is, be made insignificant in comparison with the restoring torque which corresponds to the smallest angular rate to be detected.

Most of the captive gyros we have dealt with have been of the first kind. A single gimbal

[a]Captive gyros have been variously identified in contemporary nomenclature; the name captured gyro may have become the most familiar. R. O. Yavne, who arrived in our laboratories by a devious route after a hectic international migration, and of whom this is hardly the only (or the best) reminiscence, was assigned to the technical development of captured gyros owing to his demonstrated facility with recondite dynamics. It was not at first understood why he was reluctant to work on this phase of research, it being unknown that his reluctance stemmed actually out of an imperfect grasp of idiomatic laboratory talk, until he finally summoned up the courage to say that he felt it to be a disgrace that we couldn't design our own gyros from fundamental principles, rather than to accept as a starting point a piece of equipment of enemy origin which had fallen into our hands!

axis only has been involved, and a single scalar reading has thus been provided for the absolute angular rate of the framework about a fixed line therein. Such a component, when adequately fast and accurate in response, may serve as a basic element to be used singly or in sets of two or three, being mounted in appropriate positions in a vehicle or in an articulated body, and has many diverse applications in aiming controls. A few remarks, to turn to the practical side for a moment, are warranted on the available methods for driving the rotor. The methods we have been concerned with divide first into pneumatic and electric, the electric methods split further into d-c and a-c, and a-c types are either synchronous or nonsynchronous. Alternating current drives are commonly either two-phase or three-phase. Of the pneumatic method, which is economically feasible only if associated pneumatic equipment is involved, we may say that speed regulation for the rotor has as yet only been experimentally worked out. Such work has been largely carried out by Section 7.3 and has involved both a resonant reaction to the periodic impingement of the driving jet on the cups of the rotor (the reaction throttling the jet flow) and a centrifugal brake, although several other arrangements have at least been proposed. In rotor drives involving direct current motors, one advantage is the simplicity of speed regulation, but a disadvantage is apparent in the problem of brush wear, which may unbalance the gyro by moving the center of gravity along the spin axis. This is a very real consideration, where the instrument is to have an appreciable operating life, since unbalance of this sort will give a false measurement of angular rate in dependence on the acceleration of the vehicle or platform. It is claimed, however, that such motors, or rather combinations of such motors and gyro wheels, can be made in which the abraded brush or commutator material will fly out radially and stick permanently at the original distance from the precession axis. The writer has not had sufficient opportunity to test this claim. In the employment of synchronous motors which "lock in" at the driving frequency, as with the General Electric hysteresis gyro unit, either frequency control of an inverter or speed control of a motor-generator is essential. In one case however, and the following method would apply for nonsynchronous drives if the rotor frequency is extracted, the response of a captive gyro of varying speed was corrected by the interposition of a filter in the output which altered the overall sensitivity in inverse proportion to an impressed frequency, the latter frequency being made proportional to the rotor speed.

An advantage in captive gyros generally is in the avoidance of intricate problems connected with carrying in power conductors through gimbal connections to spin the rotor. Owing to the small relative motion of the rotor axis with respect to the support, flexible connections are possible if care is taken to avoid fortuitous spring-torques. In the pneumatic case, as in the partially captive *turn gyro* of familiar renown, a nozzle fixed in the supporting case is adequate if aerodynamic symmetry is preserved.

Very briefly, the techniques for capturing and for torque measurement, in captive gyros of the first kind, are as follows. First, one may constrain mechanically the relative rotation of the spin axis, in its attempt to precess, and then may measure the small angular deflection which results and which is proportional to the constraining torque, provided of course that appreciable friction is not also contributing to the constraint. The requirements for high accuracy are: high undamped frequency of the oscillatory mechanical system, predictability of the resilient member, negligible drift of the zero position with temperature and time, means for accurate measurement of the small displacement, and enough damping to give rapid decay, on the time axis, of transient excitations. Symmetrical oscillations of high frequency are usually harmless since these are ignored by the subsequent components into which the output is fed, but random disturbances, which have a propensity for containing energy in the lower-frequency band, are meticulously to be guarded against. Such disturbances, which establish a threshold in detection, may arise, for example, from the shock impulses and noise in imperfect rotor bearings.

The second method principally in use involves a *feedback loop* whereby the error or unbalance in angular displacement of the spin axis with respect to the framework initiates application of a torque so related dynamically to the error that the latter is reduced to small magnitude and maintained there. The applied torque then measures the angular rate which necessitated it. Such an arrangement may or may not have an added resilient system which applies an unmeasured torque in proportion to the error or to its rate of change. Even in the presence of such resilient or viscous attachments, the feedback method operates substantially without error from such sources if a small and properly located null is maintained.

The problem of the two-dimensional captive gyro, of the *second kind,* as we may say, is rather different. In the first place the need for a double gimbal, to permit rotation of the spin axis about a *point* instead of an *axis* in the framework, leads to a more difficult balancing problem about the two precessional axes which must then be considered. Furthermore, as is well known, a gyro resiliently restrained in two dimensions will nutate violently if afforded the slightest opportunity. Even with zero constraint, the so-called *free gyro* will nutate forever in the absence of bearing friction or other losses. The principal obstacle is overcome, however, when the danger of nutation is recognized and provided against. Again there are two distinct capturing techniques, as above, and although they do not involve simply a duplicate application of the one-dimensional techniques for each of the two precessional axes, successful controls have been worked out which compensate for rotational tendencies.

The gain in captive gyros of the second kind, over those of the first kind, is particularly apparent only when two of the latter are replaceable by one of the former. Where three independent angular rates are desired it is almost preferable to use three of the simpler units owing to the resulting similarity of the basic units.

3.4 GYROS FOR ANGULAR RATE BOMBING

The development of angular rate bombsights as such, with a description of their place in the larger field of airborne aiming controls, is dealt with in Chapters 7 and 10. Here we shall discuss the research effort which revolved around the design of captive gyro components for such bombsights, beginning with those for the original hand-held BARB, which incorporated the theoretical principle of British-type angular rate bombing for low-altitude attacks.

Requiring a method for measuring the absolute rate of rotation of a framework which was turned about a substantially horizontal axis, and there being reason to desire an accurate, sensitive, and reproducible such measurement, the application was naturally considered of a captive gyro of the first kind, as above identified. For this and similar applications it was proposed originally by the writer (although similar proposals may have been made elsewhere) to construct a captive gyro of the first kind having extremely stiff mechanical constraint and to measure angular rate as strain through the response of wire-type strain gauges on the constraining members. This was essentially a nonfeedback proposal, the approximate null in angular displacement not being "sought" by retroaction but being assured through the relatively high stiffness of the constraint. It was considered that strain gauges of the type suggested would enable measurement of the gyro torque, and hence of angular rate, in terms of the small strain (or small dimensional changes) thereby produced in the members. Such gauges were known to respond measurably to fractional strains of the order of one part in a million, and to yield changes of resistance in extremely rapid and faithful correspondence with the dimensional alterations of the structure to which they were attached. The techniques of balancing out (by the employment of bridges, etc.) the errors owing to the temperature coefficient of the gauge wires was quite well understood, but it was not at first suspected what extreme care was necessary to guard against *drift* which arose from humidity variation. Nor was the difficult problem of *noise* completely foreseen. It was recognized at an early stage, however, that a very great advantage followed from the general arrangement suggested, in that no orthodox bearings need be involved, and hence

no sliding or rolling friction. Friction is the deadly enemy of torque measurements, whether these are to be obtained through deflection or through a feedback balancing of forces.

Half a dozen models of such captive gyros were built including one or two of the second kind, all involving the strain gauge technique. Experimental constraining members in a great variety of forms and shapes were tried out, with various dispositions of the gauge elements[b] on them. Combinations of single and double cantilevers were at first most popular, although somewhat later cylindrical members came into vogue in the laboratory. One example of the latter sort involved a pair of thin bronze cylinders placed in a line coaxially, with the gyro connected between them so that its spin axis was coincident with the cylinder axes and its case rigidly connected to the two cylinders at their inner ends. The outer ends of the cylinders were rigidly connected to a supporting framework. Gauges were applied symmetrically, four to each cylinder, and oriented along the elements thereof. Although the constraining members supported the gyro and its case against accelerations, such as that of gravity, with a corresponding deflection of the elastic constraining structure, such deflection was always "even" and was not measured by the bridge system in which the several gauges were connected. "Odd" deflections, corresponding to the angular rate torques, were measured by the system in terms of the differential resistance change of the gauges. A rather careful elastic analysis was made of the mechanical system which was fundamental to this arrangement, and corroboration was experimentally found to a satisfactory approximation.

The difficulties which were experienced with the above absolute angular rate meter were typical of those found in the case of other meters in which strain gauges were used. We have already mentioned the variation in gyro speed and the attempts to compensate for it *after the fact*, as in the eddy-current dome of a disturbed sight, so called, but although some of these attempts were ingenious (such as in the excitation of the a-c measuring bridge by a magnetic system on the gyro rotor, and the insertion of filters of transmission inversely sensitive to frequency), we shall not treat them further since similar problems are common to other types of gyroscopic rate meters, except to say that speed regulation is a comforting feature to plan on for the rotor. The problem of drift in the gauges themselves, to say nothing of drift in the attendant circuits, is a straightforward one; there are now, it is understood, standard procedures whereby this problem is surmountable in other strain-measuring applications. It may be noted in passing that the commoner role of the wire-type strain gauge is in strictly dynamic measurements, where the long-term maintenance of a zero indication is less important than in the case here discussed.

The most refractory problem was the unwanted response of the gauges to noise originating in the bearings of the gyro rotor. No trouble was found owing to vibrations arising from unbalance of the rotor, the reason presumably being that the deflections resulting were symmetrical and principally of a single fundamental frequency with a predictable harmonic content. With these circumstances, however, the rotor bearing disturbances have nothing in common. They are random and hence symmetrical only over an appreciable time interval, and further contain frequencies over the whole band. For the bearings with which we were dealing, the noise signals were such as to give a threshold which was appreciable in comparison with the smallest angular rate signals desired. In a d-c system, an obvious measure would have been to lag the response until the resulting statistical processes gave a final measurement not biased by the fortuitous disturbance, and to accept the lower performance in promptness of response. With the a-c measurements which were being tried at that time, however, a good deal of trouble was given from this source in the phasing adjustments involved in polarity sensing. Of course the cleanest cure is to improve the bearings, and certain modern gyros are of an order of magnitude better in respect to noise, even for cor-

[b]On the practical problems which arise with regard to the application of these elements, The Foxboro Company and Ruge-deForest were consulted freely and at some length. The gauges themselves and certain auxiliary calibrating equipment were obtained also from these sources.

responding sizes. (Larger gyros, in which precessional torques are relatively bigger in comparison with the bearing noises, are less troublesome in this respect.)

Several other methods were considered, however, for the avoidance of this evil, but were never tried out. One such was the proposal to add elastic members which responded identically to the vibrational disturbances transmitted from the gyro bearings, being associated with the gyro case in the same way as were the normal members, but which were not stressed by the precessional torques, owing to the fact that they were not also to be connected to the framework. Then, of course, the *difference* in indication by gauges attached to both kinds of members would be extracted. Although some very delicate questions would arise, in this connection, on the detailed dynamics of such a measure, it is still felt that thereby an appreciable improvement might have been obtained. A good many planned extensions of this sort were discarded when the pneumatically captured gyro appeared (to be mentioned again below) and it is difficult to guess what success ultimately might have been attained. As it was, the captive gyros which were developed were almost adequate for the bombing application at hand. Most of the failures in test were actually in circuital components rather than in the gyro structures or the gauge mountings.

We should add that the angular rate bombsight was not the only aiming control system for which captive gyros were designed as absolute angular rate meters. The most important application, probably, with which we have been intimately concerned has been as an input component for the PUSS system, which is described elsewhere. There the captive gyros are used to measure the angular rates of the airplane about its own axes. Other applications included one in which extremely small absolute rates were to be determined, for use in a secondary control for refining the excellence of a precision vertical. Proposals have also been made for the use of captive gyros, of the sort here treated, in guided missiles, stabilizing systems, for internal torpedo controls, and for gyro compasses.

Nor should we give the impression that the wire-type strain gauge was the only tool employed to measure the small deflections of captive gyros in which feedback was omitted. Such other responding agencies as E magnets and autosyns were also given a chance at this job, and one of the most promising alternative approaches involved electromagnetic detection of the deflection of a small magnet spinning on the axis of the gyro rotor, the rotor and case being flexibly mounted to permit small elastic rotation. An endearing property of this trick,[c] which was discarded for other reasons, was the very great sensitivity to small angular rates of which it was capable.

All of the detection methods, by which angular displacement is tangibly extracted in the nonfeedback captive gyro, are applicable as error detectors or unbalance detectors in terms of relative motion when feedback is employed. This application naturally requires less ideally characterized response in the detecting means, since the approximate maintenance of a null is all that is asked for. The accuracy of measurement is afforded by the known characteristics which obtain between some other physical agency and the balancing torque. Errors arising at intermediate points may be annihilated through the ultimate comparison of the output with the input precessional torque itself. It is not necessary to enumerate the many advantages which follow from the feedback technique. The only disadvantages lie in the possibility of greater complexity and in special problems required for the maintenance of stability, not only at any given time but throughout the useful life of the equipment.

A very compact feedback arrangement was embodied in the pneumatic captive gyro which ultimately replaced the electric forms in the angular rate bombing problem. This development was conducted almost entirely by Section 7.3, working in collaboration with Section 7.2, and thus need not be described in detail here. (See Volume 1, Chapter 4.) The capturing method led to a maximum relative angular displacement of the order of a milliradian, and consisted in a valving process, with the gyro

[c]Attributable to W. Sheppard of The Franklin Institute.

and case serving as the sole moving part, whereby pressure areas were enabled rapidly to balance the precessive torque and maintain the displacement null stably and to within close limits. The pressure difference between the valved pressure areas indicated the angular rate as output. It was found necessary to damp the motion of the moving system, and, although several equivalent pneumatic methods were proposed, a rotary dashpot with viscous fluid proved most successful practically. No attempt was made to prepare a captive gyro of the second kind by this technique, although it was generally recognized that this would have been possible. The mechanism employed appeared simple and effective, but there is reasonable assurance that a number of drastic variations would serve as well.

In the most modern of the pneumatic captive gyros, now being considered for the pneumatic form of PUSS, crossed-spring flexures have replaced the gimbal bearings. Earlier experiments indicated that with such flexures and with the pneumatic-capturing technique, no lower limit for the detectable angular rates was evident.

3.5 A PARTICULAR CAPTIVE GYRO OF THE SECOND KIND

As a basic component of the pilot's universal sighting system, the project for the preparation of which is discussed in detail in Chapter 10, and for the provision in that system of input data comprising the instantaneous angular rates of the airplane about its own vertical and lateral axes, one of the units which was proposed and developed was built around the gyro element constructed by GE for a sight stabilizer in one form of flexible gunnery director in the Superfortress. This gyro element involved an asymmetrically mounted rotor and motor in a double gimbal system of small dimensions. E magnets detected the deflections of the gimballed system in two dimensions, and pancake coils mounted at right angles on that system moved in a magnetic circuit stationary on the frame. Currents were supplied through flexible leads to each of these coils and to the d-c gyro motor, which was locally speed-regulated. Tuned, frictional, nutation dampers had been supplied for the original function of the unit, which was as follows. In tracking, an operator supplied currents from his handlebars, through dynamics which imposed "aided" performance, to the pancake coils on the gyro. These currents precessed the gyro axis over a two-dimensional field in rapid response to the manipulations of the operator, providing, incidentally, a fundamentally stabilized system. Then the whole sighting platform, on which the gyro element was mounted, was made to follow the gyro axis by appropriate servomechanism. The rest of the fire control equipment was of the nature, more or less, of a standard director, use being made of the precessing currents in the pancake coils as measures of angular rate.

It was proposed by the writer to employ this same gyro element, for our application, quite the other way around, but thereby to use almost all the features of this existing development. First, the mounting or framework of the gyro element was to be fixed in the airplane in such a way that the neutral direction of the gyro axis pointed along the longitudinal axis of the airplane. Then, under arbitrary motion of the latter, the currents in the pancake coils were to be manipulated automatically to precess the gyro axis continually into coincidence with its neutral direction in the framework and hence into coincidence with the longitudinal axis of the airplane. Since the applied currents would also accelerate the mass of the gyro rotor, together with the motor and the gimbals, means must be provided, in the regulating channels, to separate the conflicting accelerating and precessing tendencies, and to achieve stability in the null-seeking process. As before, the currents in the pancake coils were to be used as direct measures of the instantaneous angular rates.

During the course of the development which followed, and which resulted in the attainment of reasonably good success in terms of the characteristics sought, it was found best to dispense with the existing nutation dampers. By following the differential equations which were prepared, a logical sequence of control design proved feasible, being capable of cor-

roboration at every point, which led to confirmation of the degree of precision and stability indicated by the analysis. We were looking for time constants in the response of the order of 0.2 second. These were attained or exceeded. We wanted a reliable measurement of angular rate from a resolution of 0.0005 radian per second up to a maximum indication of 0.30 radian per second. We obtained such performance from 0.0003 up to 0.50 radian per second. This development continues, owing to its application in projects of Navy interest, the current work concerning chiefly the engineering design of the electronic control channels. It is evident, incidentally, that precise regulation of the electrical supplies to the latter channels is unimportant.

A word more on design details and testing methods. The general control connections were from each E magnet "around the corner" to the coil which produced the corresponding precession. A pair of cross connections, with the appropriate operational characteristics, compensated for the inertia coupling which otherwise would unstabilize the mutual operation of the two modes of control which were simultaneously present. Since the E magnet indications were at 400 cycles per second, phase reference and rectification were necessary in each channel, the dynamic networks and the final currents being preferably at direct current. Thyratrons were finally used, for space economy, to supply these final currents. With milliammeters in the output current leads, and with the gyro axis under automatic close capturing and the framework either handheld or on a turntable, an impressive demonstration of rapid, sensitive, and accurate angular rate measurement was available, and was seen by many visitors. Remaining small fluctuations in the gyro axis, and correspondingly in the output currents, are of magnitude consonant with the resolution of measurement; these are now considered to be traceable, not to mechanical friction in the gimbal bearings (approximately 0.001 inch-ounce) but either to rotor bearing noise — as above — or to torque impulses applied nearly about the spin axis by the d-c regulator of the driving motor.

Short of very completely instrumented (and ultimately essential) trials in the air, there are several good methods for testing angular rate meters in the laboratory. Although turntables are a problem in themselves, which we shall not here discuss, and are difficult to accelerate rapidly and predictably, a very convenient method for imposing and removing pseudo-rates in space is merely to add weights on the horizontally disposed axis of the gyro.

3.6 A CAPTIVE GYRO WITH CAPACITIVE DEFLECTION DETECTION

One of the more recent endeavors in captive gyros for absolute angular rate measurement, within our group, has involved a return to the nonfeedback method. Given a smoothly running constant speed gyro in a case, and a method of predictable and frictionless constraint, a very powerful means for the measurement of mechanical displacement is provided by the variation of electrical capacitance between adjacent parts which belong respectively to the gyro and to the framework. The compensation for those displacements which it is desired not to measure may thus be made inherent, rather than through the subsequent response to a difference between two appreciable quantities.

The use of capacitive methods may become quite appropriate when a number of such measurements or computations may occur throughout the system, so that a common oscillator will serve. The advantages of these methods are many, and they include low reaction on the measured structures, precision telemetering without local follow-up devices, and the opportunity for employing capacitive "slip-rings," of higher capacitance than, and in series with, the measuring capacitor. The latter may thus be placed in an electrically remote position in a mechanical system.

In the experimental apparatus referred to here, the gyro, which has been initially one of the first kind and may later be generalized, was mounted entirely on four leaf-flexures. The flexures were mounted radially, fastened along their inner edges rigidly to the gyro case and along their outer edges to the frame, and were so disposed that their common intersection (if each flexure were imagined to be cen-

trally extended) intersects and is normal to the gyro axis. The gyro axis is thus allowed to deflect principally, although very little, about the common intersection of the flexures as precession axis. On rigid members moving with the gyro case, and extending perpendicularly to the above precession axis, are mounted capacitor plates at the outer ends. Mate plates are attached to the frame, at each end and on each side, providing two pairs of series capacitors which, placed in a bridge, respond solely to deflections which arise from precessive torque. For the problem at hand, the response to angular acceleration is negligible in comparison. A gyro of about 2 inch-pound-seconds is now being used, and it is planned to allow about 0.002 radian of relative angular displacement at maximum. An individual capacitance of 10 micro-microfarads is being contemplated, which may have a total variation of 20 per cent. Computations indicate a resolution well below 0.0001 radian per second, and a time constant less than 0.02 second. This development is still in progress and may be continued by BuOrd through a separate contract (NOrd 9644) extending the work now under Project NO-265.

3.7 AN OSCILLATORY CAPTIVE GYRO

Suppose, in a captive gyro for absolute angular rate determinations, that the rotor turns periodically, first in one sense and then in the other. If under these circumstances the polarity of the angular rate measurement also were alternately reversed, in synchrony with the reversals of rotor spin, the measurement of angular rate might be altogether unaffected. This indicates the attractive possibility, which occurred to the writer some time ago, that a captive gyro might be built without conventional rotor bearings.

If the rotor were oscillated about an axis therein, say with perfect harmonic motion in angle, then it might be connected to the framework by flexure-bearings, serving themselves as resilient means which would allow precessive deflection. For an angular rate in one direction, with respect to space, the precessive deflection would also be oscillatory and of the same frequency as that of the rotor. The amplitude of the precessive oscillation would indicate amplitude of absolute angular rate, and its phase with respect to the rotor oscillations could presumably determine the sign of the angular rate. A resonant drive for the rotor seems not impossible, and the precessive mode might also be somewhere near resonance to give an increased response. Such arrangements would be limited by the phase relations of the mechanical coupling. Systems of strain gauges could be mounted on the flexing members, and bridge-connected in various obvious ways to ignore or to detect the several deflections and the corresponding torques, and hence to measure angular rates.

As to frequency, the higher the better, on the whole, but note that the flexural accelerations and the centrifugal stresses increase with the square of frequency, whereas the peak moment of momentum increases only linearly therewith. When the frequency is increased, for a given rotor, altering the amplitude to keep the energy content similar to that in a standard nonoscillating gyro, it may be shown[a] that rupturing stresses for any conceivable rotor shape or substance are reached at fairly low frequencies (such as 25 cycles per second). Now it may not be essential to maintain a high energy content, since ultimately angular deflection only is sought, and friction should be almost completely absent. Thus, in accord with the dimensional relations well known to strength of materials engineers, a very small, high-frequency gyro oscillator might turn out to be practical. This is even more attractive, since then the whole unit might be sealed up *in vacuo*, there being no bearings or other maintenance needs, much as with a vacuum tube. It is apparent that the measuring system might be the limiting feature, at least for the wire-type strain gauges, but there is no fundamental reason why such other means as capacitor gauges, where reasonably high frequencies might be locally at hand in the resonant drive, or even electronic or ionic detecting elements, might not work out. Since the direct measure-

[a]This whole question was discussed with W. H. Howe, Research Director of The Foxboro Co., and some of the considerations here expressed are due to him.

ment of absolute angular rate appears as such a universal need in modern aiming controls, and more generally yet for high performance navigation, it might well be equitable to underwrite an advanced development of this sort.

Since the earlier discussions of this proposal, we have heard that somewhat similar experimental attempts, here and abroad, have met at least with partial success. For our part, we have done as yet no experimental work on this topic whatever.

3.8 CENTRIFUGAL TENSION AS A CRITERION FOR MEASUREMENT

The centrifugal governor responds to rotational speed with respect to the foundations of a machine. This does not generally annoy the user, since the rotation of the foundation, or rather of the earth, in space is relatively so small. However, if such a governor controlled the speed of an alternator and were exclusively relied on to maintain the accuracy of clocks, a given installation, if it made clocks keep time at the equator, would run them fast in one polar hemisphere and slow in the other.

Whereas the forces on a single point mass depend on linear acceleration and gravity as well as on purely centrifugal effects, the following arrangement will isolate rotation as such. Imagine a pair of equal masses joined by a weightless and inflexible rod. Neglect the insignificant gravitational attraction between them. It is not difficult, then, to demonstrate that the total tension f in the rod, reflecting the mutual separative effort of the masses, will be proportional to the square of the absolute rate ω at which the connecting line rotates in space. It will also be proportional to the magnitude m of either mass and to the distance a —assumed constant or very slowly changing— between the centers of gravity. Actually, in consistent units,

$$f = ma\omega^2.$$

It is evident on the one hand that this tension can never be negative, and that consequently one cannot thus distinguish between rotation in a given sense and rotation in the opposite sense. We are dealing with a signless affair. Furthermore the *entire* angular rate, in the sense of Chapter 1, is measured, the system behaving like a captive gyro of the second kind in which the only available reading is that given by the sum of the squares of the two orthogonal angular rate determinations. *Components* of angular rate cannot be extracted directly. This feature is a drawback for certain applications, where, for example, the axis of the meter is used to track a given arbitrary direction, and owing to imperfections always present in the tracking operation the axis executes minute wanderings out of the principal tracking plane. In such a case the mass-pair meter would read high owing to the instantaneous and inseparable contribution from the perpendicular mode. For other conceivable applications, however, this property might constitute an advantage.

The chief disadvantage of the proposed mass-pair absolute angular rate meter is that its sensitivity drops off very rapidly for slow rotation. Thus 0.001 pound would be obtained, with one-pound weights one foot apart, for an angular rate of about 10 degrees per second. This is easily measurable. However, for 1 milliradian per second, which is significant in many branches of fire control, only about 0.00000003 pound is exerted (10 micrograms or so). This is best measured in the laboratory, at least under present day circumstances.

Of the several methods which were considered for instrumental utilization of this principle, only two were mentioned more than once or twice. One of these involved a pneumatic capturing system involving ten pressure cups and a pneumatic double bridge whereby the difference between two pressure differences could be handled. While probably feasible, the extreme requirements on precision of machining and adjustment, and the care which would have been essential in guarding against temperature gradients in such a component, appeared to prohibit the expenditure of time and effort. Another method, employing specially fabricated flexures and capacitive detection, was thought to be practical from the standpoint of machining, but the temperature problem would have been equally difficult and many second order electrical effects would have en-

tered into consideration. Furthermore, the balancing of the mechanical characteristics among the several flexure supports would have been critical in the extreme. No actual work was done on this type of meter beyond analyses and computations. The only real appeal lay in the essentially motionless nature of the physica arrangement, without spinning parts or rotating fields.

If such a general proposal is ever considered again, and it may be because it is fundamental (we refer to centrifugal tension rather than any particular mechanism), it may be worth while to consider a feedback from current to magnetic force, employing the square law between current and flux. The current might then respond approximately linearly to the absolute angular rate.

3.9 POSSIBILITIES FOR THE FUTURE

As has been pointed out above, there appears to be great likelihood that the captive type of rate meter will come into widespread use for many purposes formerly filled by free or semi-free gyros. This likelihood is deduced from the advantages in flexibility, and the inherent opportunities for improvement which have been mentioned above. The tendency also should be toward "tighter" capturing procedures. Even now a good, standard, absolute angular rate meter of reliable and high performance would probably find an easy market. A further indication is that such meters can decrease impressively in size.

A somewhat different kind of prognostication along these lines is that angular acceleration will find more extensive application as a fundamental input. Although (absolute) angular accelerometers are not common components, such instruments may be constructed to extremely high sensitivity, speed of response, and tenacity to calibration by modern methods. Similar remarks may be made on linear accelerometers, although these are better known and are already considered essential components in certain important aiming controls.

In the typical stabilizing device, the automatic control of acceleration to zero would serve as well as that of angular rate, if the control were sufficiently delicate and of the proper dynamic characteristic, and where the regime of control is continuous. This is true since either general type of device must use long-term guides based on such available landmarks as North and Down, however difficult it may be instantaneously to fix these latter directions. Witness the directional gyro used by the pilot. Now we do not recommend that *this* instrument, for example, be replaced by an acceleration-controlled body, but indicate only that for more complex structures, where servomechanisms abound anyhow, the angular accelerometer would be at least as good as the free gyro or rate meter as a fundamental component. And ultimately it could be smaller and cheaper. For control terms, integrations of the acceleration may be performed when desired, and for this purpose the operations may be approximate. Precision integration of absolute angular acceleration, for more elaborate purposes, depends of course on a practical integrator, which will certainly come but is not now generally available. Again, similar remarks apply to linear acceleration. Note that the centrifugal angular rate meter of the last section may be considered a differential type of linear accelerometer.

The important feature of absolute acceleration, whether of angular or linear motion, as a fundamental measurable, is its universal availability in pure form. Fundamentally, this is attributable to the almost perfect constancy of the property of inertia (both ordinary and rotational) in available physical bodies, and to the basic measurability of force and torque.

Chapter 4

SIMULATION AS AN AID IN DEVELOPMENT

4.1 THE PHILOSOPHY OF MODELS

So much has been written on the applicability and on the potentialities in general of physical models that it is hardly proper for the writer to give here a comprehensive exposition of such techniques, although it should be added that he would not find it difficult to articulate his enthusiasms for this extremely broad branch of technical theory and practice.

Although in this chapter the methods of electronic models, or simulators, are principally to be described, model building and model manipulation range over almost all physical media. From one standpoint, most mathematical machinery may be considered a class of such models, since there must be embodied therein a physical system obedient to the same laws which apply in the case of whatever other systems the machinery is employed to study. The differential analyzer, in this sense, and we refer to the now familiar form of this weapon, may be thought of as a synthesizer, or flexible model, as well as an *analyzer*. This question of names is a controversial issue, involving definitions rather than anything more fundamental, and is most happily resolved by recognition that the equipment under discussion is really a bridge between analysis and synthesis, bringing these two essential modes of study into closer collaboration. Specific models are more prevalent, wherein problems of a particular category only may be studied. These may be simple replicas of other physical systems, retaining even the geometry and the appearance thereof, but by a transformation of one or more of the coordinates, of space, time, energy and so on, yielding a means for experimentation in which certain limitations are removed in comparison with the original systems. The well-known principles of similitude and dimensional anaylsis are guides for the construction, operation, and interpretation of models of this sort. In fact the so-called *pi theorem* of dimensional analysis has even been taken to mean that the construction of a representative model is always possible, although it is generally held that the strictly mathematical form of the theorem is to be preferred in any venture short of philosophical reflection.

We shall deal with models of the dynamic variety only, which thus involve time as a principal variable. Furthermore the transient state is of greater interest here than either the steady state or the static state, as in *statics*, although these conditions of equilibrium are attainable in, and may be studied by, the more general transient-bearing systems under consideration. Quite commonly time in an original physical system is reflected as time in the model, albeit with a scale factor which may range from a small fraction to a large number. The interchange of some other dimension for time, or of time for some other dimension, may also be arranged for, and time may be eliminated altogether in a model, or again employed in cases where it did not occur as an important factor at all in the original. Time, as do all variables, appears as an angle in a mechanical differential analyzer. In the electric and electronic models to be discussed, time is generally preserved intact, although stretched or shrunk beyond recognition.

The most impressive models or synthetic representive structures are those in which one physical medium acts in place of another, operating thus by virtue of one or more of the many analogies which are demonstrable among components involving the known physical media. Of such analogies the better understood are those among mechanical, electrical, and thermal processes, where we include under mechanical processes such variants as hydraulic and pneumatic ones. All such processes are self-analogous under the duality transformation, with the interchange of potential and kinetic energy. With regard to energy the thermal case is exceptional in that the usual analogy which is established makes electromotive force, for example, correspond to temperature,

both quantities acting as *potentials*. This analogy leads logically to the identification of quantity-of-heat with quantity-of-charge, of which only one is truly energy. The preservation of energy, however, in such postulation of analogous correspondences in model techniques, is by no means essential. It even forms a restriction in scope. When several media combine, however, in a model or in any useful system to be studied, then it is of the greatest convenience to employ analogies which preserve energy.

A distinction must be clearly emphasized between the various model structures, synthetic and simulative devices, and physical representations (which we shall henceforth refer to, for the most part, simply as *simulators*) which are constructed and employed in the laboratory as tools of research and development, and those which are intended for applications wherein an operator deals with the simulative equipment as a substitute for another apparatus represented thereby, and by such dealings familiarizes himself with the workings of the original apparatus under conditions which are relatively easier, cheaper, or safer. The employment of synthetic trainers in fire control applications is a good example of the latter category of usefulness, and is possible whenever the effectiveness of a given man-machine interaction is important in the operation of an aiming-control system. Generally even an approximate simulation of the dynamical relations, with which the operator must associate himself in such circumstances, is sufficient for his indoctrination or for the perpetuation of his skill through practice. It is a common experience, however, for a simulator which has been developed for the first-mentioned purpose, for purely laboratory purposes, that is, in the exploration of new or proposed physical arrangements, to find incidental or ultimate application as a training device. In some such cases there has been confusion over which end was being served by a particular equipment; and not infrequently a trainer, in which certain approximations had been allowed as inconsequential to the needs of that function, has been misconstrued as presuming to embody the detailed characteristics of a complex fire-control system together with the relevant relationships among combatants, projectile, and the geometry of space. The possibility of such misinterpretation, which might appear trivial from a larger viewpoint, has been of very real importance in several developmental endeavors. It is worth guarding against even in the makeshift operations which are typical of an emergency. A sharp line should be drawn, whenever more than a single homogeneous group becomes involved, between the laboratory simulator and the trainer into which as a separate issue it may be transformed.

An intermediate category must now be mentioned, at the risk of diffusing the dividing boundary already indicated. Simulators may be constructed, in a form more permanent than is the case for the study phase in research, for the express purpose of teaching what is already known of the dynamics of a given physical system. These are not trainers, although they may certainly impart to the user a facility in carrying out such operations as the adjustment of parameters in an automatic apparatus to give maximum performance and stability. A simulator of this type might be called an *educational simulator*. In typical circumstance it may also stem out of the availability of a *developmental simulator*, as in the case of the *training simulators* already discussed. In this report an effort shall be expended to distinguish faithfully among all these separate types.

Finally, we cannot conclude this general discussion of models, of which our simulators are only a special case, without citing the purest model of all, namely the medium of the mathematician. The symbolism of variables, functions, operations, and equations, taken in company with the rules which they follow, form what is almost the ultimate in flexible models. Thus the symbol for a variable is truly the analog of its physical embodiment, an equation of some physical truth which is stated or proposed. The manipulation of the physical model is directly reflected in the manipulation of the mathematical model, or of the symbols belonging thereto. Thus it is not surprising that an underlying standard for the sufficiency of a model is aptly provided by comparing the equations of model and prototype. To go a step further, consider the wave function of the math-

ematical physicist. There he has a model without tangible physical counterpart, but which describes in some detail a complex relationship having properties which agree with ascertainable data on the "unknown" physical entity. Why ask for more? Prediction is all that counts, ideally. But this is already too far afield. We should soon be discussing words and langauge as models (which of course they are).

4.2 BACKGROUND FOR ELECTRONIC REPRESENTATION

Before turning to the use of vacuum tubes in simulative applications, we shall dispose here of several items which will not be dealt with exhaustively but which will serve to define the general field of investigation. These items shall also not appear, in any detail, elsewhere in the present report.

First, as an example of a model structure of the mechanical variety, there is the so-called pursuit-collision course plotter.[a] This plotter, which is a model of the purest geometrical sort, was needed to study the approach paths which might be obtained with the type of toss bombing equipment known as the DBS. A gyro method was involved wherein a mirror in the optical sight was positioned through linear combination of a stabilized direction and the frame of the airplane. Relatively tight coupling to the airplane was being considered. It was of interest to determine the constants of such coupling so that the maximum allowance would be realized for target motion in the air mass. The problems being reduced to a single plane, the positions of the target and of the attacking plane were simulated by moving points on a board. Each point belonged to a system which had traction with the board through toothed wheels so geared as to establish the proper ratio of velocities. The markings of the teeth on paper provided a calibrated record; the direction of motion being provided by the orientation of the tractive wheels, and a tangent to the airplane path being periodically determined by recording the position of an extending member along the orientation of the corresponding wheel. The latter orientation itself was continuously determined by linkage so as to turn, with respect to the established coordinate system, in adjustable ratio to the turning of the connecting line to the target. This "simulator" found practical use in the application intended, and aided in the determination of parameters which, according to graphical tests on the records produced, yielded an optimum correction for the errors against which the general method was proposed.

A similar device was developed at Mount Wilson, for the study of plane-to-plane attacks, which undertook to represent pursuit and firing courses, and which latterly was modified to include an embodiment of aerodynamic skid. (A description of this work is available in OSRD Report No. 4737, issued under the sponsorship of NDRC Section 16.1.) The resulting apparatus is a second example of a purely mechanical simulator which has had application in airborne fire-control development and research.

In the immediately preceding chapter we mentioned the experimental study of tracking which was conducted chiefly at The Franklin Institute by Section 7.2. This research, which was entirely for flexible gunnery and dealt principally with "classical" tracking aids and disturbed sights, will not be described here in any detail even though simulative techniques were used in both mechanical and electronic phases. One reason for this reticence is that most of this work did not come under the immediate supervision of the writer; another and more plausible reason is that it will presumably be reported upon, as flexible gunnery lore, by others connected with Section 7.2.

The simulative electronics to be discussed as part of our research involvement in aiming controls includes studies on the control of guided bombs AZON, RAZON, and ROC (Chapter 8), and that branch of the PUSS project in which simulation of the dynamic responses to airplane controls was carried out. This latter effort was only beginning to attain a practical stage at the end of hostilities. We shall treat these separate topics in sections

[a] Proposed by M. Alkan and constructed by Section 7.2 through a contract with Stanolind Corporation in Tulsa, Oklahoma. See Prefatory Comments. This device was supplied to the Bureau of Ordnance, Navy Department.

below. First a brief discussion will be given of an earlier application of electronic simulation, made by the writer in connection with studies on automatic regulation at The Foxboro Co.

Beginning in 1938, the writer was authorized to construct a series of electronic models, proposed in detail by him, for use in research on desirable dynamic characteristics for the controllers and/or regulators of industry. Although it was not uncommon then to use equivalent networks for synthetic studies on the dynamic characteristics of such apparatus as thermal and mechanical, a number of new techniques had to be invented for the regulatory study. An arrangement was devised, using individually "fed-back" components, for synthesizing the whole closed loop under automatic operation. One component represented the system being regulated, and adjustments were provided for convenient alteration of the constants to allow the synthesis of a great variety of such systems. Another component represented the regulator or automatic controller under consideration, and adjustments afforded similar flexibility there, so that not only existing types but newer proposals could be conjured up rapidly. Provision was made for appropriately forming the dual connection between these components, and for incorporation of the disturbances against which the regulator must act. Such disturbances included sudden alterations in the desired value in addition to other equilibrium upsetting factors throughout the system; any type or degree of disturbance could be chosen at will. Repetition of the disturbance periodically, and display of a crucial variable of the loop on an oscilloscope against a time sweep in synchronism with the disturbance, afforded an apparently instantaneous picture of the recovery transient followed under regulation. Parametric adjustments had immediate effect on this graph, and could be continued until the desired stability and performance was in evidence.

Since this original work was undertaken, similar developments have been reported elsewhere. We quote, for example, from a (confidential) British publication.[7]

> The study and design of cyclic dynamical systems, such as servo mechanisms, is complicated by a variety of factors amongst the most important of which are the high orders of the differential equations of motion and . . . non-linearities . . . However, even when these non-linearities are small it is still a matter of considerable difficulty to decide upon the most desirable ratios between the various parameters of the system, and the choice between different types of internal feedback systems may have to be made on rather dubious premises, due to the severe analytical difficulties.
>
> The use of the electrically equivalent network of the system to be investigated provides the basis of a method which has proved to be most powerful in the resolution of these difficulties.
>
> Briefly, the method is to construct the equivalent electrical network, modifying it as may be convenient according to chosen scales of impedance and time, and to feed it with signals which simulate one feature or another of the normal "master system," denoting by this phrase the mechanism which the servo system is to follow. A feedback path is provided, so that the signals taken from it may correspond to some characteristic feature of the "slave" or servo mechanism. The two sets of signals, "master" and "slave," are then added by appropriate means. In order to facilitate the study of the results obtained, the input signals are applied and removed regularly, so that the system as a whole is taken repeatedly through a work-rest cycle, the rest period being chosen of sufficient duration to permit of the subsidence of any transients that may be set up. If the time scale used in the construction of the network is suitable, the period of the whole cycle of operations may be made small enough to allow of the representation of the results on the screen of a cathode ray oscillograph tube, in the form of an apparently stationary picture.

The above quotation might equally well have been taken from descriptions of the writer's proposals made in 1938, except for certain differences in nomenclature. From that time until 1941 he had constructed several such simulators and had applied them in the development of improved controllers and in the successful automatic operation of certain industrial systems. One of the most unexpected results was the use of the simulative equipment in the education of technicians who had to meet the problems of analysis, specification, and adjustment in the field. A session with the simulator substituted for a long interval of arduous experience with the practical problems of the mill and plant.

Of such a synthesizer or simulator, there are several significant methods of operation other than that described above, although that was most familiar to the practical worker, who saw the transients of the recorder charts right there on the scope. Frequency analyses could be run

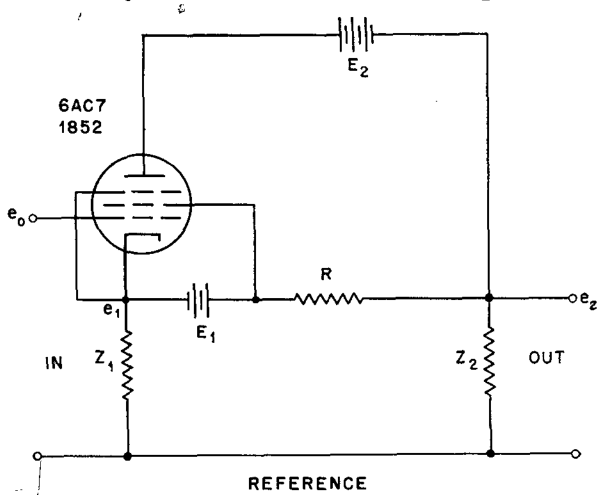
directly on the machine. Related variables could be plotted directly against one another, eliminating the time variable. Random disturbances, rather than simple periodic ones, could be imposed; this was revealing owing to their broad energy content over a wide band of frequencies. For this purpose ordinary tube noise was used. The writer naturally sought to apply similar techniques when, beginning in 1942, he met analogous problems in the design of aiming controls. That this applicability was not illusory is attested to by the several war projects in which useful ends have been served in dynamical studies by electronic simulation. Certain of the tricks employed earlier for regulatory studies have not been applied in the latter work, and the writer is rather eager to return to those studies, for servomechanism research and other applications, using the improved electric and electronic components more recently available. Somewhat more generally, it is felt that electronic simulative techniques will find growing and widespread usefulness for many recondite questions, both practical and academic, owing to the broad powers of representation embodied therein and to the speed with which exploratory manipulation may be performed in the laboratory.

4.3 FEEDBACK AMPLIFIERS [1,2,3,4,6]

The application of such amplifiers to simulative developments forms one of the most essential techniques in that art. Applied thus, the stability of the feedback circuit becomes of primary interest, leading to the remarkable situation in which stability studies as such are enhanced by the use of systems involving a set of feedback amplifiers, in the individual design of which such stability studies may be invaluable. But such a circumstance is not uncommon in research, where the talents of the detective, and dispassionate reasoning in general, find limitless opportunities.

It will be assumed here that the reader is familiar with the standard passive networks and with the corresponding dynamical systems which may be represented thereby without the necessity for feedback methods. Of the standard circuit elements only the resistor and the capacitor need be used in electronic simulation, and it is fortunate that these elements are available commercially in relatively pure, or "ideal," form.

There are two principal kinds of feedback in electronic circuits, which with their variations and mixtures have fundamental importance in simulative circuits. *Cathode feedback* and *plate feedback*, so called, comprise these two basic types, the names referring to the branches of the tube circuits in which retroactive operations are made to occur. Alternative terminology identifies these as *current feedback* and *voltage feedback*, respectively, in recognition of the electrical mode which acts most predominately in the two cases. In Figure 1 is



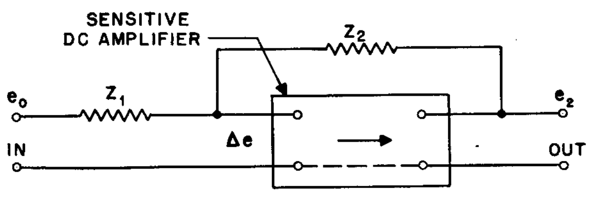
$$e_2 \doteq -\frac{Z_2(p)}{Z_1(p)} \cdot e_0$$

FIGURE 1. Cathode feedback circuit.

shown the fundamental cathode feedback circuit, or *cathode follower* adaptation, which was earlier employed by the writer in simulative assemblies. The inclusion of such material here does not imply that it carries classification. Much of the earlier work has been public knowledge, albeit not widespread. The writer had prepared and may publish descriptions of such work as applied to unclassified categories of engineering. A brief explanation should suffice. Since opposing currents circuits are produced in the impedance Z_1 by the sources E_1 and E_2, the voltage e_1 tends to follow the input voltage e_0 owing to the effect of the difference of these two voltages on the plate resistance

of the tube. For proper choice of tube (preferably of high transconductance) and circuit constants, the error between e_0 and e_1 may be kept very small, provided of course that stable operation is attained. Thus e_0 may be "followed" very closely by e_1. Whereas e_0 may occur in a low-power-level circuit, and may be surrounded by high impedances, the very high grid resistance at which the present tube can be operated insures the relative noninterference with e_0 in its natural environment. Furthermore, power may be drawn from the circuit at e_1 since the feedback works to maintain this voltage in spite of applied loads, within considerable limits. Thus we have an important tool already, namely, an *isolating* or *buffer amplifier*, with unit gain. If now no current is drawn externally either from e_0 or e_2, and the sources E_1 and E_2 are exclusively involved in the present circuit, then the current in the impedances Z_1 and Z_2 must be equal. Thus $e_2/e_1 + Z_2/Z_1 = 0$, or approximately $e_2/e_0 = -Z_2/Z_1$. Now if e_2 is repeated by a subsequent buffer amplifier, a means is provided by the representation of many operational characteristics. If in particular the two impedances are simply equal resistors, a *polarity reverser* or *phase inverter*, so called, results. For integration with respect to time, Z_2 may be a capacitor and Z_1 a resistor. Conversely, for differentiation with respect to time, the roles of resistor and capacitor may be reversed. The generalizations which are possible may easily be imagined. Initial conditions are imposed in straightforward ways, although certain ingenious processes have become useful. In the above integrator, the lower limit of integration may be established at any time simply by imposing a momentary short across the capacitor. A typical simulator may comprise a number of such feedback components, interconnected either directly or through the appropriate passive networks. In this procedure, criteria such as that of *impedance-matching* may be forgotten. Addition, or the formation of linear combinations, for two or more voltages, and hence of the variables which these voltages represent, may be carried out in a number of ways with the basic circuit of Figure 1. For example, several such feedback circuits may be arranged to possess the impedance Z_2 in common. Or again, these voltages may be connected by a high-resistance dissipative network in such a way that e_0 is proportional to the desired sum or linear combination, although care must then be taken not to load the previous systems. For subtraction one may always phase invert an odd number of times. The only failing of the cathode feedback circuit, as shown, is the necessity for at least one "high" battery (E_1). The anode source (E_2), of course, must also be "high" unless Z_2 is zero, as it may be in the simple isolating amplifier, for example. Thus each such circuit must have at least one voltage source to itself. This requirement, however, has not been found too great a burden in the laboratory, since the drain may be kept quite small. The writer used a large bank of dry batteries for this purpose, in his own work, carefully shielded to prevent capacitive interstage coupling, and found it adequate to replace them every two years. Modifications are possible which will permit the adaptation of such circuits to single-source powering, but such sources must be meticulously regulated to present zero impedance and hence zero coupling between the using channels. Furthermore, such modifications complicate the circuits so much that plate feedback might as well be used. We now turn to plate feedback.

A common plate voltage source suffices for the plate feedback circuit, shown in Figure 2,



$$e_2 \doteq -\frac{Z_2(p)}{Z_1(p)} \cdot e_0$$

FIGURE 2. Plate feedback circuit.

if the high-gain d-c amplifier which is involved (see Figure 3) is properly designed. In this connection the term "high-gain" is employed in a relative sense only, as compared, for example, to the unity-gain *isolating* amplifier. (The gain figures for typical a-c amplifiers are not referred to in this context.) Only one such

design is here shown. As suggested above, the supply sources should be well regulated. Referring to Figure 2, it should be remarked that the function of the amplifier shown there in the block is to convert the small balance voltage Δe into a corresponding but much expanded variation in the voltage e_2, and to reverse the sign. The effect is to keep the voltage Δe very near zero. As above in the current-feedback circuit, the point of operation (for the first tube) is so chosen that the grid input impedance is very high. Thus the currents in the impedances Z_1 and Z_2 are substantially equal. Hence as before, $e_2/e_0 \doteq -Z_2/Z_1$; and e_2 may be loaded within reasonable bounds. The operation of addition on several variables may be carried out, as in the second method mentioned above for the cathode feedback circuit, by substituting a number of resistive paths for Z_1, Z_2 being also resistive, and each such path initiating at one of the variable voltages to be summed or combined. The coefficients of combination are adjustable by the separate connecting resistances.

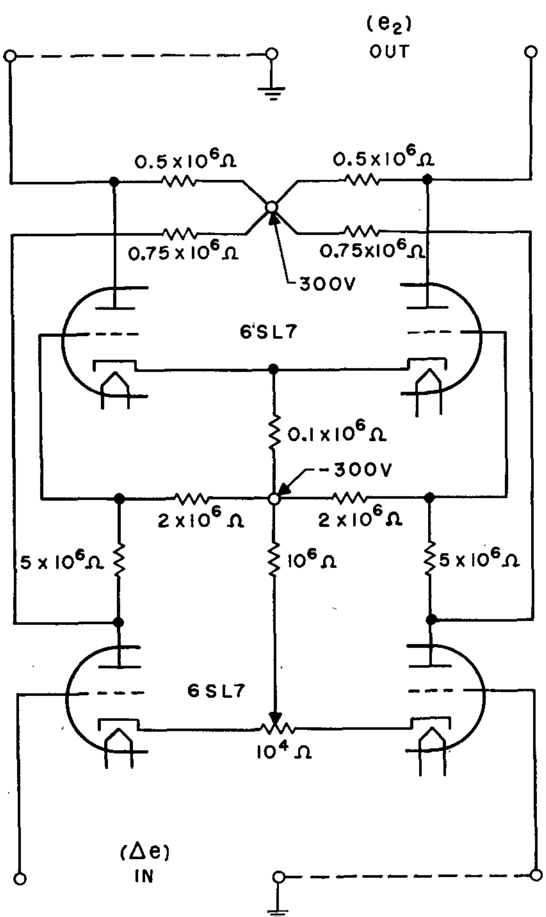


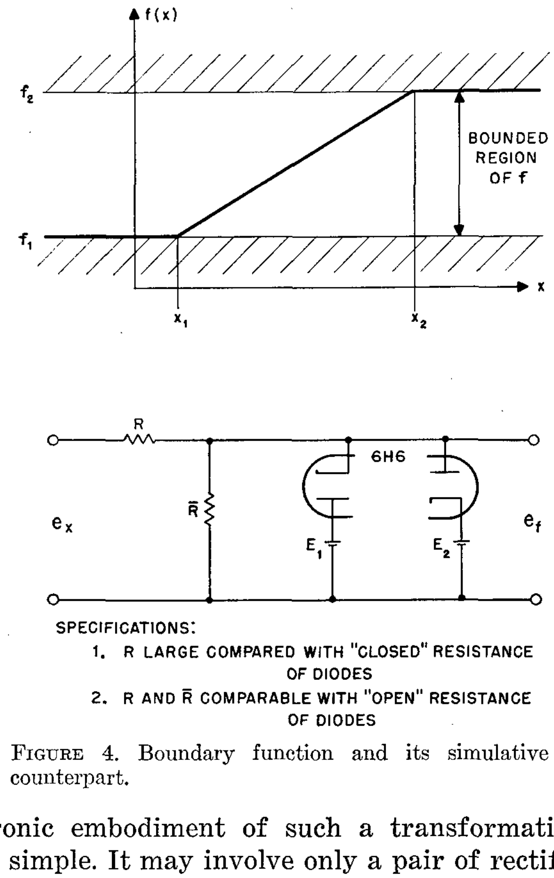
FIGURE 3. Type of amplifier indicated in Figure 2.

There are a certain number of feedback applications which do not fall into those types above classified. For example it is possible thus to provide circuits which have extremely long time constants, or long "lags" of low order, for special purposes. Such time constants are well in excess of those obtainable passively, even with the largest resistance and capacitance values available in the laboratory. Representation of the various kinds of servomechanisms or degenerative "telemeters" in prototype systems, as is to be expected, may be accomplished[1] quite straightforwardly by electronic feedback components in the process of simulative construction.

4.4 OTHER SIMULATIVE COMPONENTS AND COMBINATIONS

The amplifier circuits already referred to enable the experimenter to prepare, for his edification, extremely flexible models of dynamical systems. By simple interconnection, he may readily assemble the counterpart of any physical entity governed by a linear differential equation, of finite order, with constant coefficients. But he is not thus limited, although within this realm lie many significant dynamic relationships of sorts now submitted to analysis. For the larger class of linear relations represented by equations with coefficients which vary with the independent variable, here time, a programming of the adjustable parameters suffices. Similarly nonhomogeneous relations, with prescribed "forcing functions" (unfortunate phrase!), are representable. The greatest advantage of this kind of simulation, however, and the crux of its power, in relation to competitive methods, is the ability in simple manner to incorporate nonlinear dynamics. We need not speak of the analytic difficulties which are there involved, nor of the physical importance of nonlinear systems in general. Suffice it to say that the resulting problems preoccupy and harass some of the best available mathematical brains.

One very simple source of nonlinearity is the presence of a mere boundary in the range of a variable. Many examples may be cited, and may amply be supplied by the reader. Probably the simplest examples are furnished by transformations from a given variable to a function thereof which is either nonincreasing or nondecreasing over the entire range to be considered, and in which for at least certain portions of the range the function is unchanging. An instance is shown in Figure 4. The electronic embodiment of such a transformation is simple. It may involve only a pair of rectifying diodes, as contained for example in a 6H6 tube, so biased that voltages corresponding to variation of the first variable beyond the bounded interval produce no further change in the function of, or the transformed version of, that variable. Such an arrangement, shown also in Figure 4, must always be approximate, but the approximation may be refined to any desired degree by appropriate choice of the electrical circumstances. Only one type of nonlinear component is thus illustrated; it is evident, however, that by combination of such circuits a great variety of similar, and more intricate, relations may be embodied. Nor are we restricted to functional relations representable graphically by a single curve, or analytically by single-valued dependences. Such physical relationships as *inertness,* when for example there is *backlash* or *play* in a mechanical connection, may be simulated through the application of biased rectifiers in circuits where charge may be stored over reasonable intervals. In general, the inclusion of major nonlinearities of this sort alter fundamentally the stability considerations in control systems or in any apparatus where the dynamic performance is of interest. It is thus of great importance to be able to treat such nonlinearities without the endless drudgery which is essential when straightforward analysis is the only recourse, however otherwise educational or character-building such a process may be for the performer.



SPECIFICATIONS:

1. R LARGE COMPARED WITH "CLOSED" RESISTANCE OF DIODES
2. R AND $\bar{R}$ COMPARABLE WITH "OPEN" RESISTANCE OF DIODES

FIGURE 4. Boundary function and its simulative counterpart.

A useful tool in simulation, for that general class of dynamics, particularly, in which nonlinear relations are included (it might have been more appropriate earlier to have defined nonlinear systems as those in which the *additivity* or *superposition* principle does not apply), is the follower or servomechanism which transforms voltage into a corresponding resistance. Consider first the case in which proportional correspondence is enforced. Many such components have been built and applied in the simulative ventures we have dealt in. The usefulness of such a properly made component, which is great, is not at all limited to the representation of nonlinearities, but appears wherever the automatic manipulation of a resistor or potentiometer may be desired. The construction may be very neat and simple, and high speed and precision has been shown fairly easy to attain. Naturally the time constants of the response should be well below those which are involved, purposely, in the remainder of the simulative channel in which such a component occurs; it has been satisfactory to employ pairs of circular, wire-wound potentiometers driven by small d-c motors energized either from polarized relays or from vacuum tube circuits. The error signal, which is supplied to an interpreting network and thence

to the motor circuit, is derived both from the input voltage and from the voltage division of one of the driven potentiometers. The other potentiometer, driven for example in tandem with the first, provides a free resistor having resistance proportional, say, to the input voltage. Before turning to some applications of the proportional form, we merely mention that the introduction of any appropriate function may be accomplished, through the voltage-to-resistance follower, by embodiment of such function in the resistance versus motion law of the final resistor, or of its reciprocal or inverse in that of the follower-resistor itself.

A means for rapid conversion of a voltage into a proportional resistance enables the realization of many flights of fancy. We record only the soberer of these possibilities. It will be evident that certain of these may also be effected by logically simpler means, but we note that there is large advantage, for such purposes, in the repeated application of a given type of basic component.

Having a feedback amplifier, of either type above described, with the associated pair of impedances restricted to a pair of resistances, then it is possible, by making either such resistance proportional to a separate external voltage, to perform multiplication or division among voltages in various ways. The resulting product or quotient has also the form of a voltage. While those variables which are reflected in the resistance of either resistor pair cannot normally be made to take on negative values, or in certain cases even zero, modifications are possible whereby equivalent variation can be permitted. A general case is given by that in which the input voltage of the feedback amplifier is employed together with those which, respectively, set proportionally the "input" and "output" resistances thereof. Here the output voltage is proportional, through any desired factor of scale, to the ratio of the product of the first two voltages to the third voltage. Simple products or ratios, as well as reciprocals, may then be evaluated in a single component. Through duplicate roles, for example, a voltage, or rather the variable thereby repsented, may be squared through multiplication into itself. By including such an arrangement within a more comprehensive feedback loop, inverse operations such as the extraction of the square root may be performed. Out of necessity, we are speaking here only of *real* variables. With combinations of similar components, in series or cascade, it is evident that such delicate operations may be carried out as the raising of a variable to any fixed rational power. This type of conversion, of course, may be accompanied by any (otherwise) linear dynamic operation of the classes already mentioned.

Consider now integration with respect to variables other than time, or in general with respect to an arbitrary independent variable. An arrangement was proposed[b] for this purpose, but was not constructed practically for a variety of reasons, principally because no need persisted for it which was not more easily satisfied by other means. Consider a time integrator as described, with fixed resistive and capacitive elements, and let it be supplied as input with a voltage which is the product, obtained as already indicated, of a primary voltage e_1 (preferably, in this case, always of one sign) and the time derivative, evaluated in the regular feedback manner, of a second input voltage e_2. The output voltage e_3 of the time integrator is thus the integral of the voltage e_1 with respect to the voltage e_2. Thus

$$e_3 = \frac{1}{RC}\int_{t_0}^{t} e_1 \frac{de_2}{dt}\,dt = \frac{1}{RC}\int_{e_1(t_0)}^{e_1} e_1\,de_2\,.$$

Within certain limitations, of which it is not difficult to imagine the removal, and with precision of the order of 0.001 to 0.01, it is evident that this more general type of integration is feasible, and the potentialities of electronic simulation are thus extended to the dynamical realms of the better-known *differential analyzer*. Much practical development, however, remains to be completed along these lines, although only the fundamental elements already treated need be involved.

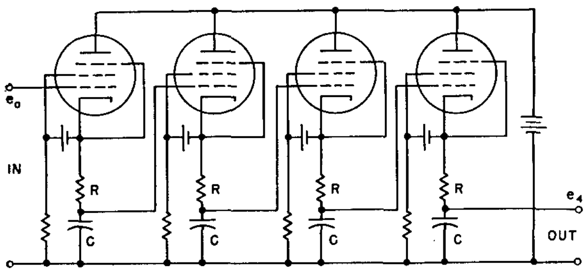
Returning momentarily to linear systems, with the continued implication none the less of nonlinear generalizations and ramifications,

[b] We should like to acknowledge the collaboration of Loebe Julie in connection with this item, and also with certain other conjectural plans for electronic simulative measures which are referred to here.

we mention two types of assembly which are of interest in simulative structures and which are singularly adapted to electronic techniques. One is the approximation of pure time lag, or of a direct shift of a function along the axis of time (or of another independent variable, by extension of the method), which may be achieved through recognition of the limit

$$\lim_{n \to \infty} \left(1 + \frac{T}{n} p\right)^{-n} = e^{-Tp}.$$

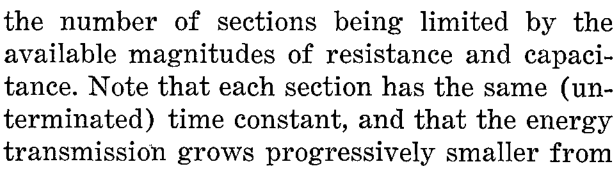
Several methods are available, each using a chain or cascade of feedback circuits to prevent the passage of energy except in one direction along the chain. A possible circuit for $n = 4$ is shown in Figure 5, where cathode fol-



$e_4 = (1 + RCp)^{-4} \cdot e_0$

FIGURE 5. Cascade of cathode followers as lag.

lowers (reference Figure 1) are employed. Naturally any number of such sections may be used, and the larger the corresponding value of n, the closer the approach to a pure time lag. A useful approximate network for cases when small n suffices is shown in Figure 6,



$e_4 \doteq (1 + RCp)^{-4} \cdot e_0$

FIGURE 6. Passive approximation to Figure 5.

the number of sections being limited by the available magnitudes of resistance and capacitance. Note that each section has the same (unterminated) time constant, and that the energy transmission grows progressively smaller from input to output.

Another useful assemblage of the elements noted above may be used to synthesize any linear relationship which may be adequately described by a rational function of the derivative operator, we assume for simplicity with respect to time. Such an operator is

$$R(p) = \frac{\sum_{i=0}^{m} a_i p^i}{\sum_{j=0}^{n} b_j p^j},$$

where it is understood that $p^0 = b_0 = 1$. For purposes of exposition assume that neither m nor n is greater than 5. Fractional differentiation and fractional integration are not considered here. Distributed systems are essential to the precise representation of such operations, but again approximations by lumped systems of low order not infrequently suffice. We make use, for the simulation of $R(p)$, of the well-known operational characteristic of a feedback system. The denominator is obtained by feeding back a linear combination of the branch outputs of a chain of differentiators, the input to the first differentiator being made the difference between an input voltage and the given linear combination. We obtain thus, familiarly,

$$\left(1 + \sum_{j=0}^{5} c_j p^j\right)^{-1}$$

as the operator describing the relation borne by the input difference to the input voltage. The separate unity term may be eliminated by setting $c_0 = 0$, thus reproducing (with the equivalence $b_j = c_j$; $j = 1, 2, \ldots 5$) the earlier denominator. The numerator, and the desired total characteristic is now created merely by taking a new linear combination from the same chain of amplifiers. Figure 7 shows the assembled system, in which plate feedback systems, of the type shown in Figure 2, are employed. The coefficients of $R(p)$ are of course directly adjustable negative values requiring the addition of a "phase inverter" in each such case. This arrangement, with appropriate means for insertions of forcing functions and for presentation, is extremely educational in the laboratory. A number of interesting modifications are possible. By division of both numerator and denominator by p^m or p^n,

whichever is the higher power, and whenever such an operation is rigorously permitted, we may obtain a rational function of p^{-1}, the definite time integrator, and proceed exactly as above except with the replacement of each dif-

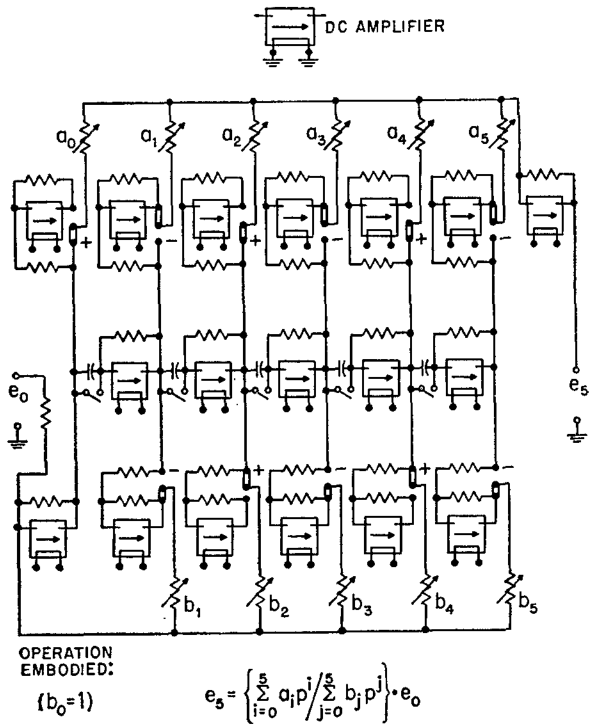


FIGURE 7. Simulative connections for class of linear operation.

ferentiator by an integrator. For many purposes this is preferable; for example it is easier then to establish initial conditions.

When certain variables in a physical system to be simulated are related by complex *static* functions, meaning by this term that time is not fundamentally involved, it may be convenient to represent such functional relations by a mechanical linkage rather than through a combination of electronic elements as such. Such linkages may involve any number of inputs, and may sometimes have more than one output. They are similar to computing linkages for full-scale aiming controls, as described in a separate chapter of this report. Angular motion is practically most convenient for the input and output variables, such motion being most readily provided, in electronic (in this case electronic plus mechanical) simulators, by the resistive follower which correlates mechanical angle with resistance, hence with voltage. Angular outputs of the linkage may conversely be accepted as resistance, since reasonable power is available from the feedback inputs, and thereby converted again into voltage. The result in effect is an extremely flexible form of "static" computer, entirely surrounded by voltages. By proper design of the linkages, in which the same engineering principles apply as with the more worldly types discussed elsewhere, high speed and reliability may be obtained. There have as yet, in our work, been few occasions for elaborate linkage components in this role, but the potentialities, say for intricate aerodynamic interrelations, are practically unlimited. A large attraction is the ease with which modifications in the nature of the functions may be incorporated, for example through alteration of link lengths and pivot positions.

It may be repeated that the general technique of electronic simulation has great flexibility with regard to the scale factors of analog versus prototype. The possible extremes of speed and sluggishness are relatively far apart. This applies, of course, only to the time scale; other dimensions are correspondingly adaptable. Further as to time, one of the principal fields of application for this type of simulation has been to systems involving a human element, and here the time scale must be religiously represented as it exists in the full-scale apparatus being represented. The man as a machine has nonadjustable time parameters, or rather he is the component which cannot be altered, except by a certain amount of learning, as a result of the research, and must therefore be accepted as is, whether trained or untrained. Since the remainder of the model must be adapted to him, particularly with regard to time scale, it is welcome that electronic structures find little difficulty in being thus adapted. Of course in any such case, with a human "component," many other provisions must be made to achieve naturalism.

4.5 SIMULATION OF FLARE-BOMB GUIDING

The projects connected with the development of aiming-control systems for the phorodromic and pseudophorodromic guiding of the bombs

AZON, RAZON, and ROC are discussed more generally in other parts of this report. Electronic simulation, however, played a crucial role in these projects, as in others, and it is considered appropriate to treat here the several such simulators which were constructed and employed by us in this work.

As a guided bomb of this general type (we refer to what are called *high-angle* bombs) fell toward the target, the guiding operator was instructed so to operate the controls at his end of the radio link, from above in the bomber, that either the flare of the bomb itself or an image thereof synthetically produced was held superimposed on the selected target beneath. This operation was one- or two-dimensional, in dependence on the class of missile and on the preferred mode of operation. Accuracy of the bomb fall depended heavily on the accuracy with which the above superposition was carried out, at the moment particularly when the missile arrived at the level of the target. Thus the stability and the other dynamic characteristics of the guiding process were of prime interest. Having a considerable familiarity with the space paths and guiding responses of such bombs (Chapter 8) through three-dimensional solutions which we had obtained from the new differential analyzer at MIT, and having seen the excellent correlation which was evident between these solutions and full-scale tests, we were in a good position to determine and to recommend the dynamical nature of electronic simulators which would represent the process sufficiently well to an operator in the laboratory. In the course of development a number of such simulative equipments were prepared, of various degrees of precision and excellence, and these were usefully applied to the research problems which arose and to the training needs which appeared in connection with full-scale tests. We shall include below a description of those features of these equipments which are of interest from the simulative standpoint.

In the case of each of the bombs referred to, the operator sends messages to it via the radio link by means directly of the angular motion of a handle in front of him. His means of *manipulation,* sometimes in one angular dimension and sometimes in two, is thus analogous both to the handlebars of a turret control, in flexible gunnery, and to the pilot's control stick or column, in fixed gunnery or in other pilot's fire control procedures. The handle in some cases operates merely a double-throw switch, and in others operates potentiometers, either one or two of each depending on the dimensionality of the control. Servo motors on the bomb are thus energized to move the control surfaces thereof. In the on-off case the surfaces have three stable positions toward which they proceed for continual occupation, by the corresponding individual handle switch, of any of its three positions. In this case the velocity of motion in proceeding from one such position to another is constant, and reverses if a new position in the opposite sense is ordered during the motion. The velocity of such motion is a property principally of the local bomb controls, and embodies an important design criterion, both for economy and for stability. On the latter question in particular, simulation proved to be an indispensable aid. To return to the manipulation, there is also a "corresponding" form of control, in which intermediate positions of the control surfaces are held for continually occupied corresponding positions of the control handle. This relationship is roughly proportional, handle position to control surface position, and is also approached substantially at constant rate in the transient intervals. The simulation of these circumstances is fairly direct. A standard control-handle is used as such, or duplicated, in the simulator, with either switches or potentiometers attached. Voltages thereby derived are followed by the voltages corresponding to the positions of the control surfaces through an electronic feedback precisely analogous to the bomb servos. The simulator circuit involves an amplifier as a sensitive relay, and an integrator to imitate the constant rate of the servo-actuators in the bomb. The latter being roll-stabilized in practice, no "phasing" controls were necessary, and either one or two operators could be involved either for the real proceedings or for the simulator in the laboratory.

We consider now the simulative presentation, by which the laboratory operator sees the re-

sult of what he is doing. For our purposes, cathode-ray tubes (referred to hereinafter as oscilloscopes or scopes) were employed in all cases. The luminous point on the screen, corresponding to the projected visual impression of the bomb flare, was observed to move in response both to the guiding operations and to disturbances, and to hover near, or in inexpert hands less near, a stationary spot corresponding to the target. In the case of AZON the target was typically a line, rather than a spot as for RAZON or ROC. Elaborations were introduced to promote realism. For example the surface of the scope screen was coplanar, in focus, with that of the image of an aerial photograph, so that to the operator the flare appeared to move over the landscape, as in the real operation. Thus he could pick out targets at will or under direction from a superior intelligence to which he was, or pretended to be, subordinate. Many games were thus possible for the promotion of guiding skill. Such further refinements were possible as the gradual diminishment of the intensity of the spot on the scope as the bomb dropped, corresponding to the weaker brilliance of the flare as the bomb receded below, with a momentary brightening at the end for the detonation. The variations are readily imaginable.

Between the voltages corresponding to the positions of the control surfaces of the bomb and those deflecting the beam of the scope in correspondence to the direction of the bomb from the bomber is incorporated the simulative electronic channels which represent the physical dynamics which obtain between these variables. The lateral acceleration of the bomb, in one or two directions, is nearly proportional,[c] within significant limits, to the deflections of the control surfaces, as well as to a function of the speed of the bomb and of its altitude. To a large degree it was shown adequate to neglect the angular dynamics of the bomb in response to the deflections, although representation of such phenomena is straightforward. For purposes of the stability study, and of the synthetic reproduction of guiding dynamics, it sufficed to consider the function of speed and altitude, affecting the spatial acceleration, as an appropriately chosen function of time, whereas in reality that function depended, although in a less serious manner, on the deflections themselves, via the drag function and the geometry of the flight path of the missile. Thus the deflection voltages, after multiplication, or division, by a selected function of the time since dropping, could be integrated twice with respect to time to produce voltages corresponding to the lateral motion of the bomb in the air mass. To obtain the direction of the bomb from the bomber, for simplicity assumed unaccelerated, it was only necessary to know the distance through which it had fallen, this being available quite closely, again as a function of the time. It should be evident how the components already described may be compounded to achieve this procedure. A programming device was allowed to introduce, as variable resistances, the time functions which entered. The independent effect of the distance to the bomb, or the *perspective* effect as it came to be called, was curiously impressive. If, in using the simulator, the bomb were left considerably off the mark in angle, and with no corrective guiding that is, its approach in angle toward the target as time wore on, owing to the simulation of decreasing parallax, was exactly as though it were a real bomb seen from a bomber, and was especially appreciated by those of us who had watched bombs go down in air trials. Further, it was a great comfort, for a different reason, to be able to guide them on to the target, but that is another matter.

An essential feature of the bomb simulators is the embodiment, in the sense outlined above, of boundaries on the variation of the control surface deflection, which ran actually against stops, and on the rates of motion of these deflections, which were limited by the capabilities of the bomb servo motors. The discontinuities thus involved were crucially important in the overall question of stability, and were incorporated in the simulators by means of biased rectifiers as already described.

Numerous auxiliary controls are necessary, for the facile operation of such simulators, but

[c]The proportionality, in the case of RAZON for example, is different in each direction, or in "range" and in "azimuth," so that a somewhat asymmetrical guiding response was employed in the simulators for a closer representation.

these may be straightforwardly provided. One must be able to start the operation precipitately, having set in the desired initial conditions, of wind, initial dropping error, and so on. At the end of the time of flight, it is convenient to freeze the bomb position for scoring, which may be accomplished by opening the capacitor circuits of the integrators, and then to return all adjustments quickly to the initial condition for the next "drop." The form of such auxiliary controls depends greatly on whether the simulator is for laboratory study or for the rougher needs of a training device. Experience was gained with both such applications, and fuller details may be found in the various contractors' reports. (See Division 5, Volume 1.)

4.6 SIMULATION OF TELEVISION BOMB GUIDING

Although the ROC bomb was, at one point in its career, experimentally equipped with a flare for line-of-sight guiding, its initial purpose had been quite different. Primarily a vehicle for radar-homing, ROC was also adapted to embody the miniature television camera and transmitter (MIMO), the latter having been developed for suicide bombing by proxy. The missile, with MIMO, was to be dropped rather naturally toward a target as a bomb, and subsequently to be guided thereto by a remote operator, over a radio link in one direction, who saw the view appearing to the forward-looking televising camera, which was transmitted over a radio link in the other direction. It is not intended here to describe this method or the ROC bomb, itself of great interest and significance, since partly this is a subject more nearly for the chapter on guided bombing and more particularly since much fuller reports are given by other agencies (Division 5 of NDRC, for example). The problems of guiding with which we were concerned, and for which we turned to simulative techniques of study, were problems of controls to give the optimum approach, in terms of the probability of hitting, and of the stability of the processes in which such controls were to be involved.

As before, a control handle is involved, both in the simulator and in the prototype system, but in this case it is exclusively two-dimensional and carries potentiometers for proportional manipulation of the bomb controls (although for certain studies a one-dimensional counterpart was considered and employed). The method of operation, otherwise, of the control surfaces over the radio link and through the bomb servos, is essentially as in the case of the normally phorodromic bombs. For these surfaces, the minimum runout time, from zero to full deflection, is longer for ROC than for AZON and RAZON, and it was partly of interest to determine the effect of this parameter on the stability of guiding. As expected, it turned out to be rather important from this standpoint. The scope presentation corresponds now to that on the screen of the television receiver. The target appears on both under the proper circumstances, in the one case as a single spot of fluorescence and in the other amid the landscape toward which the real bomb is heading. The lack of the illusion of changing perspective, in the simulator, is partly repaired by the preservation there of the increasing angular response observed, in external objects such as the target, in answer to the maneuvering controls, as the bomb continues on its course.

A second spot is presented on the simulator scope, differing geometrically from that representing the target, as the analog of a movable reticle which has been proposed as an aid in guiding. This proposal, which has common elements for the various detailed methods which have been considered, involved the manipulation of the control handle by the guiding operator in such a manner that the moving reticle was maintained on or near the image of the target. From the control handle proceeded thus two paths of influence. One to the bomb controls via the radio link; one to the moving reticle on the scope screen. The proposals for effective aids to guiding differed principally in the choice between these two influence-paths for inclusion of aiding dynamics, and in the nature of the latter dynamics wherever placed. A service was done by the simulative equipment in the choice of these dynamic control adjuncts.

Both the relative direction to the target, or *lead* angle, as represented by the departure of the target spot from the center of the scope, and the absolute direction of the target in

space, which is given also by the absolute direction of the bomb *from* the target, are simulated by varying voltages. The rate of turn of the bomb is taken as proportional to its lateral acceleration, and hence to the deflection of the control surfaces under guiding. It is also proportional to a function of speed and altitude, and hence adequately, in this case, to a function of the passage of time since release. Variation of the lead angle depends on the turn of the bomb, in the air mass, and on the rotation in space of the line of sight to the target. That is, there is a direct summation among these angles which may be simply incorporated in the simulator. In addition, the rate of rotation of the line of sight is proportional, additively, to the product of the lead angle into the bomb speed and to the lateral speed of the target, and is inversely proportional to the range. The latter, again, is taken as a function of the independent time variable. These relations are adequate, when incorporated in simulative components of the type described, to present both the lead angle and the absolute direction as voltages in response to the operation of the bomb controls, via the handle, and of course to the passage of time. Appropriate boundary circumstances are also readily included. Lateral speed of the target, in the air mass, can be set in at will, in any direction and to any desired magnitude.

It has been a simple matter also to include, in an independent channel from the control handle to the moving reticle, for example, whatever conjectural dynamical relations were proposed for effective guiding. Several such systems were thus represented. One proposed by the writer was shown to be successful, when proper initial conditions were observed, in leading to high terminal accuracy even in the presence of considerable target motion in the air mass. The principal point is that means were available, in the simulator, to evaluate the excellence or otherwise of all the various dynamic proposals which arose in development. Such assessment is not practical analytically, owing partly to the nonlinearity of the systems being studied. Although the reasoning is omitted in this context, it was found effective to transmit signals to the bomb controls which led them to deflect in proportional response to a linear combination of the lead angle, as observed on the scope or screen, and the rate at which the lead angle is changing. The constants of such combination depended on the conditions prevailing when the operation was begun. Simulative study showed that such conditions could be adequately predicted, that the constants could be consequently chosen to suit, and they were happily not critical, and that finally the dynamics of control suggested led substantially to an interception approach by the bomb to the target, under manual guiding, even in the presence of target motion in the air mass. These conclusions were later fully confirmed by large-scale tests, employing the control dynamics here conjecturally initiated.

The above dynamic relationship, between the lead angle and the instructions to the bomb-control deflections were actually embodied, first in the simulator and later in the larger equipment, by insertion of the inverse dynamics, essentially a *first-order* lag, in the connection from handle to moving reticle. Thus on the assumption that the target is tracked by the latter reticle, which thus reproduces the lead, and that the handle position directly transmits the proportional control-deflection orders, the dynamic connection from the lead angle to the latter deflection, as earlier specified, is effected. It is significant that the *collision* or *interception* approach was made possible in this case by a perverse form of "proportional navigation," without directional stabilization or space-rate determination in the missile. This was done by interpreting absolute turning of the bomb as being caused by the control surface deflections, which were continuously and directly knowable, and as being affected simultaneously and additively by the component of gravity, which was predictable through implicit embodiment in the dynamics of the control. A favorable circumstance for this sort of trickery, which is more straightforward when directional references or absolute angular rate meters are available in the vehicle itself, as seems signified for the future, a favorable circumstance, we repeat, is provided by the absence of rapid roll and of pitch and yaw in the body of the ROC bomb. This combination of properties was not fortuitously obtained, but was planned for and designed in.

In connection with assessment of the *miss* in guiding, meaning the minimum lateral distance of the bomb from the target, measured normally to the local part of the trajectory, this quantity is not easy to measure directly in terms of voltages in the simulator. A good dodge for this assessment, however, was conceived and installed in the Columbia Laboratory.[d] The derivative of the angular direction of the bomb was determined as seen from the target, this angle appearing as a voltage under simulation, at an instant when the bomb was still a short distance a from the target, and had yet τ seconds to get there. The ratio a/τ would be rather well known from the "terminal" speed v_1 of the missile. It is possible, thus, to evaluate the miss as the product of the square of the remaining range a into the above angular rate, divided by the terminal speed v_1, all these quantities being essentially constant, from one trial to the next, except the angular rate. Indication of the latter on a meter, then, provides a sensitive measure of error, and a quick and easy means for scoring, either in laboratory studies or for training purposes.

Cessation of hostilities brought this project to a stopping point in development, but it is still planned to prepare an altered version of the television bomb simulator for the needs of familiarization in the field, in connection presumably with guiding trials of the missile itself.

It is considerably to the point, with regard to the powers of simulation, that the equipment described above, through a mere change in connections and an alteration of adjustments, was employed in a stability study of underwater torpedo controls. This was pursued in our laboratories, during a brief interval, by Section 7.3 following a suggestion by the writer that a problem being studied by them would be amenable to such simulative methods, and that these methods would encompass the nonlinear properties of the controls, which were the source of some concern. The previous analysis had been based on linear theory. We mention the application only by way of indicating the flexibility of the electronic simulators which have thus come into use. It is understood that this particular study was satisfactorily concluded.

[d] By L. Julie.

4.7 SIMULATION OF AIRPLANE DYNAMICS

A much more ambitious project,[3,6] begun only recently, was for electronic representation of the complex dynamics which obtain between the flight controls of the pilot and the motions in the air mass of his craft. This work was connected with the PUSS project, under NO-265, and contemplated the inclusion also of simulative components representing the entire computing sight therein involved. Some remarks are made on this score in Sections 2.1 and 2.8. A fundamental consideration in the success of such automatic sights is the nature of the interactions among the tracking system, the computing and display system, and the gentleman at the controls. Owing to the relative newness of this project, to the fact that it is actually part of another project described elsewhere, to the expected continuation of this other project together with its simulative counterparts, and to the fact that a fairly complete set of reports have been and are still, at the present writing, being submitted by the contractor, we shall not discuss this enterprise here in any great detail. The simulative techniques are in no way essentially different from those of other projects which have been considered, although somewhat greater complexity was required; and certain refinements remain to be made before the apparatus is finally effective. It is now possible, nevertheless, to experience "electronic" flight in the laboratory, the whole illusion being accomplished by simulative components between the voltages incurred through motion of the mock controls, at one end, and the voltages which deflect the spots on scopes to represent target, horizon, and sight reticle, at the other end.

The Bureau of Aeronautics' Special Devices Division was consulted, at the inception of this project, in connection with a synthetic trainer which was then in development for single-seaters, and which surpassed, in the faithfulness of dynamic representation being sought, any previous like attempt. This trainer, which

was for combat approaches as well as for such other difficult operations as carrier landings, was to be supplied with a simulative computer under development by Ford Instrument Co. An ambitious display system was being planned for, of only partial interest for our purposes, which involved an overhead, inverted relief map in color and a periscopic viewing arrangement; extensible, rotatable, tippable and translatable. A somewhat complex optical system was to have been involved. As to the theory of the airplane motions, a workmanlike analysis had been prepared at the depot in New York[e] in which coordinate systems for unambiguous simulation were worked out in great detail. This background was helpful to us, partly because such features as coordinate systems, and the several transformations these systems must undergo in the workings of the simulator, form the largest group of technical problems thus involved. After commencement of the project, little contact was sought with this particular Navy agency, since we tended as a first approximation toward a less precise model, and since a somewhat different coordinate system was finally settled upon (the aerodynamical equations of Sir Melvill Jones were taken as standard), but it was planned ultimately to join forces when the equipment had attained a demonstrable status, since each party considered that mutual development might be beneficial. On the whole, however, BuOrd was dealt with chiefly in this simulative venture, rather than BuAer, as being cognizant of the more inclusive PUSS project.

Those parts of the simulative assembly having to do with the operator are enclosed within a model cockpit. Thus are included the potentiometric "recognizors" of control motions, including stick, rudder, and trim adjustments, signal indicators on the instrument panel, and a set of three scopes so arranged above as to present moving indices on a common focal plane, seen in the normal forward direction. These scopes carry visual indications of the direction to a target on the ground, of the altitude of a "road" passing through the target, this feature thus providing a roll-reference, and of the cross hairs of a computing-sight reticle.

Approximations are included, in this first model, which are based on its use to represent either horizontal flight or a more or less direct gliding or diving approach toward a target at ground level. Plans have been made to allow such maneuvers as circling (or "casing") the target, wing-overs as the attack is begun, and so on, in future models if such ever are desired.

The electronic channels of the simulator cover circuits in which the lags and inertias of the airplane responses, in roll, pitch, and yaw, contribute to and characterize the control transients; circuits in which the geometry of the transformations, from airplane coordinates to target coordinates and back again, are accomplished; and circuits in which are incorporated the differential equations at present proposed for the PUSS computing system. The chief approximation is in the assumed equivalence of the sequences, yaw-pitch and pitch-yaw, the error of which assumption is not influential below roll-angles of 20 degrees or so. By more complex circuitry, this approximation could be replaced in the simulator by the unapproximated circumstance. No new components were used beyond those heretofore indicated. All the equipment, and there is half a small roomful of it, mostly amplifiers, connective electrical structures, and regulated supplies, is so designed that it can be assembled on standard relay racks. Present plans are for transportation of this apparatus from New York to Philadelphia, where it may be used in continuation of the Navy Project NO-265, under the auspices of a direct BuOrd contract (NOrd 9644) at The Franklin Institute.

4.8 POTENTIALITIES FOR THE FUTURE

Frequently during several of the developments above described it has been thought, and said, that for the increasingly diversified uses to which the simulative methods were being put, requiring new construction or at least major physical rearrangement of components each time, it would be preferable to build an extremely general and flexible assembly, covering every conceivable type of system, which

[e] By Seaman Steinhardt, late of the University of Vienna.

could be adapted to any particular problem simply by the manipulation of conveniently provided organizational "controls." Knowledge of the means for achieving quite impressive generality had gradually been gained, and the proposal for such a *supersimulator,* as it came to be denoted, was considered not at all fantastic. After all, witness the differential analyzer. A considerably smaller expenditure would provide a machine, albeit less accurate, of much greater power in the order of equations which could be embodied, and of greater flexibility on the time scale. Although a human operator may be included as a component in a differential analyzer, this is not done in imitation of a corresponding activity in a situation under representation. Thus in guiding bombs, for example, the analyzer "dropped" a bomb in 15 minutes, whereas in practice it fell in 35 seconds. Of course, this is not a permanent, or inherent, limitation of such machines.

Although no supersimulator, in the full sense, has as yet been prepared, there are many who might still be found to be champions of the plan, and it would seem to have certain applicability to future developments in a variety of technical fields.

Certainly flight may be taught by models, at least through most of the learning period, and electronic simulative techniques will certainly be economically superior in this respect. The same remark is evidently valid for the special flying operations whether by local or by remote control, of combat conditions. Since the equations of rocket flight are not difficult thus to simulate, and seeking or remote-guiding procedures, manual or automatic, are representable by straightforward extensions of the techniques of this chapter, it is evident that guided rockets may be studied by such methods. Warfare among such missiles could thus be staged and observed entirely in the laboratory, possibly with statistical machinery in attendance for interpretation and assessment. Years of experience, and of trial and error, on the development of controls and dynamic components could thus be collapsed into hours. There is, in reality, no limit at all.

Chapter 5

LINKAGES FOR COMPUTATION AND MANIPULATION

5.1 GENERAL TYPE CONSIDERED

Mechanical linkages and the operations they perform are of permanent interest to the geometer, who finds them irresistibly fascinating, and they hold attraction, in addition, for the analyst who sees recondite real functions, perhaps of several variables, come to life in his hands. Throughout this discussion we refer to linkages in the purest form, in two or three dimensions, meaning thereby those linkages which consist in assemblies of moving parts which are joined at points or lines only, having no rubbing or rolling except in the neighborhood of such point or lines, that is to say only at true bearings. The so-called *friction radii* are thus very short in comparison to the size of the assembly. While we wish to speak principally of this class of mechanism, and may expound the advantages therein for certain purposes, this is by no means to detract from the benefits, in computation or otherwise, of other classes which may be named. Thus cams, either flat or three-dimensional, have natural applications and need fear no competition from linkages in such fields. Cams are important, and we have made use of them; our indifference as expressed by the omission of cams from these pages means simply that they are adequately treated elsewhere, or rather that we have nothing new to offer on the subject. The same is chiefly true of computational devices involving other means than linkages for the embodiment of their laws of operation, although other sections of the present report may be found to be obsessed, for example, with electrical computers (Chapter 10).

Linkages with sliding joints are not primarily discussed here, although there is thus included an extremely broad group, and such may occur in combination with the forms having nonextensible members. Sliding linkage is useful in many computations of partially geometric nature, for example, and the frictional forces may in some certain cases be made extremely small through designs which involve pins of small diameter. Our primary interest has been in relatively small linkages, of light weight and high resistance to the ills of vibration and unbalance, very low in the input torques required to manipulate them, and offering flexibility in adjustment and modification. The exclusive use of pivots in the design of such components has become a policy rather than a rigid rule. It is felt, however, that this policy has paid well for the effort given to promoting it.

We wish to state in general that we have some little knowledge, but no more than that, of linkage developments elsewhere.[a] We understand that our philosophy of linkages is independently shared by such workers, whose proficiency may, incidentally, date back considerably before our own efforts. It is not known whether any other activities have led to components as small as ours, where a 2-inch link is a lengthy one. Linkages of this size are far from unusual in such aircraft instruments as altimeters, and our computers were physically similar to these in some cases, but naturally of an entirely different order of complexity and of adaptability in the mathematical functions they embodied. We shall thus speak of our own developments as though no others existed, although thereby we may inadvertently give the impression, to a better instructed perusal, of unwarranted claims to novelty. No such unestablished claim, however, is intended.

Admittedly many of the advantages attached to pivotally constrained linkages are practical ones, depending on the developed techniques of design and construction. It is easier for example to obtain, from a typical design department and shop, a good bearing than a good sliding joint. This is more than a mere matter of tolerances, but depends rather on the familiarity of techniques, and on the available tools for their application. Furthermore, for a certain category of purposes, where only small angular motions are

[a]By Svoboda, *et al.*, at MIT Radiation Laboratory, for example.

needed, this type of linkage generalizes quite nicely to flexure joints, which have many additional charms. It is held that the problems of wear and dirtiness, from the important standpoint of maintenance, are considerably simplified in mechanisms of this general type.

5.2 METHODS OF DEVELOPMENT

We shall deal here very sketchily, if at all, with standard pedagogic descriptions or with the theory of well-known linkage designs. The *four-bar linkage,* so-called, or the *four-link mechanism,* which receives its name, remarkably enough, from the three moving members it involves, must be mentioned as the most fundamental component of the class considered. For some reason the flat, or two-dimensional, form is the only one widely used for computing purposes, where the nature of the functions embodied is important. Three-dimensional four-bar linkages are used predominantly for manipulative purposes, and less frequently, in typically approximate circumstances, to achieve a "characteristic." We shall refer only to the flat form hereof. And throughout we consider all linkages as being interposed, chiefly, between angular rotations as input and output, more than one of each, in general, being involved.

A basic four-bar linkage is shown in Figure 1. This component is extremely flexible, although it yields somewhat grudgingly to analysis. Although it is simple enough to write equations for this linkage, the determination of the three parameters, for example of the relative link lengths, to fit a desired characteristic is not direct by that method. Geometric criteria are found more suitable, and a number of these are available. Thus it is known, for example, that the derivative of one of the principal angles, θ_1 and θ_2, as shown, with respect to the other, say $d\theta_2/d\theta_1$, is a function only of the position of the intersection between the line joining the fixed pivots and that of the free link, and is in fact equal to the ratio of the distances from the fixed pivots to such intersection. A useful design criterion is hereby provided, determining the local slope of the connecting relation between θ_1 and θ_2, considered as plotted[b] against one another in rectangular coordinates. Having three parameters, a bit of geometry leads to adjustments which may fit three points on the curve, or two points and the slope at one, and so on. Functions of one variable which are not too badly behaved may be incorporated this way, almost to an arbitrary precision, since it is possible to connect n such linkages in a chain, the input of one driving the output of the next, obtaining thus $3n$ adjustable parameters, plus initial conditions. In the simple four-bar linkage there are numerous interesting special cases. Either or both angles may be restricted to less than complete cyclic rotation,

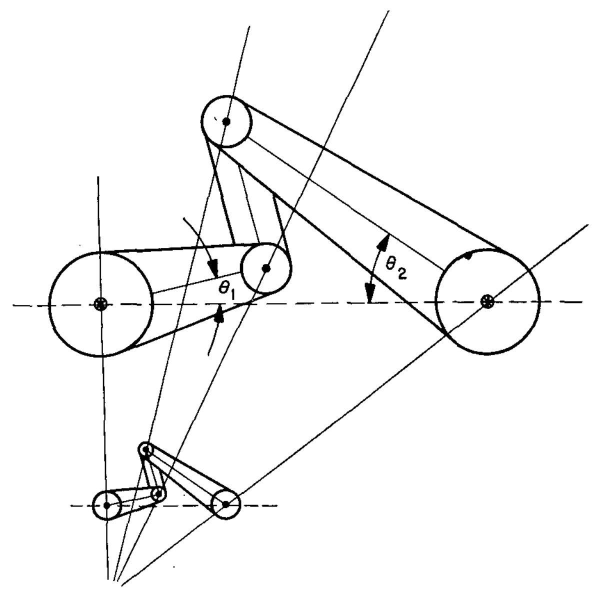


FIGURE 1. The flat four-bar linkage.

whereas periodicity of other types may also be present. For relatively close fixed pivots, both angles may rotate through 360 degrees, leading to many useful transformations in cyclic machinery and computers. In exploring the possibilities of such linkages, to perform a certain angular transformation, one may proceed by choosing a set of values, say equally spaced, for one of the angles, and assign the set of values, for the other angle, which correspond individually to the first set under the desired transfor-

[b] An earlier stratagem involved building an experimental linkage with adjustable members, with the two angles driving a recording stylus linearly on an adjacent rectilinear plot. The desired function being first drawn, the lengths are adjustable for a best compromise.

mation. Laying out the first set as points on an arc of arbitrary radius, choose successive constant lengths and draw complete circles about each point with radii all corresponding to one such length. If exploration of the pattern formed by the circles having radii equal to one such length does not disclose division of a superimposed arc in accordance with the transformed point-set, then one may try another constant length for the radii, and so on. One is thus

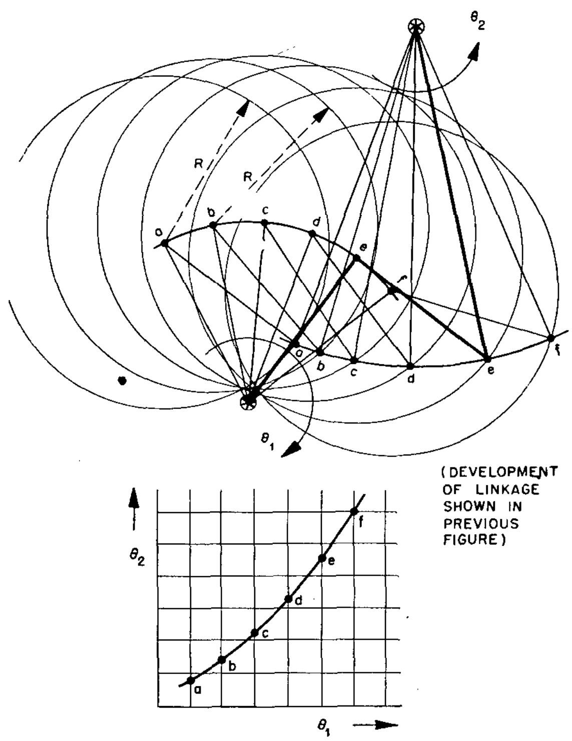


FIGURE 2. Geometrical development for linkage.

exploring a very broad field, but for reasonable precision it is surprising to find a case where the process fails to produce quickly a workable linkage. Figure 2 is intended to show the procedure thus briefly described. Approximations have been made, using such linkages in computing mechanisms, which have satisfactorily reproduced such simple functions as reciprocals, squares, and consequently of course, by reversal, square roots, and so on; but the typical application is to correct for some empirical distortion in associated equipment, or to embody a law which has been determined experimentally in response to the functional contributions of several components in a given system.

Counterbalancing of four-bar linkages may be essential where the input power is low, although under such circumstances this problem is largely conquered by making the members light and the inertias low. If each terminal link carries a counterweight whereby it is independently in static balance, then to each such counterweight may be added what is necessary to balance half the weight of the free link, considered of course at the appropriate moment arm. The problems of such balancing are n-fold if the computer is to work under ng acceleration. For the more intricate linkage assemblies, the problem of balance becomes correspondingly so, in accordance with some power of the number of connected free links, and here approximations will frequently suffice, using counterweights on the terminal links only, calibrated to resist accelerations under some chosen average condition of all inputs.

The typical function of two variables which appears in connection with airborne aiming-control computations, may be synthesized to a surprising accuracy by means of the simple 6-mem-

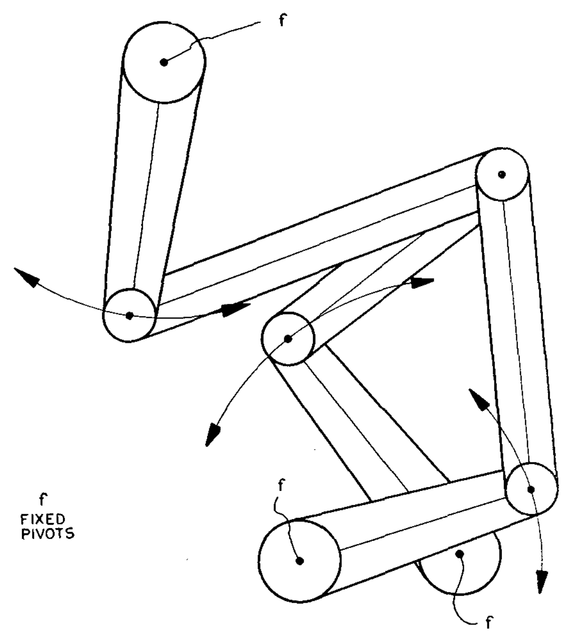


FIGURE 3. Typical linkage for function of two variables.

ber (we depart from the classical nomenclature) linkage as shown in Figure 3. An attempt is made, in Figure 4, to portray the general plan whereby such linkages may be developed. One begins with a parametric plot, say in linear and

rectangular coordinates, having a set of curves of which each corresponds to a value of the third variable, the other two being on the coordinate axes. Now it is immaterial which of these variables is the dependent one. It is only necessary that when two definite values are assigned for any two of the variables, that only one significant value appear, on the graph, of the third. Some multivalued functions may be thus represented, as when the input or output motions are cyclic, but we shall consider only the simpler cases, which indeed have found greatest application. One begins, as we have said, with a normal parametric plot. The number of parametric curves depends on the precision specified, and on the regularity of the function over the field, or more precisely on the ability to interpolate. Suppose six curves are sufficient, as in our cases has always been adequate. The next step is to choose six representative values of each coordinate, including the maximum and minimum to be considered in each case, and to draw in the corresponding coordinate rulings, specially designated in some way, as by heavier rulings. We have thus a network, as shown in the inset of Figure 4, representing a skeleton, so to speak, of the functional relation to be mechanized. The problem now is to perform conjectural warpings of this network, in which all intersections and relative positions are preserved, until one is obtained, if indeed it can be, which meets certain qualifications. The magnitude of this task, which is better demonstrated at first hand than explained thus in literary fashion, depends principally on the nature and consistency of the set of parametric curves of the network or original plot. Choosing, say, the network of rectangular coordinates initially, one replaces these by arcs of circles, in the case of each coordinate, of invariable radius and having their centers themselves on the arc of a single circle. For each coordinate, then, there is a center belonging thereto, an arc about such center with points appearing thereon, disposed according to the angular functional representation desired for that input or output of the linkage. About each such latter point is described one coordinate arc of the transformed network. The two sets of coordinate rulings have now become, conjecturally, two families of circular arcs. On this network the parametric curves are now redrawn, changed in general shape but faithfully retaining their intersections with the coordinate network. Smoothing processes, by estimation or through more meticulous geometry, may be employed in completing the network by the addition of the new parametric curves. If the parametric curves were smooth and regular in the original plot, or network, they will generally remain smooth in the transformed one. Frequently it is sufficient to show only the intersection points of these curves with the new coordinates, appropriately symbolized to identify the individual members of the family. Now comes the test. Do the transformed parametric curves appear as arcs of circles? If not, try a new transformation. This trial process is not tedious, after some practice, and a bit of experience shows how to plan each successive trial to come nearer the desired result. If the transformed parametric curves are arcs of circles but each arc has a

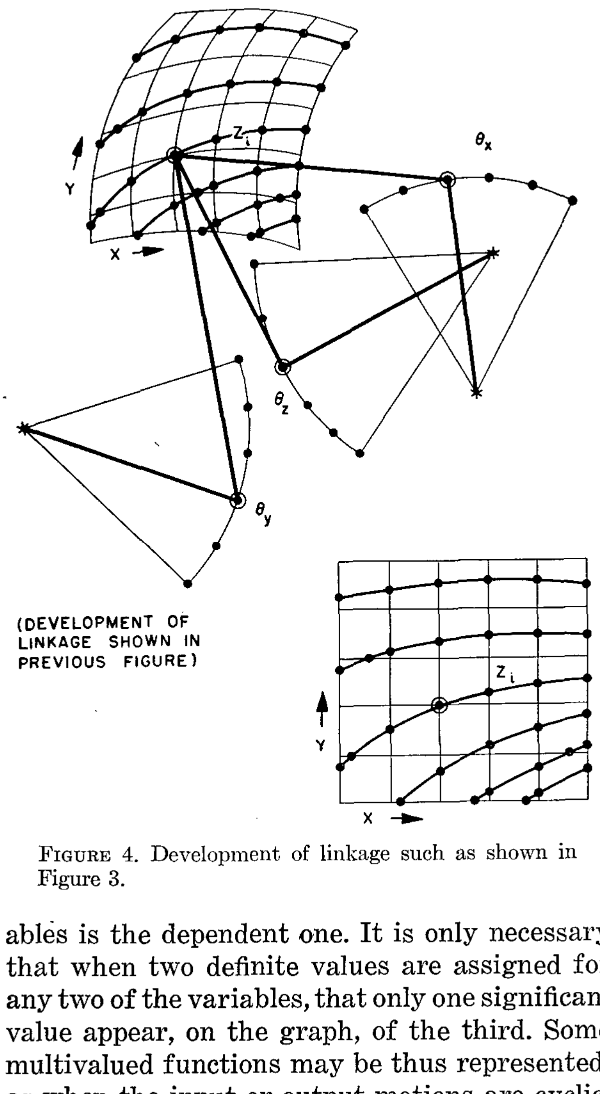


FIGURE 4. Development of linkage such as shown in Figure 3.

different radius, we must still retransform, but this is commonly not difficult, since by a relocation of pivot points the required additional warping may be fulfilled. Suppose that finally the desired condition is obtained; all curves, coordinate and parametric, are arcs of circles. There is one more criterion, but fortunately this tends almost always to be automatically satisfied, to reasonable precision. The centers of the arcs of equal curvature, which are the transformed parametric curves of the original plot, should lie on the arc of a circle. Now this circle may have a very large radius, and thus may be nearly a straight line. In such cases, as when also the locus of centers is a curve pronouncedly differing from a circle, further alteration of the coordinate transformation may help. By this time however, so many conditions have been met that one may not expect much further freedom, and it is best, as, for example, in the straight line locus, to add a few links so that the linear motion may be converted to rotary, without the use of inordinate radius arms, through a Watts linkage or other simple and approximate linear motion: here used backwards. Similarly, any output (or input) function, as it must appear in the linkage developed as above, may be transformed uniquely so as to alter the calibration there presented. Four-bar linkages, either singly or in series, may be applied here. Thus we may want a linear or a reciprocal calibration of any of the associated variables, and we may have it, no matter what internal calibration was required for the linkage design as such.

If graphical functions resist such representation, it may be advisable to replot and proceed again, replacing one coordinate by the parametric variable. In spite of such trickery, and of the free play of imagination in framing linkages, certain functions may remain refractory. We have not met any as yet. For example, when the parametric curves of the original plot cross one another, and overlap even considerably, this is not fatal at all, but yields directly to the above method. Further measures may be applied also, if part of a parametric family is warped in some local fashion, for linkage components may modify the output; and numerous generalizations are possible on the above-mentioned components, each having its peculiar powers for representation. It is safe to say that any systematic function of physical significance may be embodied, to any desired excellence of approximation, in linkages of this general type.

5.3 MORE COMPLEX FUNCTIONS

In passing from functions of two variables to functions of three, and thinking of linkage representation, we pause to note that the properties of all functions which make them easy to represent are definitely not the properties of mathematical simplicity. Better criteria would be expressible in terms of graphical configuration. Paradoxically, it is easier to make a multiplier, $z = xy$, than an adder, $z = x + y$, by the methods outlined above. Even brief acquaintanceship with such development gives the facility for predicting the representability in linkage of a set of graphical data.

We note further the tendency to make theoretically exact computers, which has led to much heavy machinery, whereas in general the precision of computers is far greater than that of other components or processes in surrounding equipment. It is gathered that equal effectiveness might have been gained more economically, in many such cases, through the use of simpler and lighter, albeit *per se* less precise, pivotal linkages for computation.

The incorporation in linkage of a function of three variables represents in general a bigger step, as compared to the case of a function of two variables, than does the two-variable case in comparison to that of a function of one variable. For many such problems which arise in practice, however, the task is simpler than might thus be indicated, owing to the tendency of engineering functions toward reasonable behavior. By canny choice of the variables to be considered in sequence, and of the sequence itself, one may frequently generalize from a linkage for functions of two variables to the complete linkage for three. For example, if the three-variable function may be considered a function of (1) one of the three variables and (2) another function of the other two variables, then the whole problem is simply solved

by twice applying the above technique for functions of two variables. An example of such a linkage, for computing a function of three variables, is shown in Figure 5. The criterion

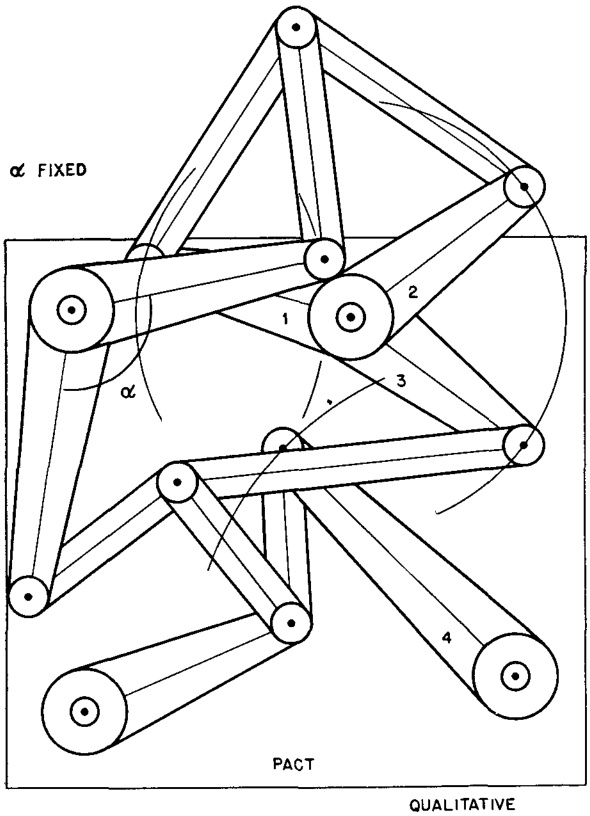


FIGURE 5. Study model of linkage component for PUSS.

for such reduction of a function $f(x,y,z)$ is that, to a sufficient approximation

$$f(x,y,z) = g(h(x,y),z).$$

First a linkage would compute h from x and y as inputs, and then g and consequently f from h and z as inputs. An elementary example of such specialization is provided where f may have a factor involving only one of the variables, as when

$$f(x,y,z) = z\phi(x,y) = \frac{z}{z_1} f(x,y,z_1).$$

The class of all functions for which the first named reduction is possible is apparently not easy to define more specifically; the question of such reducibility is evidently not mathematically trivial. When functions are not amenable to such treatment, however, linkages may still frequently be made by using this process as a guide, "expanding around" some central condition, as it were, and approaching the desired precision by a process reminiscent of successive approximations.

A somewhat less restricting property which allows representation, on the basis of the earlier method, is that the function be separable as

$$f(x,y,z) = F(u(x,y), v(y,z)),$$

for example, and it is claimed[c] that linkage design may be based on functional considerations even more general than this. The design criterion, it should be noted, of the number of links required in any particular case, is of less importance, for example, than the local compression of the output scales, which may lead to errors in the presence of lost motion, or to additional difficulties in obtaining a more desirable calibration.

Generally speaking, it is characteristic of the procedures of mechanization wherein computation by linkage is involved, to omit the step in which empirical formulas are developed to fit certain data which must be embodied, and to pass directly from such data, presented in graphical form, to the computing linkage itself. In many cases, where this procedure is legitimate, a large saving in time and effort may be realized.

5.4 SOME APPLICATIONS

There have been several occasions for applying linkage computers, of the above general type, in the design of fire-control systems for airborne torpedoing and bombing, where particularly it has been desired to prepare components which would fit into extremely small space. Such drastic restrictions on space arose either because the equipment was to be squeezed into positions which were already crowded by other necessaries, as in the case of the development which resulted in the torpedo director Mark 32, or because the major assembly was to be hand held by the operator, as were the developmental models of the project terminating in the bombsights Mark 20 and Mark 24. Further need has been present for mechanization in this form since, for similar reasons

[c] By E. Elyash of The Franklin Institute.

again, a minimum power load for operation was desired. Finally, it was held, and this is as true as ever, that linkage construction lent itself well to mass production techniques as represented in the facilities of the manufacturer not ordinarily practicing in the fire-control field.

A relatively simple case in which such a linkage was forced into service with evident benefit was in connection with a guided bomb simulator, where in production it was found difficult to procure potentiometers having a square law in resistance versus rotation. Several such units, ganged, were to be involved in each production unit. It was made possible to use the commoner linear potentiometers by driving them through an inexpensive linkage which approximated the square law remarkably well. In this case completely cyclic operation was also required. There may certainly be many such cases. The most serious applications of such methods of computation, however, occurred in the development of aiming controls for rocketry and for toss bombing. There was PARS, for example, an ambitious project for the construction of a miniature rocket sight.[d] In this instance the plan was to include the whole system, except for the pilot's sighting index itself, within the case of an aircraft altimeter, barometric altitude being one of the inputs. Indicated airspeed, normal acceleration, and temperature, were also inputs and were also locally supplied, all inputs taking the form of (angular) motion. A quite small, flat linkage, of the type described, computed the "super-elevation" as output from the above input motions. This project led to an experimental model and to preliminary tests, but is not considered to be finished, or to have resulted as yet in a completely workable system.

In the PUSS project there has been the most thoroughgoing employment of all-pivot linkages with which we have been connected. In the computing component of the aiming-control system involved here, a multipurpose linkage serves in connection with the rocketry and tossing bombing functions. The linkage computer, employed as a "static" computer, as we may say since time is not explicitly there involved, is connected intimately with the components of the dynamic computer, the form of which differs in the various versions, pneumatic and electric, which are being experimentally prepared. To the amalgamated linkage go eight inputs, two of which are manually effected and the rest automatically. Four outputs are derived, setting parameters in the dynamic computer and, in one case, determining a release condition. Most of the inputs apply at several places in the linkage assembly, being joined mechanically then by common shafts, and several intermediate computations are performed internally. The design of the latest experimental form is in four layers, being mounted on a framework involving correspondingly many plates. This present design is to fit into a 6-inch cube, complete with counterbalances, etc. Approximately five dozen individual links are contained.

[d]Pursued on a developmental and design level, principally between H. Whitney of AMG-C and R. W. Pitman of The Franklin Institute.

5.5 A THREE-DIMENSIONAL LINKAGE

We shall speak next of a particular type of linkage as an element of mechanism, first quite aside from its potentialities for usefulness. It was initially[e] demonstrated in this connection by the writer, however, as the exact solution to a problem of so moving a mirror as to give a certain desired behavior of the reflected ray. As soon as the linkage was proposed, however, it was recognized that it had properties of several different sorts, suggesting a number of other applications, and that furthermore it appeared to be geometrically fundamental and thus to warrant existence in its own right.

The linkage thus lavishly introduced involves five linear pivots, or *hinges*, by which its moving members are joined, there being no sliding or rolling connections aside from the bearings or hinges thus included. While there are many possible variants, the basic linkage may be defined most readily in terms of the relations among the axes, each possessed in common by two members or links of the linkage, of the hinges or bearings. A complete defini-

[e]As to originality, this is evidently disputable since E. L. Rose has remarked that a similar structure was arrived at by E. Root III for some mathematical function, using the reasoning of projective geometry.

tion is supplied by the statement that, in the linkage, each of the five such axes, all five being concurrent or having a common point of intersection, is maintained perpendicular to each of two others. It is the function of the joining links, of course, and we include the "stationary" link as in the term *four-bar linkage,* to maintain the orthogonality between the axis pairs. An equivalent statement, which may seem at first paradoxically so, is that there are five concurrent axes which may be considered in a "ring," each being perpendicular to the next in the ring. It is, incidentally, rather interesting to consider the special cases, for axes having this type of mutual orthogonality, for four axes and for three. The present linkages reduce to these by application of certain constraints, as may be expected. In recognition of the numerical and angular properties of the hinge axes in this linkage, it has been called the *orthopentax.*

An axis held in double gimbals of the well-known type is a three-dimensional generalization of a simple two-dimensional hinge. It is, however, an asymmetrical such generalization, since one input axis is carried along when rotation about the other is introduced. The orthopentax is a symmetrical such generalization, both input axes, in the generation of a doubly hinged and spatially moving output axis, remaining stationary.

In Figure 6 is illustrated an elementary form of the orthopentax linkage, displayed in three typical conditions. In the particular form here shown it is seen that there are four movable, right-angle links, two of which pivot in stationary bearings and the remaining two of which pivot in bearings which are fixed in the first pair of links, the second pair joining one another in a fifth bearing. The axes of the five bearings are the five axes referred to above. If the shaft affixed to the fifth bearing, in the center of the linkage, be regarded as a manipulable *gnomon,* or index in space, its motion, under applied rotations of the terminal links about the stationary axes, is of interest. For consider the parallel projections of the gnomon on each of two planes which are normal to the stationary axes. It will be recognized that the rotation of each such projection of the gnomon is precisely equivalent to the input rotation of the corresponding terminal link about its stationary axis, and is completely unaffected by the input rotation of the other terminal link about the other stationary axis.

If we imagine a sphere of any radius, centered in the common intersection of the axes of the orthopentax, then the locus of the pierc-

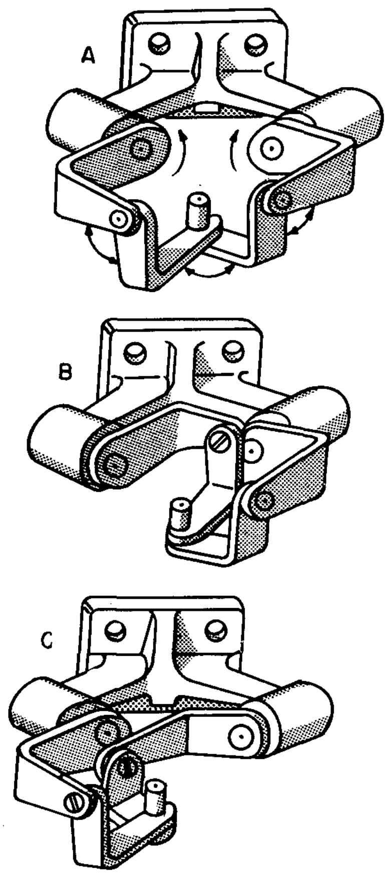


FIGURE 6. Fundamental three-dimensional linkage; so-called orthopentax.

ing point of the extended gnomon in the surface of this sphere, on rotation of the terminal links, has special properties. If either terminal link is unrotated, then rotation of the other link causes the gnomon to trace a great circle on the surface of the sphere. All such great circles intersect in two poles which are themselves the two intersections of the stationary axis, of the unrotated terminal link, with the sphere. There is thus established on the sphere a coordinate system given by the network of two families of great circles, each having a pair of poles in common, and the two pairs of

poles perpendicularly disposed on the surface of the sphere. The corresponding coordinate systems for a double-gimbal arrangement is similar to that of latitude and longitude, consisting in polar great circles and zonal circles; it is an orthogonal system on the sphere.

Again, if one considers the intersection of the gnomon of the orthopentax with a plane parallel to the plane of the two stationary axes, it is seen that this intersection follows straight-line loci when the individual terminal links are manipulated. The coordinate system thus formed is orthogonal, and is linear in the tangents of the angular rotations of the terminal links out of their zero positions, which latter correspond to normality between the gnomon and the plane of projection. This property of the orthopentax linkage implies significance for many special applications, where independence and symmetry of the two modes of manipulation is important in the generation of a direction in space.

To avoid misunderstanding, it must be pointed out that a geometrical similarity may be shown between the function of the orthopentax and the combination of a double gimbal system and a so-called *bail ring*. In the latter combination, however, one of the projected angles does not appear explicitly, in tangible form for indicative purposes or other employment that is, and furthermore either sliding or rolling must be resorted to in a mechanical guide, rather than the exclusive application of purely pivotal bearings, as in the orthopentactical linkage.

5.6 MANIPULATION OF A MIRROR

In connection with the presentation to a pilot of the output of an aiming control system, in the form of a reticle image, which image is to be made to coincide as aiming criterion with the target direction, it is desired to produce such an image by reflecting, from a single moving mirror, the parallel light rays from a collimator which is stationary in the fighter airplane. It is desired further that the departure of the reticle image, which actually is seen by the pilot after reflection from a sloped and semitransparent surface before him, from a zero position approximately straight ahead, shall be under the symmetrical control of two deflection variables so that these variables produce rectangular motion of the apparent position of the reticle on an imaginary plane normal to the zero direction and having rectangular axes along the airplane's "horizontal" and "vertical." For constructional convenience it is planned that the normal or neutral position of the moving mirror be at 45 degrees, say, to the incident and reflected beams. These requirements are similar to those which arose in connection with the so-called Project PUSH (Chapter 10), for a pilot's universal sight head, within the more general PUSS project.

It became recognized that the problem, namely that of preparing a linkage to move a single mirror in the manner prescribed, can be broken into two subsidiary problems, first to so manipulate a mirror that the reflected ray was along the direction of a single manipulating rod, and second to direct the latter rod, in response to two input rotations, in the manner desired for the reflected ray, or, what is tantamount to the same thing, for the image of the reticle. Now the first such component problem is answered by an astronomical mechanism known as *Foucault's siderostat,* which had been used to reflect continuously the image of a star into a stationary camera. The second component problem is solved by a direct application of the orthopentax linkage. This view of the overall solution was not suddenly conjured up, a good deal more groping having been involved than might hereby be indicated, but it is true that division of the problem in this manner was appropriate as soon as the general form of the answer was once seen.

The siderostat already referred to involves a slanting mirror held in a double gimbal, the stationary axis whereof is along the fixed ray to, or from, the mirror. The mirror rotates about an axis in the "outer" gimbal, which latter axis is normal to the stationary axis of the gimbal. A link mounted as "inner" gimbal on the first gimbal, and having an axis in the latter parallel to that of the mirror, carries a fixed rod perpendicular to the latter parallel axes and in its neutral position normal also to the stationary axis of the first gimbal.

Then by so connecting the rotations of the inner gimbal and the mirror, as by the well-known *half-angle* linkage, so that the mirror rotates, with respect to the outer gimbal, half as fast as does the inner gimbal, the reflected ray or beam may be made to coincide identically, under all conditions of manipulation, in direction with that of the said rod.

It remains merely to employ the orthopentax linkage for the manipulation of the direction-determining free rod. In such employment, parts of the normal orthopentax combine naturally with parts of the siderostat, so that the

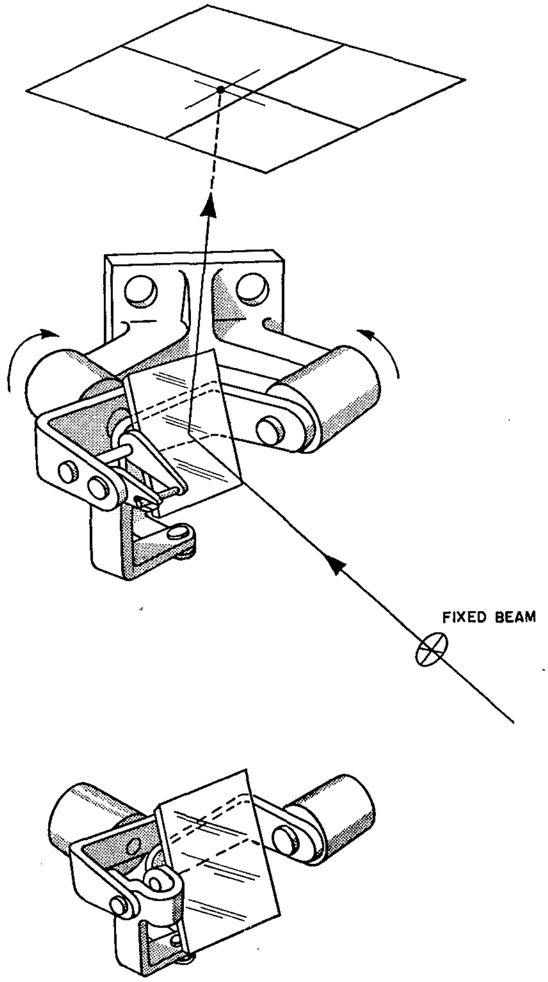


FIGURE 7. Orthopentax as mirror manipulator for oblique incidence.

resulting assembly, as shown in Figure 7, is not simply the sum of the two component mechanisms, and indeed can be justified by less roundabout, albeit somewhat more special, arguments. The arrangement shown in the latter figure is based on the alternative position of the purer linkage shown in Figure 6C. For small angular departure of the reflected beam from the neutral, the projective orthogonality,

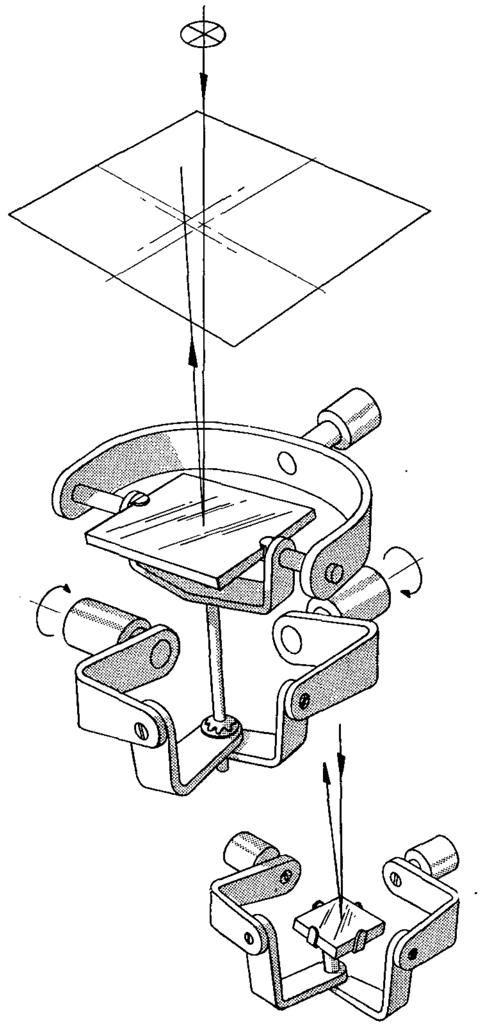


FIGURE 8. Orthopentax as mirror manipulator for oblique incidence; same as Figure 7, but for normal incidence at neutral.

which is *exactly* embodied in the linkage already described, may be preserved to a remarkable approximation by omission of the half-angle mechanism. The somewhat simpler method of mounting the mirror on the inner gimbal, which is thus permitted, is shown by the inset in Figure 7. Many other variants are possible. Another exact method of manipulation, when it is desired that the incident and reflected beams be parallel in the neutral case, for zero deflection, is shown in Figure 8. Here

a three-dimensional form of the half-angle linkage is applied, by displacing equally the centers of the mirror linkage and the free bearing of the orthopentax (cf. Figure 6A) from the plane of the stationary axes of the latter linkage.

One property of such manipulations, involving only one mirror, is that of rotating the reflected image about the axis of the reflected beam, as manipulation takes place. If this is objectionable, one must then resort to more than one mirror or to corrective rotation of the primary image-forming pattern.

5.7 TRIGONOMETRIC COMPUTATIONS

Many broadly inclusive geometric and trigonometric functions are embodied among special lines in a rectangular octant. Consider thus one such octant, or the corner enclosed among three perpendicularly intersecting planes, and suppose lines are allowed to move, one in each of two of the planes, so that they pass always through the triple point of planar intersection. These two lines, each of which is perpetually perpendicular to one of the coordinate axes formed by the figure, may coincide with one another as a special case in the third such coordinate axis. Let us consider the angles between each such line and the said third coordinate axis, these angles being both zero when the two lines coincide, and each angle having values between plus and minus one right angle. Consider further, in each of the two planes to which the above lines are restricted, second lines each perpendicular to the first lines. We now ask: what is the angle between these second lines? It is not difficult to show that this is the angle of which the cosine is the product of the sines of the two angles first introduced for the rotating lines. Consequently the *sine* of the departure from a right angle of the angle between the second lines is the product of the sines of the two input angles. Now by reference to Figure 6 it may be seen that the above geometry is completely embodied in the orthopentax linkage. The final angle, of course, corresponds to that between the two free links, and describes their relative rotation about the gnomon. There is thus tangibly afforded a precision method for the multiplication in terms of the sines of angles, and consequently of such other operations as those involving ratios instead of products, by an inverse operation, or cosines rather than sines, through readily applied alterations of the procedure.

To return for a moment to the geometrical picture already employed by way of clarification, one may wonder why the angle directly between the lines originally introduced was not employed, rather than that between the lines perpendicular to and respectively coplanar with the originally introduced ones. Well, this is not hard to explain in terms of mechanism. Consider what mechanical contrivance would be required to maintain indication as the latter lines passed through coincidence, their included angle then passing through zero and changing sign. Thus, although the sine multiplication may certainly be carried out between these original lines, or with the corresponding linkage, which might be entirely different from the orthopentax, their replacement by the perpendicular references, or the journey twice "around the corner" as in the orthopentax, avoids the embarrassing circumstances in the neutral position, and makes a simply constructed all-pivot linkage do the computation perfectly. For some applications, of course, the mechanical embodiment of an important variable in the relative rotation of a bearing which has such positional freedom may be highly objectionable, and may invalidate the appropriateness of the method. The methods for removal of such apparent disadvantages, and, for that matter, the remaining computational potentialities of the orthopentax linkage have not been exhaustively explored.

5.8 OTHER FUNCTIONS AND FORMS

Owing presumably to the fundamental character of the orthopentax linkage, a number of interesting applications have been discovered to widely diverse fields. Many such applications, all of which incidentally cannot be here given in any detail, stem from the appropriateness of this linkage as a *general deflector* in space, whereby two input rotations, with respect to stationary bearings in a rigid framework, are permitted symmetrically to manipulate and

uniquely to determine the direction of an index member. In this sense the linkage may be thought of as a modified universal joint, and has other properties which may be inferred from the latter type of function.

If the free index, or gnomon, of the orthopentax (Figure 6) be constrained to rotate in a plane perpendicular to that of the two stationary axes (as may be accomplished by inclusion on the index member of a second bearing or pivot which permits rotation about a new stationary axis lying in the latter plane, and which is perpendicular to the original bearing axis of the index), then rotation of either original terminal link about its stationary axis results in a corresponding rotation of the other terminal link. The correspondence is one of equality when the plane of constraint referred to is normal to the bisector of the original stationary axes. Thus is formed a right-angle drive without gears, or a universal joint with uniform rotational properties, if such is desired. The freedom of rotation, however, of either input or output, that is to say of the terminal links, is necessarily limited to something under 360 degrees, owing to blocking action of the component links. For small rotations of both input and output, a torque amplifier results. Unity gain corresponds to the case latterly mentioned, when the plane of constraint is symmetrically placed with respect to the stationary axes, or when the new stationary axis, about which the gnomon member must rotate, bisects the original stationary axes. If, however, the new bearing axis is itself rotated, in the plane of the stationary axes, and is maintained concurrent with these and all the other axes, then any transmission ratio of torque from nearly zero to nearly infinite may be selected at will. This should provide a large improvement over levers. Some simpler forms, of such a torque multiplier, may be derived for special applications.

The orthopentax has a further most surprising property which may be illustrated by assuming that a source of torque is available, with respect to the framework, for application about the gnomon or index axis to the two links hinged therein, both such torques having strengths independent of the angular motion of each such link. Figure 6A may be referred to, for example, in this demonstration. Under the application of the torques, it is evident that in the neutral position corresponding to what we may call the zero position of rotation of the terminal links, when the gnomon axis is normal to the plane of the stationary axes, no resulting torques will appear on the terminal links, or about the stationary axes. When, however, either terminal link is rotated from its zero position, as above agreed upon, a torque will appear on the *other* terminal link, about its (stationary) axis, since a component of the full applied torque is then allowed to exert itself there by the rotation imposed. This inverse relation holds simultanously in both directions, from each input rotation to the corresponding torque about the other axis, and the relation is undamaged when both rotations take place together. Such resulting torques are proportional to the sines of the corresponding rotations, and of course also the magnitude of the full torques originally applied. We have thus the spectacle of a pair of springs, so to speak, the reaction of each depending uniquely on the deflection of the other. It is not known what applications this may have in general, but one such application, which was seriously considered, has been to the implicit stabilization of a semiconstrained gyro system. Recall that a gyro normally spring-constrained in gimbals will nutate badly. Consider thus a symmetrical gimbal formed from an orthopentax, with the gnomon coinciding with the gyro rotor's axis. Now let the round-the-corner rate versus torque characteristic of the gyro be matched by the corresponding round-the-corner torque-versus-deflection characteristic of the orthopentax linkage, through application of the axial torques above introduced. It may be shown that this process leads to stability in the semiconstrained gyro assembly, which in fact then has the properties of a lead-computing sight. The "time constant" of the kinematic computer thus formed depends on the moment of momentum of the rotor and on the applied torques, and might be made as long or short as desired. A number of advantages have been thought to be possible of realization in this type of mechanism over the classical types. The attractive possibility which suggests itself, of applying torques to the inner

gimbals of the orthopentax mounting through eddy-current drag between them and the rotor, is spoiled when it is recognized that *negative* time constants are thus obtained, although indeed the motion is aperiodic. A reversal in rotation, between the rotor and whatever drag-disks were used, would be required. Such a procedure was entertained at one point in the PUSS development, and although shelved as a tangible plan was never wholly discarded.

The writer has found that the orthopentax might serve in the construction of analog assemblies for geometrical models in the synthesis of airplane flight. In such synthesis the complex relations among dive, pitch, turn, bearing, roll, and bank must be embodied precisely for complete effectiveness. Automatically, these relationships are displayed by the orthopentax. Suppose, as usual in such synthesis for flight simulation, we choose axes in the airplane, so that the framework of the linkage, as we have considered it above, is fixed with respect to the vehicle. Let one of the stationary axes be aligned with, and thus represent, the longitudinal axis of the airplane. Let the other stationary axis of the linkage be parallel and thus represent the "line of the wings," normal to the airplane's axis of symmetry. This orients the linkage in the airplane completely. The axis of the gnomon should now be made coincident with, and consequently to represent, the true vertical. Under these conditions it will be observed, particularly if a model is constructed and manipulated, that dive and bank, which are angles with respect to the vertical and horizontal, are represented directly by rotations involved in, and measurable or manipulable in, the bearings between the terminal members and the inner members of the linkage. Further, roll and pitch of the airplane, which are generally considered, but need not be, as indefinite integrals of roll rate and pitch rate, are embodied directly in the bearing rotation of the terminal links in the framework. Turn and bearing, somewhat differently, are embodied by the geometry between the linkage as a whole and an external horizon-compass reference system. Since these quantities are mechanically available in the linkage, it was contemplated that such an assembly might be directly installed in synthesizing structures, where it could carry out the "computations" inherent in its nature, as a substitute for more complex analytic assemblies of components. The *exact* character of the representations would there provide an attraction as well. The method was not employed, however, in the projects thus far completed, since in them certain somewhat more rudimentary approximations were felt to be adequate in the initial stages. It is considered that in future work, now being planned on such simulative projects, for airplanes and less orthodox vehicles, the orthopentax may actually be pressed into service as a comprehensive mechanical computer.

Chapter 6

AIMING OF TORPEDOES FROM AIRPLANES

6.1 THE WEAPON ITSELF

As a self-steered projectile, of which class there are many contemporary airborne counterparts, the underwater torpedo[a] was early and famous. A rich literature describes its development and relates its illustrious history. The aiming methods are well known, for launching both at and below the surface of the water. In United States Services, practically all torpedoes launched from airplanes have been of the so-called straight-run variety, which maintains the direction of the missile in effect at release, or when the gyro is uncaged. Substantially all our experience in the preparation of sights and computing systems has been with this type, and specifically with early "mods" of the torpedo Mark 13. Militarily, the history of the tactical use of the torpedo in World War II is well known. Having made a bad start, with very heavy squadron losses in several engagements, somewhat of a renaissance was evident in the use of the torpedo from airplanes during the last year of the war in the Pacific. Improved projectile stability may have been a heavily contributory element in this trend. The weapon itself, however, has unrivalled qualities against major fleet units. It is believed by many in our branch of activity that the employment of a good computing sight might have made torpedoing from the air a much more attractive mode of warfare, not only in the execution of operations but, in particular, to the warrior himself. The prejudice against torpedo sights, which has been evident among operating personnel in general, may have had basis partly in the violence of the approach tactics which were felt necessary, and which allegedly disallowed the delicate matter of sighting, but it is now generally believed that this prejudice arose from experiences with the rather inadequate sighting systems which were at first provided.

[a]The term denotes self-propulsion as well, in its modern connotation; earlier usage preferred the *automobile torpedo.*

For the purposes of this report, the projectile may be considered to fall as a bomb *in vacuo* from the launching vehicle into the water, holding a constant body heading, and thence to proceed, with respect to the water and submerged at a set depth, in a direction determined by the above heading and consequently by the compass heading of the torpedo airplane at the instant of release. Although at first the speeds and altitudes at which the torpedo might effectively be released were somewhat limited, these limits were considerably relaxed by subsequent developments. Entry angles in the vertical plane, governed by a relation between altitude, speed, and glide angle at release, were similarly made less critical. It is not our purpose, however, here to discuss such properties of the projectile, nor indeed are we qualified so to do. We merely record such background as is supposed the minimum for a rapid grasp of the instrumental developments.

The water speed V_t of the torpedo is fixed, and is known to within better than 10 per cent on the average. In the projection of a torpedo from any point in the air toward an immovable point a range R ahead, the average speed V_{at}, including both air and water travel, is valuable to know. This is evidently the ratio of this range to the total time consumed, or,

$$V_{at} = \frac{R}{t_f + \dfrac{R - V_a t_f}{V_t}} = \frac{1}{1 - \dfrac{t_f}{R}(V_a - V_t)} V_t$$

where also t_f is the time of fall in air of the torpedo, and V_a is the "ground" speed, or strictly speaking the water speed, of the airplane. For horizontal flight this time is given substantially by $\sqrt{2h/g}$, h being the airplane's altitude over the water. Consistent physical units are everywhere assumed to be used.

It is seen that when the range is infinite, as it cannot be except mathematically since in particular the projectile runs out at 4,000 yards or so, the average speed is merely the water speed. At the other extreme, when the

range is merely the speed of the airplane into the time of flight, it is seen that the average speed of the missile is simply the airplane speed. This reduces the torpedoing mission to one of bombing, which is inadvisable since an arming run in the water, normally some 200 yards, is essential. Thus between the limits set by practical conditions no singularities exist and we may safely use the relation:

$$\frac{V_{at}}{V_t} = \frac{1}{1 - \frac{\sqrt{2h/g}}{R}(V_a - V_t)}. \tag{1}$$

6.2 BRIEF HISTORY OF OUR DEVELOPMENTS

Beginning in the earliest days of the airborne fire control contract (OEMsr-330) at The Franklin Institute, Section 7.2 of Division 7 was associated with the development of a series of computing sights for airborne torpedoes. In the Services this work began under Navy auspices entirely, with Project NO-106 for the design of such sights, but in later efforts it became partly recognized under Army projects as well.[b]

Torpedo directors[c] of the commonest general type involve an estimate of the target's speed and relative course, to be made by the pilot before the attack is begun. We shall refer only to those in which the relative course setting, once made, is automatically corrected for turning of the airplane, or in which the compass course of the target is, as we may say, *stabilized* in the director. The manner in which it has been preferred to set in the target course is through manual alignment, of a body in the airplane representing the target, into a position parallel to the heading of the target itself. Certain unstabilized directors, notably torpedo directors Mark 28 and Mark 30, were in production prior to our activities, and are mentioned here only by way of comparison, although one of the projects latterly undertaken was the provision of automatic target-course stabilization for the torpedo director Mark 30.

All the computers involved in the directors which were developed under this project were of the vector type, involving the target velocity vector, presumably set in magnitude and direction by reconnaissance, a potential torpedo speed vector along the airplane's heading and consequently fixed in direction in the vehicle, and a computed unit vector to be directed toward the target by appropriate maneuvers in fulfilling the aiming criterion.

Early efforts dealt with the construction of mechanical models involving manual settings of target speed and course, airspeed and altitude of the airplane, and the torpedo run, for which the range to the target at the time of release, the so-called *present range,* was taken as a sufficiently valid approximation. Stabilization of the target-velocity vector was accomplished by servo-repeater from a directional gyro. The output of the computer rotated a reflecting sight bodily, about a vertical axis in the airplane, establishing the line of sight to be directed at the target.

An analogous director was also constructed experimentally using electric computation, with alternating currents representing vectors in amplitude and phase. Electrical servos were employed for introduction of airplane course from the directional gyro, and for manipulation in azimuth of the pilot's sight head. Such experimental systems were installed in a mock-up in the laboratory in order to assess their possible effectiveness in terms of the occupation of space and of the ease with which operations might be made on them in practice. The principal task appeared to be that of reducing the size and weight, and this was made clear by the military personnel who were consulted and to whom proposed forms of the apparatus were demonstrated.

As an attempt markedly to reduce size and weight, a miniature director was then conceived and designed, which was entirely mechanical and which occupied only a 4-inch cube in the

[b] In most of the work on these projects undertaken since 1943, A. L. Ruiz has been the responsible overseer rather than the present writer. (See Part II of this volume.)

[c] A computing sight for torpedoes, typically a *lead computer* in the most literal sense of the term, has traditionally come to be called a *director.* Special types of aiming controls for torpedoes, to be described later, more nearly merit such terminology; but in spite of the illogical nature of the term in the present connection we shall use it here in an effort to conform.

airplane. This director derived target-course stabilization directly from the directional gyro, on which it was physically superposed. Fortunately this was possible since that flight instrument was normally placed at the top and in the center of the instrument panel, so that by cutting an opening in the horizontal cowling a mechanical connection could be made from the director, placed directly between the windscreen and the head of the pilot, via an adaptor on the directional gyro to the outer gimbal of that instrument. A clutching arrangement was provided so that the operator might quickly connect or disconnect the stabilizing link between gyro and director, thus making it possible to remove from the directional gyro even the minuscule load imposed by the stabilization on that flight instrument when the director was not in use. The pilot set into the director the several variables mentioned above by the manipulation of a set of small dials which faced him. A stationary post and a movable bead established the variable line of sight in azimuth, which, by maneuvering the airplane, the pilot was to align on the target as an aiming criterion. All computation was by a condensed system of cams and links.

After several experimental models were constructed of this instrument, of which the final units were tested by the Navy, it was placed in production as the torpedo director Mark 32[1] and became standard equipment on some torpedo airplanes.

Several modifications of the torpedo director Mark 32 were experimentally pursued under Section 7.2 projects. Versions were worked out for Army employment, wherein somewhat different operating requirements had to be met. It was necessary, for example, to alter the calibrations in terms of which the input variables appeared, this being admittedly a rather trivial development. In one case, however, the computing mechanism of the Mark 32 was adapted to remote indication of the lead to the pilot, involving both mechanical and electrical repeater components, and with special arrangements for stabilization from existing army equipment and means conveniently to establish the estimated target course. This latter operation, wherein the pilot must be enabled to adjust manually an index of some sort into a direction analogous to that of the target ship, came in for considerable research effort in all the torpedo projects we dealt with. The problems of foolproof clutching, to satisfy both the requirements of lightness and speed in declutching, and also to retain sufficient strength to avoid breakage when wrongly manipulated, were not insignificant. It was desired to permit this input setting to be made by the pilot, with one hand and without needing to see the adjustment he is making, that is while keeping his eyes on the target. Numerous experiments were thus made with stylized ship models which could be grasped in the hand and turned about a vertical axis in the airplane, and which would immediately and uniquely impart, by touch alone, their orientation in comparison to a visual reference.

Projects which required less original development, but which were undertaken in view of specialized experience which had been gained, included those for applying target-course stabilization to other torpedo directors. One such project involved the torpedo director Mark 30. As source of stabilization the reference was generally the directional gyro, adapted somewhat as before for the Mark 32, but now with the attachment of followers such as the Telegon and the Magnesyn connected through a variety of experimental servo channels. These stabilization projects usually appeared in combination with the setting of target course, since the stabilizing action usually *added* to that input, once made.

Stabilization was also provided for a motor torpedo boat director of existing design. This director itself was of simple vector type and was used visually. The target course was incorporated by a mechanical part therein, and was stabilized by appropriate connections with the flux gate compass circuit on the vessel. Following tests in which the equipment was found to function as desired, the whole outfit was delivered to BuShips. In the course of this work, which constituted rather a digression from airborne activities, several suggestions were submitted for alternative modes of operation, to remedy certain defects in the sequence of operations, but these need not be described in detail since they lead to methods precisely analogous

to those proposed for airborne versions. Such proposals are described in Section 6.5. Further on torpedo boat applications, a certain amount of consultation on aiming controls was provided to other groups who were concerned with the "blind" attack problem, employing the SO search radar, for example.

In response to a Navy request, a modification of the torpedo director Mark 32 was worked out which permitted manual stabilization for applications in which this director was intended for standby operation in patrol airplanes, and in which automatic stabilization was not considered feasible. The attraction here was merely compactness of the computer and the presentation, and the extended development involved only the design of an inverted mounting and certain recalibrations.

Although no equipment of such type has been constructed, much thought and certain theoretical work has been done on what were called *two-man directors* for torpedoes. The general scheme of these differs radically, with regard to the sequence of operations, from the standard forms, although in fact the aiming principles, or the equations solved, are identical. Whereas in the regular director the airspeed vector, and hence the torpedo speed vector are inherently incorporated through the fixed orientation of the vector computer in the vehicle, in the two-man version the direction of this vector is the output of the system and provides an index of heading to which the pilot maneuvers the airplane. Further, instead of so flying that the generated line of sight, which is the output of the standard director, points at the target, in the two-man version one operator tracks the target and supplies that direction as an input. Several advantages obtain under this method of operation, simplification of the pilot's task being the principal one. As already pointed out no such directors have been built for airborne use, although possibly this might be of interest for the future, particularly where the newer torpedo bombers contemplate pilot and copilot, sitting close together and side by side. For the PT boat director, however, an experimental adaptation was made in the laboratory for such two-man operation. In the actual operation the helmsman was to steer to a compass course automatically established on his panel, or keep an indicator of course error on or near zero.

In the earlier directors the run of the torpedo was estimated in terms of the range to the target. For relevant cases these variables differ significantly. Thus it was desired, since also the "present range" was latterly available from ARO radar, to develop a director in which this variable, rather than the torpedo run, would be accepted directly by the computer. It had been evident earlier that the transformation, from range to run, could be made instrumentally by a mechanical process of successive approximations. No adequately simple such mechanism was arrived at. Somewhat later it was pointed out by the present writer, however, that a simple linkage existed which accomplished this transformation implicitly. On the basis of this linkage, and of the several variants which it allowed, a series of so-called *present range* directors were constructed, under Army and Navy projects. For descriptions of these directors, see Part II. A brief exposition of the fundamental geometry involved in the computing linkage is given in a subsequent section of the present report.

The vector computer in its simplest form is based on the somewhat dangerous assumption that the target course and the target speed are unchanging quantities from the moment of their estimation until the torpedo gets to its mark. In general this is not the case, and consequently significant error may result from such an assumption. Failure of the course and speed of the target, evaluated at a given instant, to predict the subsequent track thereof in space and time is due to accelerations which are present in the intervening regime. These may begin during the attack, in the form of incipient evasion, or may have been in process in expectation of attack. The smallness of available acceleration and the delay in initiating it, from the enemy point of view, makes the latter type of evasion preferable in typical cases. In any case a serious problem in counterevasion measures is presented. Early features of torpedo directors, intended for outsmarting the prospective evader of torpedoes, included such simple means as an optional modification of the target-speed vector, imposed under choice of the pilot on

observation of existing or impending evasive tactics. Such modification, for application in average circumstances, consisted, for example, in a 15 per cent reduction in target speed, as set into the computer, coupled with a 10-degree rotation of the target course applied in the appropriate sense. An extensive program of study leading toward a more articulate counterevasion development program was undertaken in cooperation between our contractor at The Franklin Institute and the Statistical Research Group under AMP at Columbia. These studies included theoretical investigations, based on available knowledge of the capabilities of enemy targets, and have been embodied latterly in a program for the design of a computer which would incorporate an optimum handling of the evasive situation for practical purposes. (See Part II.)

6.3 THE ELEMENTARY VECTORIAL SOLUTION

The problem[d] of so projecting a torpedo as to hit a stationary target is one largely of putting the torpedo into the water and of directing it straight at the target. From the moment of release until the trip is ended the course thus set will be maintained by the directional controls of the torpedo itself. The distance of the target from the point of release — its range, that is, — need not be accounted for if it is within the maximum range of the torpedo; and neither does the torpedo's own speed. For the conventional torpedo these observations apply equally well when the target has a real velocity but is either receding or advancing on a straight line from the observer, although of course the presentation of the target may not be the most desirable under such conditions.

In directing a torpedo at a moving target it is hoped that the motion of the target will continue unchanged, at least during the run of the torpedo. If the travel time of the projectile were very short, as with a ray of light or other radiation, this hope would be nearly fulfilled. Since the speed of a torpedo is one of the order of that of the target, however, its travel time is not inappreciable but is an important consideration, especially if the motion of the target is of the more general sort.

In general a prospective target, say a ship, will be headed on a course which lies at an angle α to the line of sight from an observer, and will have a certain speed or velocity V_s on that course. Its motion may thus be described by a vector having this speed for magnitude and directed in the above manner. From the same observation point a torpedo is to be projected in such a way as to meet the ship if the latter continues on its present course at its present speed. The torpedo has a known speed V_t in the water (we assume *pro tem* that the torpedo makes the entire trip in the water; air travel corrections are elsewhere considered), and may be directed at some angle β to one side of the line of sight. There is thus formed a "torpedo speed vector." Assume for the present that the torpedo enters the water and attains its speed immediately upon release. Knowing the torpedo speed V_t and having estimated the ship speed V_s and course angle α, the problem is to obtain and employ the appropriate directing angle β, the "lead" angle for the torpedo. (A better symbol would be λ, but again we should run counter to one established convention in trying to follow another.)

Assumed successful, the travel time of the torpedo, subsequent to the moment of release, will be the same as that of the target. The distances traveled, then, by ship and torpedo respectively, will be V_sT and V_tT, T being the travel time, and will form two of the sides of a triangle as shown in Figure 1. The angle α, and if properly chosen the angle β, will be angles of the triangle as shown. These relations suffice to give β in terms of the other quantities, for by application of the law of sines

$$\frac{V_sT}{V_tT} = \frac{V_s}{V_t} = \frac{\sin \beta}{\sin \alpha},$$

and hence

$$\sin \beta = \Omega' \sin \alpha, \qquad (2)$$

where

$$\Omega' = \frac{V_s}{V_t}. \qquad (3)$$

If the target is stationary, V_s vanishes and

[d]Some of the text of the present section, and also of the remaining sections of this chapter, has been adapted from earlier informal reports and notes of the writer; such reports have not otherwise had wide distribution.

so also does β. When the target is moving on the line of sight, as considered before, then α is either zero or 180 degrees and again β is zero. Otherwise β will have a nonzero value

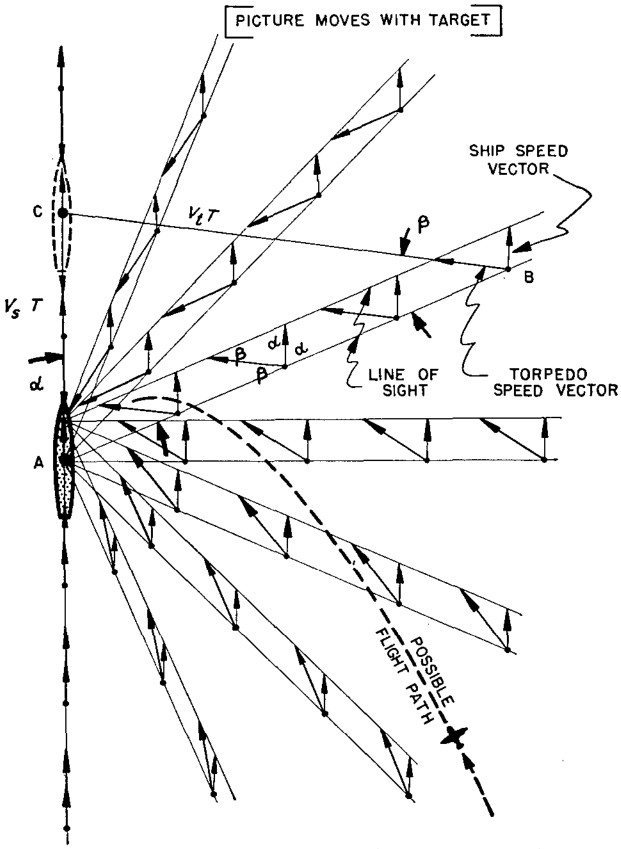


FIGURE 1. Illustrative case showing vector solutions.

and will in any case show the proper direction for projection of the torpedo.

In this solution of the directing problem, it will be noted that the range itself does not enter. Thus in Figure 1, where for a particular case the appropriate torpedo vectors are shown at a variety of surrounding points, these vectors are shown to be unchanging along radial lines from the target. Furthermore the speed of the vehicle is assumed here to be ineffective, the torpedo rapidly assuming a velocity of its own after being released. All information is thus contained in the lead angle β. Since with conventional methods the torpedo's direction upon and following release is that of the longitudinal axis of the airplane, and not the direction of the flight path, it is only necessary that this axis be directed at the angle β from the line of sight. In the torpedo director this is arranged for by automatically turning a sight in the opposite direction through the same angle β with respect to the thrust axis of the plane. When the target is in the sight the plane is properly directed. If it is kept in the sight release may occur at any chosen instant.

In general the desired lead angle β, which is also the sighting angle, in the opposite sense, will not be constant but will vary as the plane comes in on a course such as that shown in Figure 1. The existence of such a course presupposes that the airplane's axis coincides with the instantaneous direction of its flight path, and is one of a number of possible courses obtainable by following the torpedo speed vectors of the figure. This is with the assumption of no sideslip. If there is sideslip the accuracy of aiming is not affected but a different sort of course will be followed. For example the plane may be flown on a path directly toward the target, if it is continuously sideslipped so that the target always appears in the sight. Under this condition the lead angle β will be almost perfectly constant, since the plane speed itself is relatively so great.

The torpedo director in the airplane, which gives the lead angle β by rotation of the sight, must do so through a continuous knowledge of the quantities α, V_s, and V_t. The latter is known in advance, being a property of the torpedo. The two former may be accounted for, assuming the target will continue on an existing or predictable course and speed, by means of a ship speed vector involved in the director mechanism. Having once been set, as by preliminary observation, the magnitude and direction in space of this vector are not allowed to vary. Thus the direction must be stabilized, as with a directional gyro. This ship speed vector, in combination with the torpedo speed vector, the direction of which is merely that of the plane axis, gives the sighting angle automatically. Figure 2 shows three equivalent graphical solutions, set up as though at the point B of Figure 1. In Figures 2A and 2B the direction of sight is shown as the direction of the vector difference between the torpedo speed vector and the (stabilized) ship speed vector. In Figure 2C the sight direction is that of the

sum of the torpedo speed vector with the negative of the ship speed vector.

Naturally there are a number of ways in which this solution may be carried out physi-

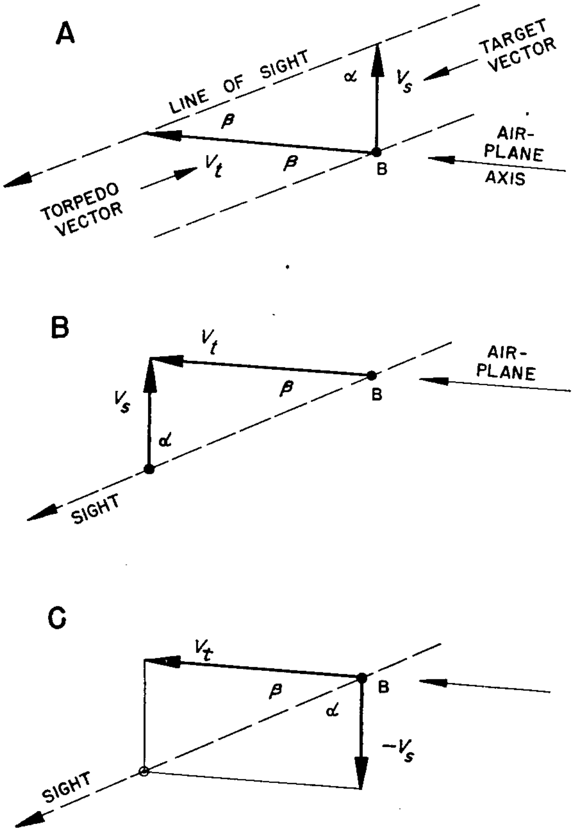


FIGURE 2. Relations at point B of previous figure.

cally. One important problem is to provide for errors which may occur if the plane banks, for it is inconvenient to stabilize the whole director mechanism in space. Optical arrangements are possible so that the target always appears dead ahead in a stationary sight. For example one of a pair of mirrors may be rotated through an angle $\beta/2$. Such proposals, however, have hitherto been unpopular.

It will be noted that it is not absolutely necessary to adjust for both V_s and V_t, since only their ratio Ω' is ultimately involved. It might be thought mechanically simpler to adjust V_t only, so to speak, leaving V_s fixed at some reference length, but this would give an infinite length on the V_t scale when V_s were zero, as for a stationary target.

One may notice, either geometrically or from equation (2), that when $\Omega' > 1$ two solutions for β may result. Both produce hits, although that for the longer torpedo run is ruled out by a mechanical computer. Other troubles arise in the region about $\Omega' = 1$, but fortunately the lower values are predominant in practice.

6.4 RESULTS OF ERRORS IN TARGET MOTION

It is necessary, since this type of director is based on visual estimation of course and speed of the target, to consider the effect of errors made in estimating these properties or in setting them into the director. This study will not completely apply to such errors when due to changes in the properties of the target since last observed, or which are due to such causes as imperfect directional stabilization, over a period of time, within the director itself.

In Figure 3 the general case is shown, where $\mathbf{V}_s'$ is an erroneous setting of the exact target speed vector $\mathbf{V}_s$. In this case both target course and target speed are assumed to be in error. As a result, even though the target is perfectly sighted, the torpedo is incorrectly directed along the supposed torpedo speed vector $\mathbf{V}_t'$, which differs in course by $\Delta\beta$ (radians) from the appropriate such vector $\mathbf{V}_t$. The same analysis will apply for the corrected torpedo speed $\mathbf{V}_{at}$. If the range for the torpedo, or the so-called run, is R as shown, the torpedo would miss a point target having constant speed and course by the amount, approximately

$$M = R\Delta\beta \,. \tag{4}$$

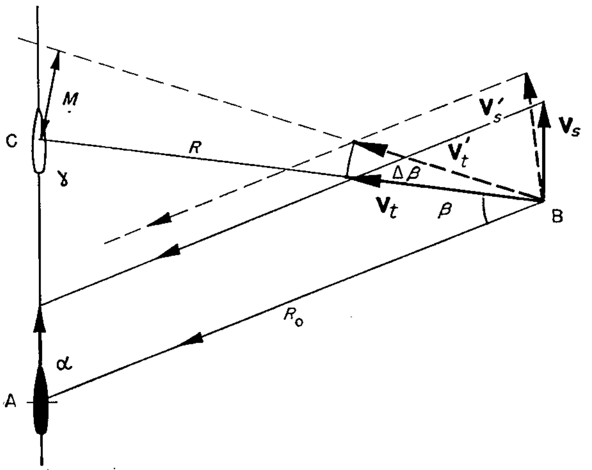


FIGURE 3. Error in estimate of target motion.

The whole problem will be split into two parts; (1) the effect of errors in setting target speed only and (2) the effect of errors in target course only. Figures 4 and 5 are drawn for these two cases respectively.

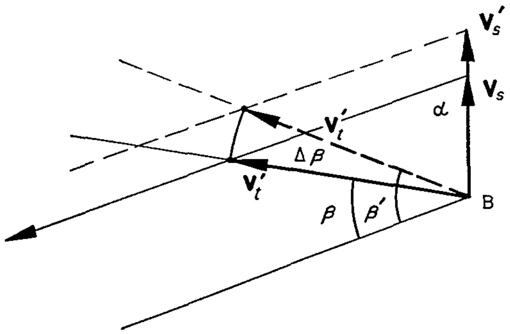


FIGURE 4. Error in estimate of target speed.

The fractional error in setting the length of $\mathbf{V}_s'$ may be called ν

$$\nu = \frac{V_s' - V_s}{V_s}, \tag{5}$$

where the speeds V'_s and V_s are merely the scalar magnitudes of $\mathbf{V}_s'$ and $\mathbf{V}_s$.

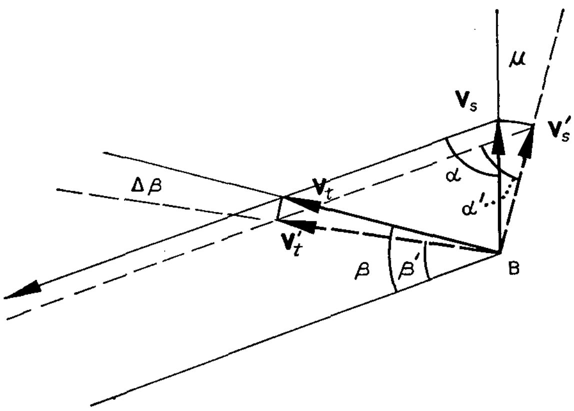


FIGURE 5. Error in estimate of target course.

We find, to a good approximation,

$$\Delta\beta = \frac{\Omega' \sin\alpha}{\sqrt{1 - \Omega'^2 \sin^2\alpha}}\,\nu. \tag{6}$$

This gives the error in torpedo course due to a fractional error in the target speed setting. Let us obtain the corresponding value of $\Delta\beta$ for an angular error μ in the target course setting. Referring to Figure 5, then,

$$\mu = \alpha' - \alpha. \tag{7}$$

We find, corresponding to equation (6),

$$\Delta\beta = \frac{\Omega' \cos\alpha}{\sqrt{1 - \Omega'^2 \sin^2\alpha}}\,\mu. \tag{8}$$

Equations (6) and (8) show errors in torpedo directing due to the fractional errors ν and μ, as functions also of Ω' and α. The amount of the resulting miss depends on the torpedo range as given by equation (4), or by

$$\Delta\beta = \frac{M}{R}.$$

Since a convenient dimensionless form for the miss is M/R_0, R_0 being present range, we may write

$$\epsilon = \frac{M}{R_0} = \frac{R}{R_0}\,\Delta\beta, \tag{9}$$

and consider ϵ_ν and ϵ_μ as the errors, expressed as fractions of present range, due to the initially committed fractional errors ν and μ respectively. Thus

$$\epsilon_\nu = \frac{R}{R_0}\,\frac{\Omega' \sin\alpha}{\sqrt{1 - \Omega'^2 \sin^2\alpha}}\,\nu. \tag{10}$$

$$\epsilon_\mu = \frac{R}{R_0}\,\frac{\Omega' \cos\alpha}{\sqrt{1 - \Omega'^2 \sin^2\alpha}}\,\mu. \tag{11}$$

Now since

$$R \sin(\alpha + \beta) = R_0 \sin\alpha,$$

we have finally:

$$\frac{\epsilon_\nu}{\nu} = \frac{\Omega' \sin\alpha}{1 - \Omega'^2 \sin^2\alpha + \Omega' \cos\alpha \sqrt{1 - \Omega'^2 \sin^2\alpha}}. \tag{12}$$

$$\frac{\epsilon_\mu}{\mu} = \frac{\Omega' \cos\alpha}{1 - \Omega'^2 \sin^2\alpha + \Omega' \cos\alpha \sqrt{1 - \Omega'^2 \sin^2\alpha}}. \tag{13}$$

The functions ϵ_ν/ν and ϵ_μ/μ, of Ω' and α, may be considered as error-transfer factors which show how the errors in setting target properties carry over to a miss on a point target.

By means of the relations

$$\sin\beta = \Omega' \sin\alpha,$$

and

$$\sin\gamma = \sin(\alpha + \beta).$$

The factors in equations (12) and (13) may also be expressed as functions of Ω' and β, or of Ω' and γ, and so on. Results of computation may be graphed or tabulated against these variables for convenience in inspection.

In connection with errors such as those discussed above, especially if the results are to be used to show favorable types of approach, there is an important modification which arises from the varying width of a given target as seen by the torpedo. The effective "diameter" of the usual target, from the torpedo point of view, varies with the angle γ between the target and torpedo courses, and changes from a minimum when $\gamma = 0$ degrees or 180 degrees to a maximum when $\gamma = 90$ degrees. It is the particular shape of the target which determines the variation of this diameter for intermediate values of γ.

If an ellipse is taken as resembling sufficiently closely, say a battleship, then it may be shown (Figure 6) that the ratio of effective diameter to the length is given by

$$\sqrt{1 - \eta \cos^2 \gamma}.$$

An average eccentricity η, for warships, is about 0.988.

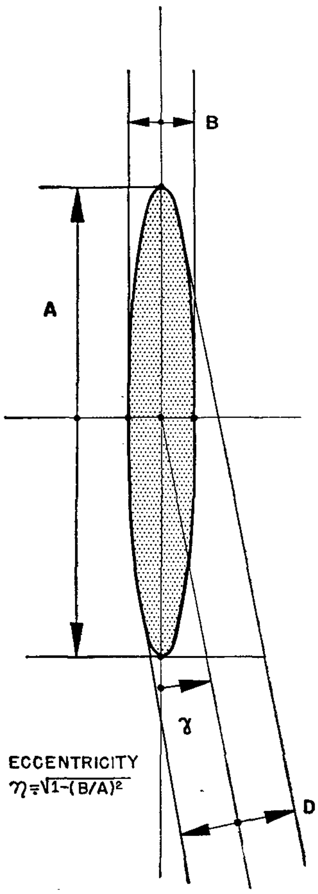


FIGURE 6. Intermediate diameter of ellipse.

6.5 TWO-MAN OPERATED DIRECTORS

With the pilot-operated director described, the principle of which is illustrated for example in Figure 7, the airplane heading is so varied that an automatically computed vector lies along a line of sight to be aligned with the

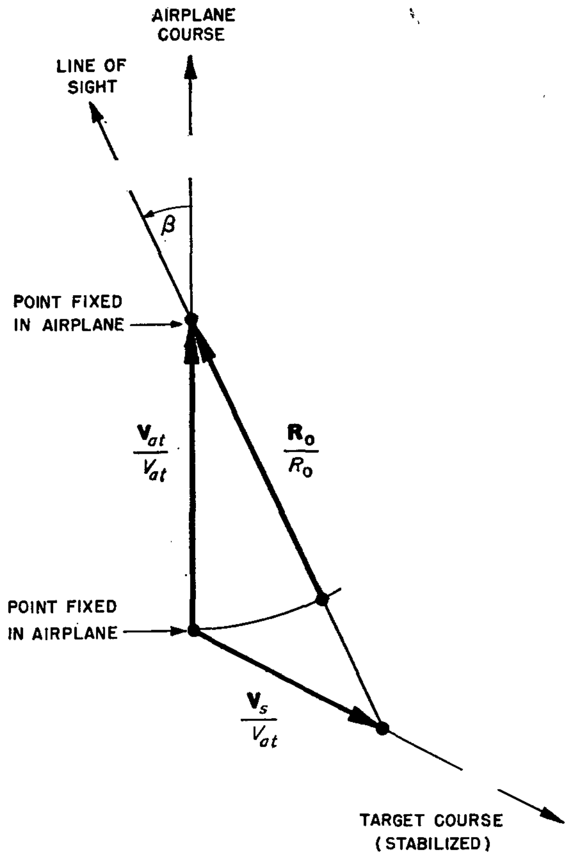


FIGURE 7. Vector form of classical solution.

target. This vector is merely the difference between a unit vector directed as the airplane (and hence as the torpedo) and another vector directed (and stabilized) as the target and of length equal to the ratio Ω of target speed V_s to average torpedo speed V_{at} in air and water.

The latter vector is manually set after estimation of the corresponding properties of the target motion.

Since direction only of the computed sight vector is important, this vector may be replaced by a unit "range" vector $\mathbf{R}_0/R_0$ as shown, making the angle β with the airplane heading.

If, for convenience, the symbol ψ is used for a unit vector, then $\mathbf{R}_0/R_0$ may be denoted by

ψ_r. Similarly $\mathbf{V}_{at}/V_{at} = \mathbf{V}_t/V_t$ may be represented by the symbol ψ_t, and $\mathbf{V}_s/V_s$ by ψ_s. The simplified vector picture is shown in Figure 8. Mathematically,

$$k\psi_r = \psi_t - \Omega\psi_s,$$

k and Ω being scalar quantities. (In fact, k is the ratio R_0/R of target range to torpedo run.)

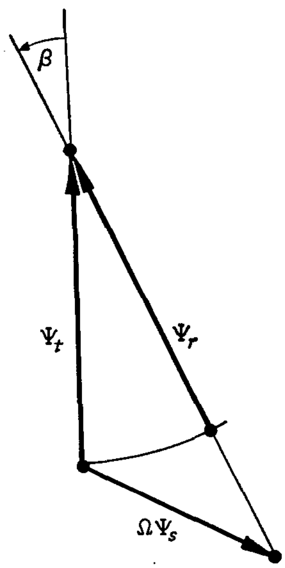


FIGURE 8. Expression of classical solution in unit vectors.

The angle between the sighting line (determined by ψ_r) and the airplane or torpedo direction (determined by ψ_t) is the angle β by which the target must be "led" to secure a hit. It is not constant but varies during any approach of general sort. It may be noted that there are two "sight lines," and hence two different angles which might be called β. One is the actual line of sight to the target and the other is a line in the direction of the pilot's physical sight in the rotation of which the director results. If it is considered that the symbol β describes the latter, the former may be called β'. The pilot's efforts during attack are directed toward making $\beta = \beta'$, which is his release condition. It is important to notice that β varies while the airplane is being operated in an attempt to make good this equality.

It has been proposed that a director might be considered, operating on the same principles as this one, but in which the work of keeping a sight on the target and of directing the airplane is divided between two individuals. The pilot-operated director assigns these duties to one man. This, it should be explained, is one of its merits, in that it is *possible* thus for a single operator to perform these functions. A two-man director would merely be used to simplify the pilot's routine and also to relieve the stringent space restrictions around the cockpit. In the bombing airplanes, for example, the director mechanism might be located at the bomber's station, communicating with the pilot only through the medium of a pilot's direction indicator [PDI]. It is interesting to note that in such a director it would be possible to employ the same mechanical computer and stabilizer as was developed for the pilot-operated version.

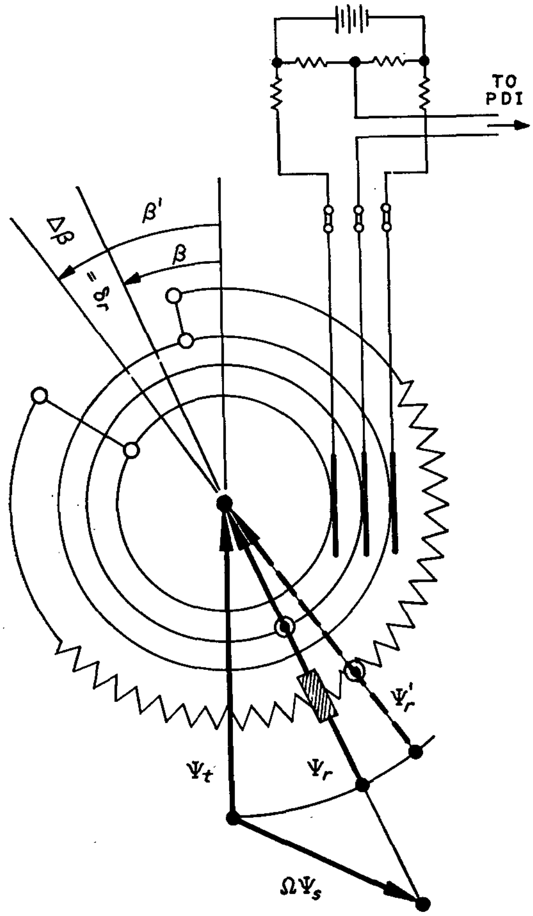


FIGURE 9. Transmitter for two-man director.

Application of the director principle already described to a two-man instrument might be made as shown in Figure 9. With the unit torpedo (or airplane) vector ψ_t still fixed in the

airplane, and the relative target speed vector $\Omega\psi_s$ set in as usual, the unit range vector ψ_r along the resultant shows where the target *should* appear if a solution is to be obtained. If the actual direction of the target is tracked

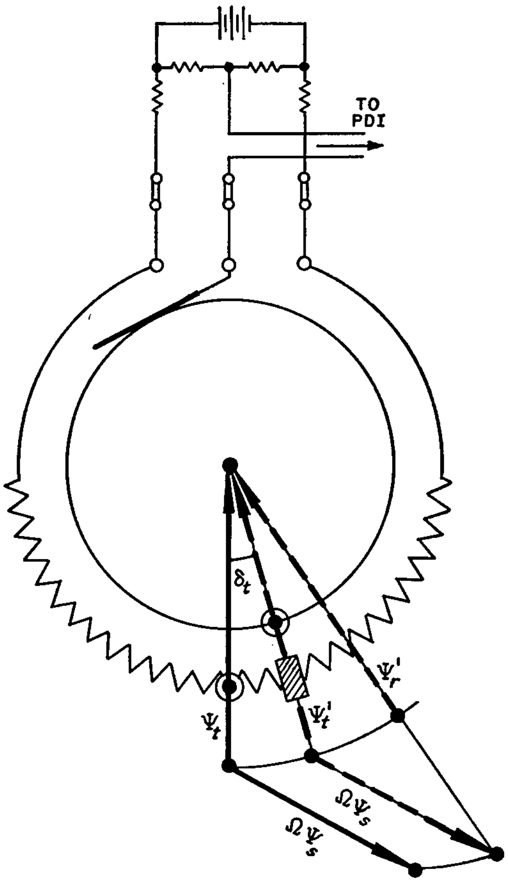


FIGURE 10. Transmitter for two-man director; Form 2.

manually, as determined by another unit range vector ψ_r', the real unit range vector that is, then the angle δ_r between ψ_r and ψ_r' gives a measure of how far the computed sight line is from the actual sight line. This error angle δ_r, which might also be called $\Delta\beta$, as is evident from Figure 9, is an indication likewise of which way the airplane should be turned, though not truly how much, to bring about a solution. That is to say, if $\delta_r = \Delta\beta = 0$, then the airplane is properly directed. If the winding and contact, respectively, of a potentiometer are attached to the unit vectors ψ_r' and ψ_r, the deflection of a meter on the pilot's instrument panel can show him how to direct his airplane.

The mechanism shown in Figure 9 requires three slip rings and a potentiometer winding to be attached to the director mechanism, as well as a downward periscope directed along ψ_r, replacing the pilot's sight. Rotation of the periscopic sight and of the stabilized target vector $(\Omega\psi_s)$ takes place about fixed points: the terminals of the unit torpedo vector ψ_t.

If it were allowable either for the stabilized target vector or for the sighting arm to rotate about a point not perfectly stationary in the airplane, then two alternative forms of two-man director immediately become possible. These are built around the same basic mechanism considered above, but differ from the director schematically shown in Figure 9 in two respects. First: only one slip ring is required, since the potentiometer winding can be stationary. Second: the indication given the pilot may be always directly proportional to the amount δ_r by

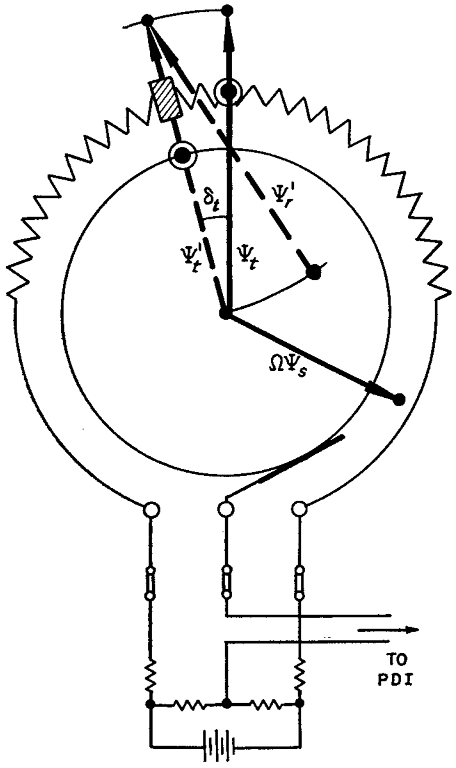


FIGURE 11. Transmitter for two-man director; Form 3.

which he is away from the desired course. In these two variants, which are shown in Figures 10 and 11, the whole director mechanism rotates as the target is tracked.

Returning to Figure 9, here the operator tracks the target directly with ψ_r', having set

in target speed and course which are absorbed in the henceforth stabilized vector $\Omega\psi_s$. Since the unit resultant vector ψ_r will in general not lie along ψ_r', as it should for a solution, there is an angle, having a definite sense or polarity, between ψ_r and ψ_r'. This angle unbalances the bridge circuit shown and thus indicates to the pilot the direction, and in some measure the degree, in which the airplane must be turned. If, however, the airplane is turned merely through the angle δ_r, a residual indication will remain, since ψ_s and ψ_t do not maintain a constant relationship during a turn of the airplane. This should not necessarily prevent reduction of δ_r to zero, but it does indicate action in a closed loop, where stability questions may always arise.

Operation of the two mechanisms of Figures 10 and 11 may be described together. Here the whole vector triangle is rotated about one end or the other of the vector ψ_t' in such a way that ψ_r' tracks the target. The target speed-and-course vector $\Omega\psi_s$ has already been so set that ψ_t' shows the proper heading for the airplane. The actual heading being given by the stationary vector ψ_t, the angle δ_t between ψ_t and ψ_t' is the true angle through which the airplane should be turned. To indicate this angle continuously to the pilot, a potentiometer with stationary winding and a contact point driven by ψ_t', as shown, will suffice.

Since the only continued manipulation required of the operator is that of tracking the target, he would probably find time to keep well up to date the settings of target speed and course. Release of the torpedo, say by the pilot, could be made when he and his "confederate" were both satisfied, as indicated by appropriate signals or interphone discussion, with the general prospects of a hit.

6.6 PROBLEMS OF THE CONVERSION OF PRESENT RANGE AS INPUT

In the torpedo director the ratio

$$\Omega' = \frac{V_s}{V_t}$$

between ship speed and torpedo water speed must for precision be replaced by the corrected ratio

$$\Omega = \frac{V_s}{V_{at}}$$

between ship speed and an average torpedo speed applying to the combined air and water trip of the torpedo. Transformation from Ω' to Ω may be made reasonably well according to the formula

$$\frac{\Omega}{\Omega'} \doteq 1 - \frac{\sqrt{h}\,(V_a - V_t)}{7.12\,R} \qquad (14)$$

where h is the altitude in feet at release, V_a is the component in knots of the plane's ground speed in the direction of the plane's heading, V_t is the torpedo water speed in knots, and R is the torpedo run in yards.

From the standpoint of the mechanism involving these quantities, it is convenient to consider

$$V_{as} = \frac{V_s V_t}{V_{at}} \qquad (15)$$

as a sort of corrected ship speed, so that

$$\frac{\Omega}{\Omega'} = \frac{V_t}{V_{at}} = \frac{V_{as}}{V_s}. \qquad (16)$$

Thus a mechanical lever length in the director corresponding to V_t may be left fixed, and the air travel correction applied to another lever length representing V_s, which is hereby changed to a length V_{as} in the ratio given, say, by equation (14). Still further, the length of the V_t arm may be fixed and defined as unity and length of the V_s arm set at Ω' (or at Ω for the corrected case) on the numerical scale thus defined.

In practice a dial is calibrated in V_s itself and the correction given by equation (14) is included automatically; h, V_a, and R being set in normally on other dials.

The altitude may be difficult to obtain with any precision, unless special instrumentation is available. V_a, it will be noted, is not the airspeed itself unless no wind is experienced, so that a precise setting here may depend on knowledge of speed and direction of the wind. However, if these quantities are at all exactly obtainable it may be worth while to consider how the estimation of torpedo run R may be

improved. Substitution of present range R_0 for R can be in considerable error. It has been said that R may be estimated as exactly as may R_0, but this means only that the usual estimates of R_0 are poor. If it is possible, by auxiliary means, to determine R_0 closely, it will be appropriate to consider how conversion from R_0 to R may be made without losing this precision. This is probably only advantageous if also reasonably close determination of h and V_a are also possible.

The director mechanism possesses a variable dimension which corresponds in magnitude to that of the second term on the right-hand side of equation (14), which may equally well be written

$$\rho \frac{\sqrt{h}\,(V_a - V_t)}{7.12\,R_0}$$

where

$$\rho = \frac{R_0}{R}.$$

If this dimension is set on the assumption that $\rho = 1$, or in other words if present range R_0 is substituted for torpedo run R in the correction adjustment, and if then this dimension be altered in the ratio ρ, the result will be the same as though the run R had originally been employed. It remains only to determine and employ the ratio ρ.

If the air travel correction for torpedo speed be temporarily neglected, a value for ρ may be obtained which depends on any pair of the quantities: α, β, γ, Ω'. Such a value would not be far from the precise value, which latter could not be explicitly obtained, by the arrangement described above, because a knowledge of ρ itself is needed for the correct determination of ρ (cf. Section 6.8).

A more nearly precise value for ρ would result when the air speed correction were made by substituting R_0 for R, and this is presumably the way the mechanism might work. A series of operations would occur within the director, each operation leading to a more exact value for ρ. It is evident that a closed causal loop is involved, as in a normal regulatory circuit. The success and effectiveness of its operation depends on how rapidly convergence takes place, if it does.

Suppose, then, that with some angle α in effect, the speed ratio Ω and the angles β and γ have been determined on the basis of an air travel correction which substituted present

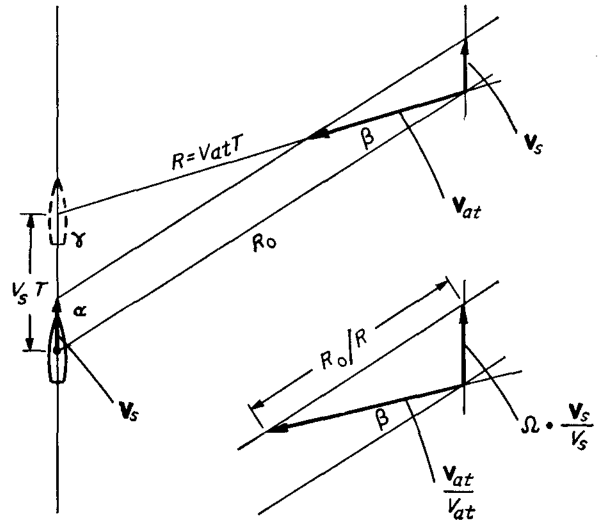


FIGURE 12. Embodiment of range-to-sun ratio.

range R_0 for torpedo run R. It is evident from Figure 12 that the vector difference

$$\mathbf{V}_{at} - \mathbf{V}_s$$

has a magnitude equal to

$$\frac{R_0\,V_{at}}{R}.$$

In the normalized form then, as shown by the inset in this figure, where the vector $\mathbf{V}_{at}$ is replaced by the unit vector $\mathbf{V}_{at}/V_{at}$ and where $\mathbf{V}_s$ is replaced by $\Omega\mathbf{V}_s/V_s$, the corresponding vector difference

$$\frac{\mathbf{V}_{at}}{V_{at}} - \Omega\frac{\mathbf{V}_s}{V_s} \tag{17}$$

has a magnitude which is simply

$$\frac{R_0}{R} = \rho.$$

Thus in the director mechanism as planned there is a mechanical distance which may be calibrated directly in the ratio ρ. It is merely the distance, along the sighting direction, between the pivot points corresponding to the two velocity vectors involved. In other words it is the length of the vector given by equation (17).

It is actually possible to accomplish this correction explicitly, without the successive approximation involved in the above method. See, however, the simpler solution given below. By the above relations, we have

$$\frac{\Omega}{\Omega'} = 1 - \rho\xi$$

where

$$\xi = \frac{\sqrt{h}\,(V_a - V_t)}{7.12R_0}.$$

The factor ξ involves only known and explicitly determinable quantities.

From Figure 12 we have

$$\xi = \frac{R_0}{R} = \frac{\sin\gamma}{\sin\alpha} = \frac{\sin(\alpha+\beta)}{\sin\alpha},$$

and of course

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta.$$

The director is based on the exact relation

$$\sin\beta = \Omega\sin\alpha.$$

Thus

$$\rho = \sqrt{1 - \Omega^2\sin^2\alpha} + \Omega\cos\alpha,$$

and consequently,

$$\frac{\Omega}{\Omega'} = 1 - \xi\sqrt{1 - \Omega^2\sin^2\alpha} - \Omega\xi\cos\alpha.$$

This leads to the following quadratic equation in Ω:

$$\left(\frac{1}{\Omega'^2} + 2\,\frac{\xi\cos\alpha}{\Omega'} + \xi^2\right)\Omega^2 - 2\left(\frac{1}{\Omega'} + \xi\cos\alpha\right)\Omega + 1 - \xi^2 = 0.$$

6.7 THE "COMPLETE SOLUTION"

All previous solutions have been based on certain simplifying assumptions. The necessity for evaluation of certain instrumental and tactical approximations creates the need for a closed and explicit solution, without these assumptions, against which proposed approximations could be tested in a precise and quantitative manner. No difficulties were encountered in obtaining such a solution, although mechanization on the basis of this solution was not primarily intended.

Only in a relative sense is the term *complete* applicable to the solution given here. It should be understood that the following assumptions are retained.

1. The target continues on a straight and unaccelerated path during the entire attack.
2. The torpedo water course parallels the aircraft heading at release, terminal water speed and direction being attained immediately upon entry.
3. The aircraft is in level flight at the instant of release.
4. Aerial ballistic effects produce no distortion from a vacuum trajectory for the torpedo.
5. The heading or thrust-direction of the aircraft defines the instantaneous direction of its path in the air.

It is unimportant, for the type of director concerned (see Figure 13), whether α or $\bar{\gamma}$

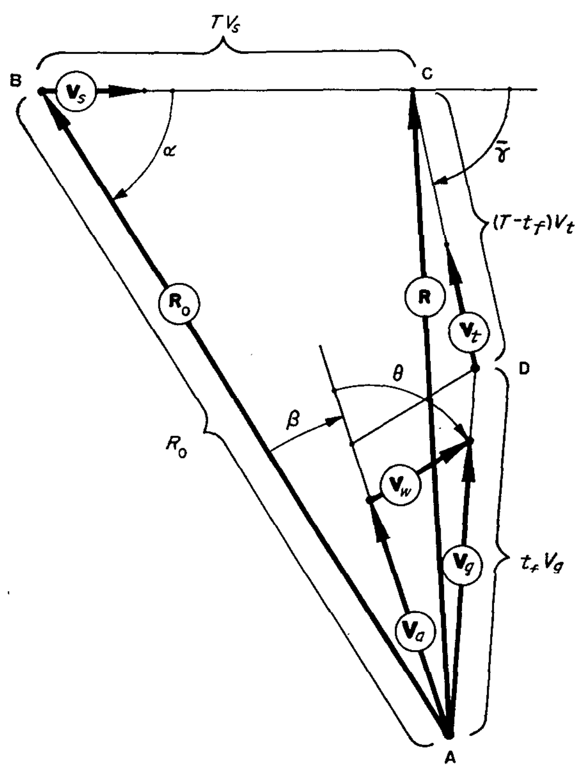


FIGURE 13. Space diagram for straight-run torpedoing.

is considered as the primary known target angle. Each is uniquely determined when target aspect is estimated and set, and each is automatically readjusted by the stabilizing agent. Both cannot be assumed known, since jointly

they determine β, which is the principal unknown. Solution for β may be expressed in terms either of α or of γ, and in this case the latter will be taken. The run R is an unknown, but is here assumed to be computed inherently in the process resulting in a true value for β. The initial range R_0 is assumed known. Wind velocity, airspeed, torpedo water speed, and target speed, or V_w, V_a, V_t, and V_s, are all assumed to be known, as is also the wind bearing θ with respect to aircraft heading.

In connection with θ, it should be pointed out that this is the angle between the aircraft heading and the *positive* direction of the wind. The supplement of θ gives the angle from heading to the direction of the wind *source*, to which reference is usually made in denoting the compass bearing of wind. It will be noted from the figure that the angle between target heading and the compass bearing of the wind is simply $\alpha + \beta + \theta$ or $\gamma + \theta$.

From assumptions 3 and 4 we may relate altitude h and time of flight t_f by the formula

$$h = \frac{g t_f^2}{2},$$

or, in feet and seconds, approximately

$$t_f = \frac{\sqrt{h}}{4}. \tag{18}$$

From the figure and by assumption 2,

$$\alpha + \beta = \bar{\gamma} = \pi - \gamma. \tag{19}$$

Again by assumption 2,

$$\mathbf{V}_t = (V_t/V_a)\,\mathbf{V}_a. \tag{20}$$

Noting that

$$\mathbf{R} = t_f \mathbf{V}_g + (T - t_f)\,\mathbf{V}_t,$$

and that

$$\mathbf{V}_g = \mathbf{V}_a + \mathbf{V}_w,$$

and adding vectors around the circuit,

$$\mathbf{R}_0 + T\mathbf{V}_s = t_f \mathbf{V}_a + t_f \mathbf{V}_w + (T - t_f)\mathbf{V}_t;$$

or, by virtue of equation (20),

$$\mathbf{R}_0 + T\mathbf{V}_s = t_f \mathbf{V}_w + [TV_t + (V_a - V_t)t_f]\frac{\mathbf{V}_a}{V_a}. \tag{21}$$

For solution in terms of γ it is most convenient to take components along and normal to the aircraft heading, or the torpedo water run. Thus

$$R_0 \sin\beta = TV_s \sin\bar{\gamma} - t_f V_w \sin\theta, \tag{22}$$

$$R_0 \cos\beta = TV_s \cos\bar{\gamma} + t_f V_w \cos\theta + TV_t + t_f(V_a - V_t). \tag{23}$$

Now using the abbreviations, and appropriate physical units,

$$\xi_0 = \frac{\sqrt{H}\,(V_a - V_t)}{4R_0}, \tag{24}$$

$$\Omega_s = \frac{V_s}{V_t}, \tag{25}$$

$$\Omega_w = \frac{V_w}{(V_a - V_t)}, \tag{26}$$

$$\tan\beta_0 = \frac{\Omega_s \sin\bar{\gamma}}{1 + \Omega_s \cos\bar{\gamma}}, \tag{27}$$

$$\bar{\Omega} = \xi_0\left(1 + \Omega_w \frac{\sin(\bar{\gamma} + \theta)}{\sin\bar{\gamma}} + \frac{\Omega_w}{\Omega_s}\frac{\sin\theta}{\sin\bar{\gamma}}\right), \tag{28}$$

we obtain, from equations (22) and (23),

$$\tan\beta = \tan\beta_0\left(1 - \frac{\bar{\Omega}}{\cos\beta}\right), \tag{29}$$

or

$$\tan\beta = \tan\beta_0\left(1 - \bar{\Omega}\sqrt{1 + \tan^2\beta}\right). \tag{30}$$

The forms of equations (29) and (30) should be convenient for certain purposes, but they must be reworked for an explicit solution. Transforming equation (30), and squaring,

$$\tan^2\beta - 2\tan\beta_0\tan\beta + \tan^2\beta_0 = \bar{\Omega}^2(1 + \tan^2\beta)\tan^2\beta_0,$$

or

$$\left(\frac{\tan\beta}{\tan\beta_0}\right)^2 - 2\frac{\tan\beta}{\tan\beta_0} + 1 = \bar{\Omega}^2(1 + \tan^2\beta),$$

or, again

$$(1 - \bar{\Omega}^2\tan^2\beta_0)\left(\frac{\tan\beta}{\tan\beta_0}\right)^2 - 2\frac{\tan\beta}{\tan\beta_0} + 1 - \bar{\Omega}^2 = 0.$$

Solving this for

$$\frac{\tan\beta}{\tan\beta_0}$$

we have

$$\tan\beta = \tan\beta_0\frac{1 - \sqrt{1 - (1 - \bar{\Omega}^2)(1 - \bar{\Omega}^2\tan^2\beta_0)}}{1 - \bar{\Omega}^2\tan^2\beta_0}. \tag{31}$$

When $\Omega_w = 0$, for no wind, $\Omega = \xi_0$ and

$$\tan\beta = \tan\beta_0 \frac{1 - \sqrt{1 - (1 - \xi_0^2)(1 - \xi_0^2 \tan^2\beta_0)}}{1 - \xi_0^2 \tan^2\beta_0}$$

When $\xi_0 = 0$ there is no correction for air travel, and both equation (31) and this one reduce to

$$\beta = \beta_0,$$

except for the relatively unimportant case where $V_a = V_t$, which must be specially handled.

Equation (31) gives $\tan\beta$ in terms of $\tan\beta_0$ and the composite wind and air travel correction Ω. It is also possible, and in many cases more opportune, to offer $\sin\beta$ in terms of Ω and $\sin\beta_0$, where

$$\sin\beta_0 = \Omega_s \sin\alpha \qquad (\Omega_s = \Omega').$$

Returning, then, to equation (29), and multiplying by $\cos\beta$ we have

$$\sin\beta = \tan\beta_0 (\cos\beta - \Omega), \tag{32}$$

or

$$\sin\beta + \bar{\Omega}\tan\beta_0 = \tan\beta_0 \sqrt{1 - \sin^2\beta};$$

squaring and rearranging,

$$\sin^2\beta + 2\bar{\Omega}\sin\beta_0\cos\beta_0\sin\beta - (1 - \bar{\Omega}^2)\sin^2\beta_0 = 0.$$

Solving this equation as a quadratic in $\sin\beta$,

$$\sin\beta = -\bar{\Omega}\sin\beta_0\cos\beta_0 + \sqrt{\bar{\Omega}^2\sin^2\beta_0\cos^2\beta_0 + (1 - \bar{\Omega}^2)\sin^2\beta_0};$$

and finally

$$\sin\beta = \sin\beta_0\left(\sqrt{1 - \bar{\Omega}^2\sin^2\beta_0} - \bar{\Omega}\sqrt{1 - \sin^2\beta_0}\right).$$

For no wind, $\bar{\Omega} = \xi_0$. For no air travel correction whatsoever,

$$\bar{\Omega} = \xi_0 = 0$$

and

$$\sin\beta = \sin\beta_0.$$

6.8 LINKAGE FOR IMPLICIT RANGE CONVERSION

We referred above to a simple linkage solution for the present-range problem. Having shown the proposals for computation by successive approximations, and having given the more complex explicit expressions of the last section, we now show the basis of the simpler procedure which was subsequently employed for all directors having present range as an input.

We refer to equation (31) above, and to the accompanying Figure 14. The uncorrected lead

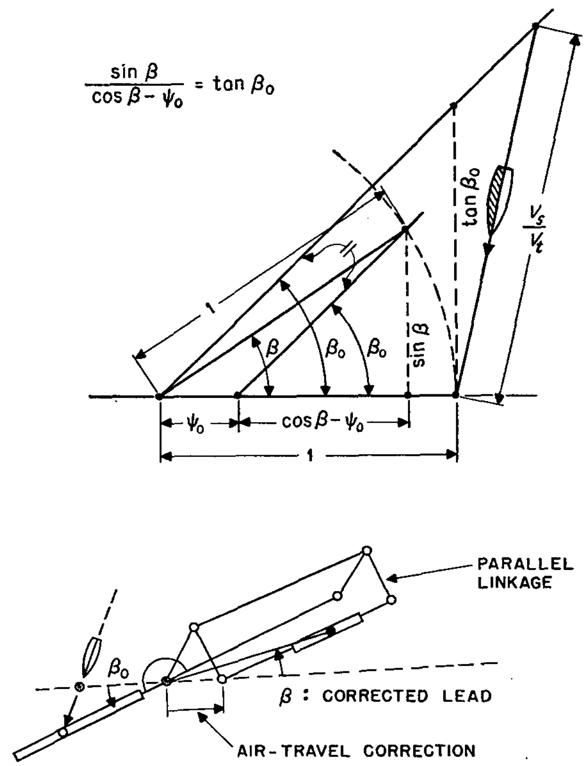


FIGURE 14. Diagram for present-range linkage.

angle β_0 is assumed to be obtained in the usual way, as for example from equation (26) above. Assume, for convenience, that the wind correction is unnecessary, so that Ω reduces to ξ. It is now geometrically evident that by the choice of unit length as indicated in the figure, and the enforced parallelism, that equation (31) is embodied by virtue of the similar right triangles completed by the construction lines. The computing linkage follows readily from this diagram.

Chapter 7

AIMING OF BOMBS FROM AIRPLANES

7.1 OUTLINE OF DEVELOPMENT PROJECTS

We list first, approximately in historical order, and aside from the relative emphasis they will receive elsewhere in this chapter, all except the most trivial of the bombing projects in which we have engaged for NDRC. Subjects related to guided bombing are omitted, since these are treated exclusively in Chapter 8. Our activities have been on the following topics: bombing aspects of the airborne torpedo, and of similar experimental projectiles; hand-held bombsights, principally for low altitude, including those specially prepared for antisubmarine attacks, as well as those for use in blimps, and extending to the theory and development of the special angular rate methods; computers for ground speed as an aid in bombing operations of several types; computers for slant range at release; computers of the auxiliary type known as *preset* for advance adjustments in synchronous high-altitude bombing; computers for statistical evaluation of the optimum length of train in terms of tactical and instrumental circumstances; modification of high altitude techniques for increased effectiveness with incendiary projectiles; instrumental developments for toss bombing; theoretical work on the errors incurred in low-altitude bombing by comparable techniques; theoretical work on flight paths on which release may occur over an interval, on the angular-rate principles which may then be used as criteria, and on the special cases for climbing approach and for vertical dives to which these paths reduce; and on certain model equipment for training purposes.

The most prominent projects, on which effort was expended on a considerable scale, were those for the various hand-held bombsights, for error-analysis of low-altitude methods, and for a computer to be used in a toss bombing system (as a component, in fact, of the pilot's universal sighting system: Project PUSS, NO-265). These projects will be discussed more fully than the rest, and Section 7.4 will therefore be devoted to them in this chapter, in company also with certain related theoretical work not hitherto promulgated. The rest, comprising a good number of smaller endeavors, coordinated in some cases with the work of other groups, we shall discuss first in somewhat broader terms and with one eye on historical sequence and significances.

7.2 VARIOUS RESEARCHES – A COMPENDIUM

We had not discussed, in our prefatory comments, or elsewhere heretofore, the general plan of this report, beyond pointing out that it was to be of flexible arrangement, being composed substantially of a set of independent monographs. For a specific reason this is not the case in a strictly literal sense; and we explain: Chapters 1 through 5 have dealt with techniques and components which have applied more or less horizontally over the various fields of aiming-control development. In Chapters 6 through 10, the individual developments and problems of these latter fields are considered in sequence, vertically. References are natural between these two groups of chapters, representing as they do the two distinct modes of presentation. Thus, in several cases, a technique, treated in the earlier group in general, has been there illustrated as applying to a development justifiably assignable to the latter group. It is natural, in such a case, that reference be later made to the previous discussion or exposition. The attempt will be maintained, however, especially in connection with material which is considered worthy of preservation, to avoid the obvious failing whereby such subject matter may be inadequately treated through reciprocal reference to another context for amplification; and on the other hand we shall not indulge the temptation bodily to reproduce a given exposition in more than one spot on the thesis of complete independence or self-sufficiency for each monograph.

A slight apology is offered, incidentally, for including this explanation here, but it serves thus as a reminder to the writer at a juncture where a rather scattered account of general activity is begun.

To return to bombs, it seems as though we had been preoccupied right from the beginning, from our beginning at least, with trajectories, their calculation, their approximation, or contriving to suit them in some sense or other. Bombs are only one example. Torpedoes falling through the air furnished several problems in which we became involved. Questions on this score concerned the relations between their air trajectory with respect to the air and the same trajectory measured in water coordinates. These relations were important partly because the water-entry characteristics depended on the alignment of the torpedo itself with its water coordinate trajectory, and partly because the aiming problem properly must be solved in recognition of the entry position as well as of the compass direction to which it is automatically steered following such entry. Again, the effects on the air travel of glide and skid must be considered, since neglect of these produces errors which are found significant in practice. The ability of the torpedo to resist impact and turning torques on entry further placed restrictions on the bombing aspects of its use. Underwater trajectories, or orbits as they have come in some cases to be called, are naturally crucial for torpedoes and relate closely to their air travel behavior; such properties after entry as the "hook," so called, and the turning radius by which the torpedo comes over to its set course, are important to the aiming problem. The details of behavior underwater arise also for the airborne depth charge, such as used against submarines, and for the plunge bomb.[a] In rockets, as used against shipping, a predictable and effective underwater trajectory is one of the most important desirables, and the character of such trajectories, as well as those in the air, affects the design of aiming controls. We are discussing such controls, for depth charges and for rockets, in another place. The case of the plunge bomb is less related to other work, however. This missile was not to be self-propelled, either in the air or in the water, but was to be so designed aerodynamically and hydrodynamically as to have most nearly ideal properties in both media. Model studies had shown that very long and flat underwater orbits were attainable by special shapes, and the plan was to make a projectile of large pay load ratio which would also be simple to handle strategically, or logistically. High-speed launching was considered, in either horizontal or climbing approaches, to attain great range.[b]

In any bombing problem the speed of the launching vehicle with respect to the air is far less important than its speed with respect to the ground, or ultimately with respect to the target. Yet it is the airspeed which is available by local measurement. The synchronous bombsights invoke an implicit measurement of speed in target coordinates for the principal part of their solution, applying airspeed only for the aerodynamic trail corrections and so on. In other methods of bombing, wind and target speed may require estimation by the operators, so that their joint effect, with airspeed, on the ground speed or on the speed in target coordinates may be measured. Even where wind, for example, may successfully be determined in intensity and direction, and this requires either a saltier proficiency than is typically at hand or a time-consuming series of maneuvers, the corresponding process of vector addition is very difficult in an emergency, and seldom, in any case, leads to high accuracy. For such reasons we were led, as others have been, to the development of mechanical computers for these quantities, that is for airplane speed with respect to ground and target, which would be compact and simple to operate, and which would offer improvements over the then available rather large and clumsy plotters. One such project was for a ground speed computer to be used in connection with handheld low-altitude bombsights which were simultaneously in development. This computer accepted airspeed, wind speed, airplane heading, and wind direction; it gave not only ground

[a]Proposed by Slichter.

[b]We worked with Dr. Slichter on the question of aiming controls for such a bomb, and computed probable errors using a variety of the available sights, standard and experimental. The whole proposal appeared to be worth more than a little attention, but no construction was ever resorted to in our branch of this item.

speed, but by a later attachment, the drift angle. Four experimental versions were built, leading to the final one which was produced to the extent of twenty units. The resulting instrument was considered quite successful in meeting the specifications laid down. It was mountable almost anywhere, even in the most restricted confines, and could be operated very quickly with one hand, even when wearing a heavy glove. A geometrical vector principle was involved, with a single sliding linkage, and the setting scales were cylindrical and coaxial. In one form a miniature, rotatable ship model could be attached to aid in computing the additional course change required by target speed. A report on this item is listed in the bibliography appended to the present chapter.

Another ground speed computer, of very different sort, was designed by the writer in answer to an expressed need by Wright Field Armament Laboratory and H2X radar researchers. The problem was to provide, in a horizontal bombing approach at known altitude, the ground speed — or target speed — by means of two successive measurements of the slant range to the target and of the time intervening between them. A novel nomogrammic method was applied, and drawings for the resulting manual computer were supplied.[c] A model was built and tried out by the 20th Air Force.

For low-altitude blind bombing, in which BuAer desired a rough and ready means to bomb on the radar range signal, a manually operated computer was built which provided the slant range at which release should occur. Inputs were altitude and closing speed, both measurable by "blind" methods. Altitude came from the radio altimeter, and closing speed from the range rate at ranges great in comparison to the altitude. The instrument which resulted, being extremely small and flat, and operable with one hand, was considered successful as specified, but was antiquated almost immediately by the development of other techniques.

For higher altitudes, that is to say above 5,000 feet, a somewhat more ambitious development program was undertaken to construct computers as auxiliaries to the synchronous bombsight. In this work we cooperated in a more general program with the Aircraft Research Section of BuOrd, with whom in fact most of our airborne developments were coordinated. The principal auxiliary instrument involved thus has been the so-called *preset* computer, of which three separate developmental models were prepared. In operation, these computers were either attached to the auxiliary (vectorial) ground speed computer of the Norden bombsight or, as in one case, they were to be of the hand-held variety. The original idea was to enable a preliminary and approximate solution of the bombing problem, on the approach, so that either this solution could be employed as such if the synchronous method proved impossible or inappropriate, or to permit initial settings in the synchronous mechanism which would lead more rapidly and/or more precisely to an effective attack therewith. An added requirement, of the preset-computation, was to inform the operator of the moment at which, after maximum delay for evasion and so on, he must begin his synchronous operation. To the existing ground speed computer, which normally accepted airspeed, wind speed, and wind direction (the latter being subsequently maintained by the azimuth stabilizer of the bombsight), and which was sometimes employed for low altitude bombing independently of the synchronous instrument, was added a target speed component wherein the course and speed of a ship target could be set. Stabilization applied then both to the wind and target speed vectors. From this component of the system an approximate drift angle could be derived, for initially guiding the pilot, and, most significantly, a measure of the closing speed was obtained. The latter, as a mechanical displacement, fed the preset computer. Manual settings were made of altitude, time of flight, trail, and bomb type; reference to bombing tables being admitted since this was necessary, anyhow, for the synchronous bombsight itself. (Several projects for mechanizing the tables were known of, and it is evident in retrospect that this would not have been difficult. A complete linkage computer for this purpose, combined with the preset computer, could probably have been readily prepared, with the application of techniques described elsewhere.) The output of the preset

[c]To Captain F. Best.

computer, which was essentially a geometrical, or triangular, representation of the well-known bombing formula, consisted in two scale readings. One was the tangent of the probable dropping angle, to be set into the bombsight via the "rate" knob. The other was the tangent of that target angle, as observable on the bombsight angle-indicator, at which synchronous control should begin. Computation of the latter was made through recognition of the (adjustable) time interval during which synchronization could effectively be performed, and interpretation of this interval in terms of the corresponding (and displaced) value of the tangent of target angle. The first models having been large and flat, subsequent models were very much more compact. Tests were made at NAS Banana River.[d] The results were reportedly satisfactory, but production was not planned at last knowledge.

In collaboration with AMP,[e] and the Statistical Research Group at Princeton, a study was made of instrumental application of work they had done on the optimum length of train, or the interval between successive bombs in a train. Several computers were experimentally prepared, based on slide rules which were presented, which permitted settings of: number of bombs, altitude, probable dropping error, dimensions or type of target, and the angle of approach thereto. The answer appeared as the displacement in space between adjacent bombs, this being set, together with the airplane speed, on the intervalometer. Various models of the computer, in which several compromises occurred between ease of setting and compactness, were prepared. One model was directly attachable to the intervalometer itself, so that the spacing which was computed would be delivered automatically to that instrument.

Following requests from more than one source, a certain amount of work was done on the question of dropping incendiaries from high altitudes. The classical bombsights were unusable owing to the excessive trail of these bombs, which surpassed the limits of the mechanism. It was desired in particular to drop both high explosives and incendiaries on the same spot. This is evidently possible through the regular bombing procedure, applied, say, by a synchronous bombsight for the standard projectile, followed by a regulated maneuver and subsequent release of the incendiaries, which, then, if the solution was correct, arrive at the target somewhat later. Knowing the time of flight and trail for the standard bombs, and also those properties under the circumstances of the attack for the incendiaries, it was shown how to compute, by a determinate vectorial solution, the course change and the added time interval to come to the secondary dropping point after the initial release was made. This solution was shown personally to several interested persons. Subsequently a similar solution to the problem was heard of, apparently found by independent search.

A certain amount of thought was given by the writer to a particular method of bombing.[f] This method, which at first glance may seem somewhat naïve, came to be called *Zenith-Bombing* or "Z-Bombing," and involved a precipitate diving approach from directly above the target. The plan was to make the path with respect to the target extend vertically over the latter, for then the *in vacuo* solution would be exact, and the trail would be quite small owing to the small horizontal component of airspeed. Of course the path in the air mass would be inclined, in the presence of wind, and consequently the heading itself would be out of the vertical in general. One problem is how to assure, if indeed it is essential, that the airplane is not "upside down" in such circumstances. On the instrumental side, a good (although special) gyro horizon would be the basic mechanism, accuracy otherwise depending on the variables of the problem only to a second order. Dynamically, it is most important to arrange for stable subsidence to the vertical dive, and numerous measures have been considered conjecturally for this purpose, including those involving the application of angular accelerometers. One advantage, probably only temporary, was considered to be the practical difficulty experienced by typical gun emplacements at the target, say a warship, in firing straight up. This would not

[d] In cooperation with (then) Lieutenant H. G. Cooper.

[e] Principally with H. H. Germond.

[f] Discussed at length with Commander E. S. Gwathmey of BuOrd.

deter support fire in quite the same manner, however.

A project was undertaken at The Franklin Institute on the development of a maneuverable target for the standard cart type of bombing trainer. In such training it had been common practice to simulate a ship target by means of a "bug" which was driven along the floor in a straight line at adjustable speed. It was considered that a generalized "bug" could be made which, under the choice of an instructor, could execute turns as would a ship under attacks. Such turns are not strictly arcs of circles, unless a continuous series of turns is made, or unless the rudder has been held still for some time, so that the intermediate transitional character of the evasion had also to be represented.[g] Six models of the final version, which was rather luxuriously supplied with selective controls, were recently completed and delivered.

7.3 CLASSICAL METHODS AND INSTRUMENTS

The adjective *classical* is meant here to imply horizontal bombing at medium and high altitudes, although all techniques in this field are not old ones. Generalization to bombing in nonhorizontal flight, while not trivial, is straightforward for reasonable climbs and glides, provided the motion is unaccelerated. The distinction between low and high altitude bombing is principally determined by whether or not trail may be neglected. We may state the classical principle of bombing in terms of what happens to a standard bomb when it is dropped. In air coordinates, it falls down and forward in the vertical plane containing the instantaneous flight path at the instant of release. Its downward progress is slower than *in vacuo,* either the total time from any altitude, the time of flight, that is, or the difference between this and the time of flight *in vacuo,* called the *differential* time of flight, being given by known functions of altitude and airspeed. Its forward progress in the air mass is merely the product of airspeed and time of flight, from which is rectilinearly subtracted the so-called trail distance, or trail, at least for the whole drop, this trail being also known as a function of airspeed and altitude. In ground or target coordinates, one need only add the motion of the air, or the wind. All standard bombing theories are the direct result of these facts, but a few special results may be pointed out. *In vacuo,* and when flying horizontally, it is only necessary to fly in the invariant vertical plane containing the target and to drop the bomb when the target appears at an angle from the vertical whose tangent is the square root of $2V^2/(gh)$, V being the airplane speed in target coordinates. The arrival at this condition may be determined on the basis of many other relations, as when the slant range is the square root of $h^2 + 2V^2h/g$, or when the absolute angular rate of the target, in radians per second, is the reciprocal of $(2V/g) + (h/V)$. Other complications enter, even *in vacuo,* when one wishes to hit a point by aiming at an auxiliary point somewhat removed therefrom, or when it is desired to aim the center of an automatically spaced train of bombs by releasing the first such bomb.

Removing the vacuum restriction, which is a poor first approximation at all but the lowest altitudes, we may see that the travel of the bomb, as viewed from above, is given in ground coordinates by the vector sum of the following displacements: the vector velocity of the airplane in the air, or the airspeed vector, multiplied by the time of flight; the vector velocity of the air (assumed uniform) with respect to the target, or simply the wind vector, multiplied also by the time of flight; and the trail distance, or else the trail angle multiplied by the altitude, taken in a direction opposite to that of the airspeed vector of the airplane. The trail or trail distance is the horizontal displacement of the bomb behind the unaccelerated bomber at impact; the trail angle is this displacement divided by the altitude. This angle is usually measured in mils, whereas properly speaking this is the tangent of an angle. Both trail, whether distance or angle, and time of flight, for any given projectile horizontally released, are functions of altitude and the scalar airspeed.

For any given target, as may be shown directly from the above facts, there is a vertical

[g]Lieutenant H. G. Cooper, at NAS Banana River, who had taught bombers special methods for bombing targets evading in this manner, tried out the products of Project BUG as they were prepared.

cylinder which determines the locus of release points and directions of flight for a hit. The axis of this cylinder, at target level, is upwind from the target a distance equal to the wind speed multiplied by the time of flight of the bomb. Its radius is the product of the bomber's airspeed and the time of flight, minus the product of the trail angle and the altitude. At the instant of dropping, the airplane must be head toward, or have its airspeed vector directed toward, the axis of this cylinder, and it must then also be just piercing the surface of this cylinder. These conditions are both necessary and sufficient for a hit on the target, following the premises already given. Simultaneously, the ground track of the bomber must pass through a point at target level which is also upwind from the target, but displaced from by a distance given by the product of the altitude, trail angle, and wind speed, divided by the airspeed. It will be seen that this latter point and the previous cylinder, as bombing "directrices," are independent of the angle of approach of the bomber.

The solution of the Norden Bombsight Mark 15 is based inherently on the above geometry, with one major approximation however. In this bombsight, the full trail distance is taken geometrically along the ground track rather than along the airspeed direction. Since the one is the other multiplied by the cosine of the drift angle, usually less than 15 degrees, the approximation is a good one. While it might have been equally simple, as indicated once by J. B. Russell of Section 7.2, to build an exact mechanism, the point is not too serious. In the Norden Mark 15, the closing speed, or ground speed for a stationary target, is obtained as an angular rate, being inherently divided by the altitude, by synchronously tracking the target through a tangent screw. The "range" solution is expressible, on the basis of data already given, as

$$\tan \phi_0 = \left(\frac{v}{h}\right) t_f - \tau, \qquad (1)$$

where ϕ_0 is the dropping angle, v/h is the ratio of closing speed to altitude (or the absolute angular rate of the target as though directly beneath), $t_f = t_f(h,V_a)$ is the time of flight, and $\tau = \tau(h,V_a)$ is the trail angle. In some other types of bombing, particularly where solutions are obtained by correcting the first order vacuum solution, the "range lag" is used instead of trail or trail distance, being defined as the distance the bomb falls behind a vacuum trajectory at impact. The relation between range lag R_e and trail distance $h\tau$ may readily be shown by equating two expressions for tangent of dropping angle as above. Thus

$$\tan \phi_0 = \left(\frac{v}{h}\right) t_f - \tau = \left(\frac{v}{h}\right) t_{fc} - \frac{R_e}{h},$$

and hence

$$R_e = h\tau - v\,(t_f - t_{fc}) \qquad (2)$$

where $t_{fc} = \sqrt{2gh}$ is the vacuum time of fall.

7.4 THE EXTRAPOLATING ANTISUBMARINE BOMBSIGHT

The instrument referred to is a bombsight specifically intended for low altitude use against submarines, allowance being automatically included for underwater travel of the target in case of submergence at any instant during the attack. While the sight is meant to be hand held, it is not necessarily restricted to such operation.[h]

A number of experimental models have been built, under our contract at The Franklin Institute, of which many were given preliminary tests in the air against a dummy target. Production of 75 units of the final model, which became bombsight Mark 20, was undertaken. Wide distribution to Fleet units was made.

Representatives of BuOrd and BuAer were freely consulted on this project, which was assigned the project-control designation NO-129.[i]

The geometrical development of the extrapolating solution for underwater travel of the target is shown in Figures 1 through 8. In Figure 1, an attacking airplane A is approaching a surfaced submarine S; the submarine is headed at an angle α to the line of sight. If

[h]The principle on which this instrument and its method of use are based is due to Captain A. B. Vosseller of ComAirAsDevLant. Section 7.2 cooperated with him in a development project for reduction to practice of this principle.

[i]Commander E. S. Gwathmey of BuOrd and Captain Vosseller were liaison officers for the Navy.

the submarine remains on the surface, at least until the airplane arrives to within bombing range, then the problem is relatively simple. This possibility will be considered below, as a special case of the more general problem. If the submarine submerges, for example at S, then it is necessary on the one hand to be able to fly a course passing over it and on the other

FIGURE 1

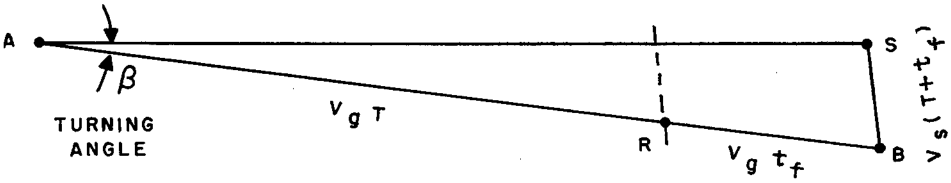


FIGURE 2

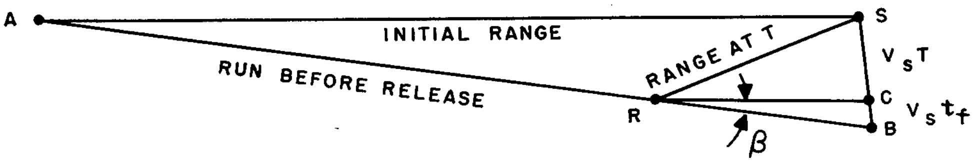


FIGURE 3

to have a criterion for the release of bombs at the appropriate point on such a course. After submergence the only visible reference available for these purposes is the temporary but stationary swirl left on the surface by the submarine.

It will be assumed that the airplane had been navigating directly toward the submarine at the moment of submergence, that is, the track of the airplane was directed toward the target at that time. It will also be assumed that the following conditions are known: target aspect angle α at submergence, target speed V_s underwater, altitude H, and ground speed V_g of the attacking airplane (although in different contexts, the symbols h and H, for altitude, are used interchangeably throughout this chapter and, probably, elsewhere). The underwater target speed is taken as an empirical constant of the instrument. The supposition that the target does not appreciably alter its direction during or shortly after submergence is similarly justified. If the above assumptions are made and the variable conditions known, then it is evident (Figure 2) how the airplane course may be altered, through a discoverable angle β and at the time of submergence, so that subsequent flight on a straight track will result in the desired "collision course" passing over the submerged target at B. While the triangle ASB, of which α and β are angles, is a space triangle, these angles are equally well represented in any similar triangle. It will be evident below how the turning angle β is given directly by the mechanism.

While a knowledge of the range at submergence is not necessary, usable values being limited principally by persistence of the swirl, bomb release must take place at a definite range from the collision point B in Figure 2. The dropping range RB depends on the ground speed V_g and altitude H, assumed constant, of the attacking plane at release, for the forward distance covered by the bomb is $V_g t_f$, where t_f is the time of flight. Thus if distance is everywhere measured in feet, and time in seconds, we have

$$RB = V_g t_f = \frac{V_g \sqrt{H}}{4}.$$

Since there is nothing to distinguish the point B on the water surface, and thus to guide the

release of bombs, it is an essential feature of the method to determine the range *RS* (Figure 3) from the swirl at which release may occur.

If time is measured from the moment of submergence (and of the assumption by the airplane of a collision course), release may be said to take place after the arbitrary interval *T*. The total time intervening between submergence and arrival of the bomb will be $T + t_f$, and the distances traveled respectively by airplane and target will be $V_g(T + t_f)$ and $V_s(T + t_f)$. At the time of release the target will be at the point *C* (Figure 3), having traveled a distance V_sT and with V_st_f yet to go. It is evident that the triangle *RCB* is similar to the triangle *ASB*. If the moving point *D* is introduced (Figure 4), leaving *C* at the instant of submergence and moving backward toward *S* at submarine speed, it will arrive at *S* at the precise moment that point *E*, representing the airplane and leaving *A* simultaneously at airplane speed, arrives at *R*. Thus the line *RD* (which might be considered as the *fictitious dropping range* from the swirl), and the line *ES* (the *real swirl range*), become simultaneously equal to the actual dropping range from the swirl. Equality of *RD* to *ES*, then, may be used as a criterion for release. Thus if the triangle *RDB* were mechanized, to scale, with the leg *BD* growing out from *BC* at the appropriate rate, and the angle *RDB* appropriately set, it will generate the "range" *RD* which, by its momentary equality to the real range from the swirl, will signal the instant of release.

In practice, single bombs are not generally used, so that in order to lay the stick symmetrically across the target the first bomb must be released at a point one-half the stick length back along the track. Furthermore, the underwater travel of the bombs, as they penetrate from the surface to the depth set for detonation, must be accounted for by an additional backward displacement at the release point. The small amount of trail may adequately be allowed for by a correspondingly small delay or forward displacement. The accumulated corrections for these effects are contained in the removal, by the distance *L*, of the point *R* (Figure 4), to the new point *F*. The line segment *FD* is then taken as the fictitious range, which becomes equal to the real range *ES* when *E* arrives at *F*, both segments being then equal, within a good approximation, to the desired dropping range *FS* from the swirl.

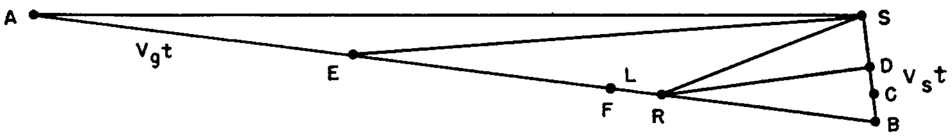


FIGURE 4

Mechanization of the principle involves two triangles, a "horizontal" one and a "vertical" one. The horizontal triangle, as shown in Figure 5, is derived directly from the geometry of Figure 4. In this triangle the leg *FB* is known, as is the portion *BC* of the leg *BD*. The remaining portion *CD* grows uniformly in time. The angle α' (Figure 5), the supplement of angle *FBD*, is set to the observed course angle α, to which it is a satisfactory approximation since the difference β is always small. It will be noted that the angle β, represented as angle *CRB* in Figures 3 and 4, is still available in the known configuration of Figure 5. The vertical triangle of the mechanization, shown in Figure 6 as *GFD*, has a common leg (*FD*) with the horizontal one. It serves as a means for comparing the fictitious range with the real range. If the altitude leg *GF* is made equal (or rather proportionate) to the

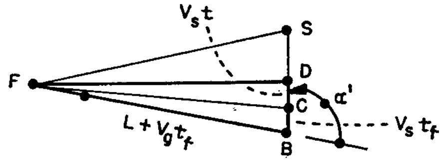


FIGURE 5

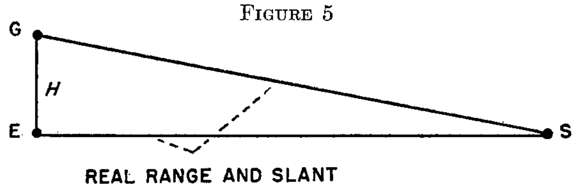


FIGURE 6

real altitude as shown in the "real" vertical triangle *GES* of Figure 7, the equality of *FD* and *ES* is indicated by equality of the fictitious

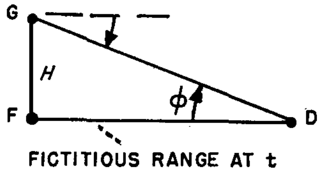


FIGURE 7

depression angle $\phi = GDF$ of the mechanization to the real depression of the swirl. Thus if the triangle is held (even approximately) in

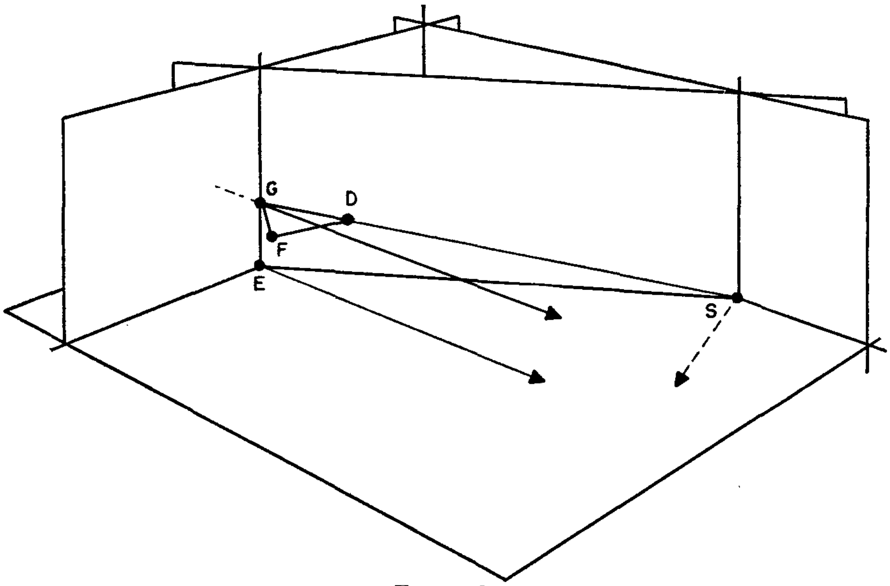


FIGURE 8

a vertical plane containing the swirl, and *FD* is accurately horizontal, release may occur when the fictitious slant range *GD* points directly at the swirl. A less accurate method, although easier to explain, is to point *GD* at the swirl and merely note when the leg *FD* becomes horizontal. This procedure is diagrammatically shown in Figure 8, although it is not usable with a bubble as level indicator owing to the response lag.

For convenience of adjustment and for compactness of instrumentation, the two triangles shown separately in Figures 5 and 6 and together in Figure 9 are subjected to a scalar transformation which consists merely in division of their linear dimensions by the time of flight t_f, or by $\sqrt{H/4}$. The resulting pair of triangles (Figure 10) completes the mechanization, at least symbolically. Altitude (*H*) and

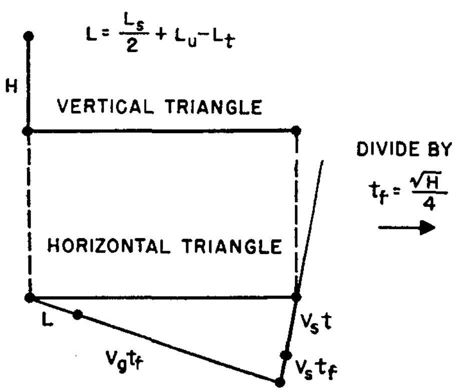


FIGURE 9. Components of bombing triangles for mechanization.

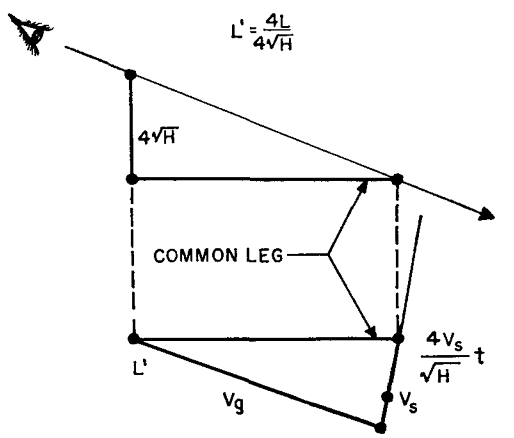


FIGURE 10. Components of bombing triangles for mechanization.

ground speed (V_g) are set in manually, submerged submarine speed (V_s) being assumed constant; the angle α or α', described above, is set in by direct comparison with the target being approached while on the surface. The range correction, L' in the transformed version, is taken to be preset in dependence on stick length, bomb characteristics, etc. Beginning upon manual operation of a trigger at submergence, the variable leg of the horizontal triangle grows at a rate determined by set altitude as shown in Figure 10. The hypotenuse of the vertical triangle defines a "fixed" line of sight to be directed at the swirl. The release depression is determined through a bubble assembly attached to the base of that triangle.

As to actual operation, this method is based on a straight run over the target, which is entered upon at the moment of submergence. It is required that the ground track for this run be oriented by the angle β (Figure 2) with respect to the line of sight to the submarine (or swirl) at the time of submergence. If ground speed (V_g) and the target angle α were already known and set into the instrument, and if the plane's track were headed toward the submarine up to the time of submergence, it would merely be necessary to turn in the direction of target heading through the angle β, as indicated by the instrument, and thenceforth to hold a constant course. A change in heading itself by the amount β would accomplish this to within sufficient accuracy. In the presence of wind, however, it may be difficult for the pilot to "navigate" in this manner, that is, to direct his ground track at will. Wind also enters the problem as it affects the ground speed as compared with airspeed, the difference between these two values varying drastically with compass heading. Ground speed is a necessary adjustment in the bombing instrument. There are a number of ways to satisfy these various conditions, each involving some sort of guide by which the pilot may head his plane to prepare for and carry out the bombing run if and when submergence takes place.

If wind speed (assumed known in magnitude and direction) is added vectorially to airspeed, as in Figure 11, the true ground speed is given. Thus if the wind vector is stabilized, automatically or by compass matching, the proper heading for any given ground track may be directly determined. If, furthermore, a "submarine" vector is added to these two (wind and airspeed), and similarly stabilized after orientation, the proper heading may be directly selected by which to "lead" the target right up to the time of submergence. A simple computer of this sort was prepared for use by the pilot in connection with this method, and also provided the value of ground speed for setting in the bombsight.

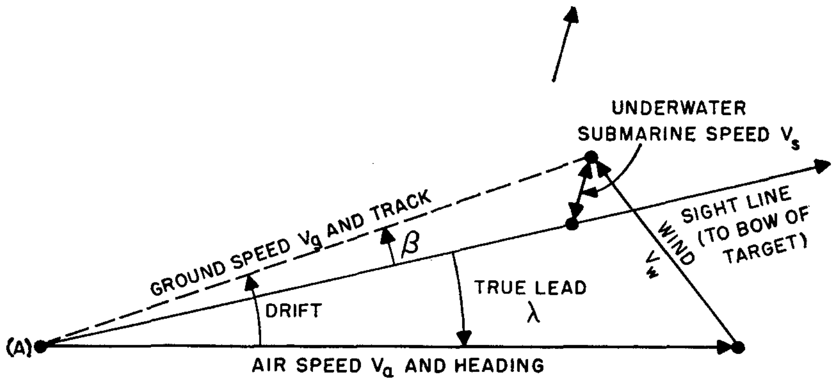


FIGURE 11. Mechanization of guiding apparatus for approach in azimuth.

The initial models were fairly straightforward physical adaptations of the geometry of Figure 10. The combined triangle pair formed three consecutive edges of a tetrahedron (see also Figure 12), the "vertical" edge being normal to the plane of the other two. The assembly appeared and was operated somewhat like a sextant, and was built around a timing clock of which the rate was adjusted to an altitude scale and which was triggered off upon sub-

mergence. Two link lengths were adjustable, to altitude and ground speed scales (and a subsidiary stick length scale), and the angle between another pair of legs was set according

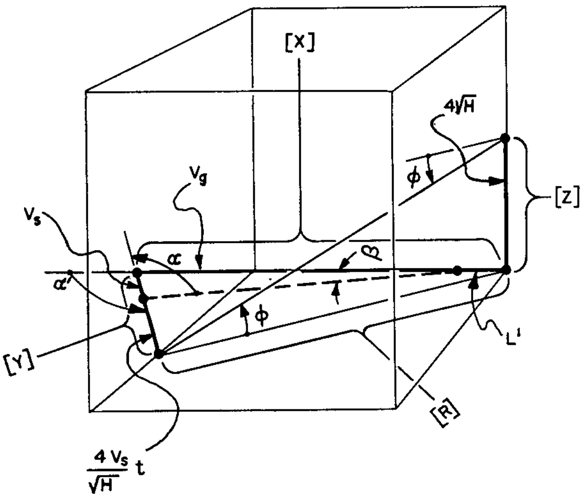


FIGURE 12. Three-dimensional form of bombing triangles.

to target aspect. A ring and bead sight was used, each component at one end of the connected series of links. Ring and bead separation was variable as the adjustments were made and as time elapsed. The signal for release was given by a bubble attached normally to the vertical leg. In one of the models the bubble image was collimated and reflected into the line of sight by a somewhat elaborate mechanical and optical structure. None of the early models were particularly easy to use, especially in an airplane under working conditions. Two hands were continually required, and determination of lateral verticality and bubble transit left much to be desired. Several desirable features were sacrificed in order to minimize load on the timer, and some intricate and bizarre mechanism was resorted to for the avoidance of blind spots. Withal these first two-models showed the workability of the principle and formed reductions to practice from which much information and experience was gained.

In subsequent models it was decided to retain the principle intact but to change the design radically. It appeared possible, and at length practical, to incorporate the whole sight in a hand-held, pistol-grip affair which might be completely operated, including adjustment, with one hand. In particular it was found to be possible to fix the line of sight with respect to the body of the instrument, making the task of sighting considerably simpler. This involved building the mechanism "around the line of sight," and separating the two triangles at their common leg. Aimed exactly like a pistol, the signal for release appeared automatically, as the passage of the target by the aiming point, with the instrument so held that the transit of a bubble indicates the proper depression. A bomb-release key is incorporated as a pistol grip for the sight, so that release is made by a trigger in full analogy with the firing of a pistol. Adjustment of target aspect, altitude, and ground speed, and initiation of the timer operation at submergence, could all be carried out with the thumb of the same hand.

As was mentioned above, this antisubmarine bombsight, which became Navy bombsight Mark 20, was placed in limited production on the basis of the final experimental bombsight. Extended flight tests prior to such production had established that, with care, horizontal low-altitude bombing, in the altitude range from 100 to 500 feet, could be carried out to a probable range error of 20 feet, which was as good as could be done in line, or laterally. This was with a nonsubmerging target. Few tests actually were carried out, owing to mechanical difficulties, under simulated conditions of submergence, and it is not known whether enemy submarines were successfully attacked, either on or below the surface, with this equipment, although a number of models were distributed to the various operating theaters. For this purpose a set of instructions was prepared[1,2] and certain Navy officers were assigned to educational trips in this connection.

While the practicality of the general method was established, a number of modifications were proposed and carried out which omitted the underwater extrapolation feature. One such model included an inherent vector computer for the combining of airspeed and wind speed and/or target speed for a more effective solution. Auxiliary devices such as the miniature ground speed computer, mentioned elsewhere, were prepared for cooperative functions with

such bombsights. In some of the experimental hand-held bombsights, the ranges of airspeed and altitude were considerably extended, the latter up to 1,000 feet. One of these sights, particularly for use in blimps, extended to very low airspeeds. This model, which became bombsight Mark 24, was tested at Lakehurst with better results, it seems, than might have been expected.

A project which resulted in a technique much used in other fields was that for automatic setting of altitude, in the Mark 20 type of sight, from the radio altimeter. Here a d-c servo was employed, with motor and resistive follow-up mounted on the hand-held instrument, accepting as altitude a voltage from the AYD altimeter. Since the "triangular" bombing principle employed in the sight depended rather crucially on this variable, this accurate and automatic setting from the AYD altimeter gave very impressive bombing results during tests. The tests, incidentally, were over water.

7.5 METHODS INVOLVING ABSOLUTE ANGULAR RATE

Development of hand-held bombsights based on the "triangular" principle was gradually discontinued for several reasons, although the results were not to be disparaged. In the first place better principles were at hand. Furthermore, the submarine emergency was waning, and rockets were the more popular weapon anyhow. This is not to say that low-altitude bombing was not important, for it remained so throughout and its previously unexpected tactical application has been one of the phenomena of the war. It was a curious circumstance which led high-altitude bombing to be well developed, at least instrumentally, when World War II began, whereas the apparently simpler art of low-altitude bombing, which in fact was put to extensive use, was almost completely unequipped.

Many competitive bombing methods had also appeared, such as the FM radar *Sniffer*, which gave a solution in terms of range and altitude to the target, even on blind approaches, and which was later generalized to gliding attacks, with the range rate and the rate of change of altitude as input variables. There were also other bombsights of the visual triangular variety. The most impressive new principle, perhaps, was that of angular rate bombing, which we have also discussed in Chapter 3. This principle was of British origin, being employed in their Mark III LLBS, in which a grid of lines at infinity were rotated downward in space by means of a luminous rotating helix and a stabilizing mirror system. With the target observed through the image of this grid, release was made automatically when relative motion momentarily became zero. It was suggested by the present writer that this principle could also be embodied the other way around, by tracking the target accurately and measuring continuously the angular rate in space of the physical index with which the tracking was carried out. We were thus led to apply, at first to hand-held sights and later to those which were supported in various ways, rate meters based on captured gyros of the type previously being developed for lead-computing applications. This type of instrument came to be known as the BARB. Under our immediate direction a project was conducted in which strain-gauge torque measurement was applied to captive gyros for this purpose. A number of such models were built and tested (see Chapter 3). The most successful such method, however, has been that involving the pneumatic capturing technique, which indeed has been experimentally applied in other fields of fire control, to captive gyros and to other components, and which was developed principally under Section 7.3 by intersectional arrangement. In the research on the pneumatic angular rate bombsight, which ultimately became the bombsight Mark 23, and of which a subsequent elaboration — called SuperBARB — having a pneumatic form of aided tracking has recently been designated the Mark 27, several agencies contributed.[j] (See Chapters 3 and 4 of Volume 1, Division 7.)

Whereas in Chapter 3 this angular rate principle of bombing is described somewhat, we may give here a slightly different interpretation of the basic doctrine. We refer thus to the

[j] Excellent theoretical work was contributed by L. Goldberg of the McMath-Hulbert Observatory.

accompanying Figure 13. In target coordinates, it is evident that the angular rate of the approaching bomber, as seen from the target, which is the same as that of the target from

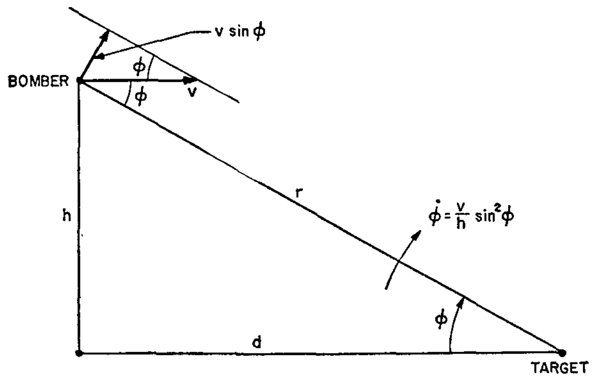


FIGURE 13. Illustrating angular rates on bombing run.

the bomber, is $(v/h)\ \sin^2\phi$. As a function of time, this is evidently

$$\dot{\phi} = \left(\frac{v}{h}\right)\left[1 + \left(\frac{v}{h}\right)^2 t^2\right]^{-1},$$

where v is the closing speed and t is the time before crossover. The release condition, in this simplest case, is obtained by substitution of the time-of-flight t_f, in terms of altitude, in this equation. The more general cases[9] develop when an offset in aiming-point is desired, when climbs and glides are contemplated, and so on. It is found, for example, that this method, at the lower altitudes particularly, is remarkably insensitive to errors in the altitude input, and is also less sensitive, relative to other methods, to errors in the closing speed.

At one time it was planned to base a synchronous bombing method, in which the whole prerelease tracking interval would be utilized to develop accuracy, on matching the time function given above for the angular rate to an analogous function of frequency, which is well known in an RC electrical circuit. It is clear, however, that the tracking scheme of Super-BARB, though based on another synchronous principle, has been quite successfully executed, as indicated by recent test results. With what has been said here, together with the various references given[8,9] the interested future investigator will find enough to carry him reasonably far into this subject. For the present writer, the most significant application of the absolute angular rate principle, to instrumental solutions for gravity drop, is in the combined system of PUSS, where, particularly in the rocketry case, this principle is applied simultaneously to the gravity drop correction and to the lead computing solution, employing thus the same fundamental gyro component.

7.6 PATHS OF CONSTANT RELEASABILITY

For conjectural bombing tactics it is of interest to study those paths on which a bomb may be released at any instant, each such hypothetical release scoring a hit. We consider here the problem *in vacuo,* and with respect to a coordinate system fixed in the target. It is further assumed that the airplane speed, in target coordinates, is constant. The results obtained have been used as flight criteria in several other investigations.

It is evident that any given such path must be completely contained in a vertical plane through the target. Let (x_0,y_0) denote the coordinates of a point of release resulting in a hit. Then

$$x_0 = -\frac{dx_0}{dt}t_f, \tag{3}$$

$$y_0 = -\frac{dy_0}{dt}t_f + \tfrac{1}{2}gt_f^2, \tag{4}$$

where t_f is the time of flight of the bomb and dx_0/dt, dy_0/dt are the values of the components of the projectile's velocity at the instant of release. Since dx_0/dt and dy_0/dt are also the components of the plane's velocity at this instant, then for constant velocity v on the flight path (in target coordinates)

$$\left(\frac{dx_0}{dt}\right)^2 + \left(\frac{dy_0}{dt}\right)^2 = v^2. \tag{5}$$

Elimination of t_f between equations (3) and (4) gives

$$y_0 = x_0\frac{dy_0/dt}{dx_0/dt} + \frac{gx_0^2}{2}\left(\frac{dx_0}{dt}\right)^{-2},$$

or

$$y_0 = x_0\frac{dy_0}{dx_0} + \frac{gx_0^2}{2}\left(\frac{dx_0}{dt}\right)^{-2}. \tag{6}$$

Since from equation (5)

$$\left(\frac{dx_0}{dt}\right)^{-2} = \frac{1}{v^2}\left[1 + \left(\frac{dy_0}{dx_0}\right)^2\right], \tag{7}$$

the expression (6) can be written with no time derivatives as

$$y_0 = x_0 \frac{dy_0}{dx_0} + \frac{gx_0^2}{2v^2}\left[1 + \left(\frac{dy_0}{dx_0}\right)^2\right]. \tag{8}$$

The desired constant speed dive bombing paths are the loci of the point (x_0, y_0) satisfying equation (8). Hence the subscripts are now dropped and equation (8) becomes the differential equation of the paths.

$$y = x\frac{dy}{dx} + \frac{gx^2}{2v^2}\left[1 + \left(\frac{dy}{dx}\right)^2\right]. \tag{9}$$

This equation is inconvenient to integrate as it stands. Substitution of

$$\eta = \frac{dy}{dx} \tag{10}$$

gives

$$y = x\eta + \frac{gx^2}{2v^2}(1 + \eta^2). \tag{11}$$

Differentiation of equation (11) with respect to x results in

$$\eta = \eta + x\frac{d\eta}{dx} + \frac{gx}{v^2}(1 + \eta^2) + \frac{gx^2}{v^2}\eta\frac{d\eta}{dx},$$

or

$$0 = \frac{d\eta}{dx} + \frac{1 + \eta^2}{\dfrac{v^2}{g} + \eta x}.$$

This can be written as a linear first-order differential equation in x:

$$\frac{dx}{d\eta} + \frac{\eta}{1 + \eta^2}x = -\frac{v^2/g}{1 + \eta^2}. \tag{12}$$

Solution of equation (12) by the standard formula gives

$$\begin{aligned} x &= -\frac{v^2}{g}e^{-\int\frac{\eta d\eta}{1+\eta^2}}\left\{\int e^{\int\frac{\eta d\eta}{1+\eta^2}}\frac{d\eta}{1 + \eta^2} + c\right\} \\ &= -\frac{v^2}{g}(1 + \eta^2)^{-\frac{1}{2}}\left\{\int (1 + \eta^2)^{-\frac{1}{2}}d\eta + c\right\} \qquad (13) \\ &= -\frac{v^2}{g}(1 + \eta^2)^{-\frac{1}{2}}(\sinh^{-1}\eta + c), \end{aligned}$$

where c is a constant of integration. To determine c let (x_0, y_0) be the point at the beginning of the dive where $\eta = 0$. Then

$$c = -\frac{g}{v^2}x_0, \tag{14}$$

and thus

$$x = \left(x_0 - \frac{v^2}{g}\sinh^{-1}\eta\right)\left(1 + \eta^2\right)^{-\frac{1}{2}}. \tag{15}$$

Equations (15) and (11) constitute the equations, with η as a parameter, for the dive bombing paths.

By setting $\eta = 0$ in these equations there is obtained, as a special case, the level bombing criterion *in vacuo*

$$2y_0\frac{v^2}{g} = x_0^2. \tag{16}$$

For a given speed v and altitude y, equation (16) gives the point for entering the dive path continuously. If ϕ is the depression angle of the target viewed from the plane then

$$\cot\phi = v\sqrt{\frac{2}{gy_0}} \tag{17}$$

gives the depression angle at which the dive may begin. Release may occur at this or at any subsequent point on the path.

Since

$$\eta = \tan\gamma \quad (-90^\circ \leqq \gamma \leqq 90^\circ) \tag{18}$$

where γ is the angle from the horizontal to the tangent to the flight path. Equations (11) and (15) may be written as

$$y = x\tan\gamma + \frac{gx^2}{2v^2}\sec^2\gamma \tag{19}$$

$$x = \frac{x - \dfrac{v2}{g}\ln(\tan\gamma + \sec\gamma)}{\sec\gamma}, \tag{20}$$

or

$$e^{\frac{g}{v^2}(x_0 - x\sec\gamma)} = \tan\gamma + \sec\gamma. \tag{21}$$

The flight path terminates at the target, as might be expected, for by equation (19) y approaches zero with x. At the origin the flight-path angle γ has a value γ_1 given by

$$x_0 = \frac{v^2}{g}\ln(\tan\gamma_1 + \sec\gamma_1), \tag{22}$$

or

$$e^{\frac{g}{v^2}x_0} = \tan \gamma_1 + \sec \gamma_1 . \tag{23}$$

Thus

$$\sec \gamma_1 = \frac{e^{\frac{g}{v^2}x_0} + e^{-\frac{g}{v^2}x_0}}{2} = \cosh\left(\frac{g}{v^2}x_0\right).$$

From equation (16)

$$\frac{g}{v^2}x_0 = 2\frac{y_0}{x_0} = 2\tan\phi_0,$$

and hence

$$\sec \gamma_1 = \frac{e^{2\tan\phi_0} + e^{-2\tan\phi_0}}{2} = \cosh 2\tan\phi_0.$$

Similarly

$$\eta_1 = \tan\gamma_1 = \sinh\left(\frac{g}{v^2}x_0\right) = \sinh 2\tan\phi_0. \tag{24}$$

Since

$$\tan\phi = \frac{y}{x},$$

equation (11) can be written as

$$\tan^2\phi = \eta\tan\phi + \frac{g}{2v^2}y\,(1+\eta^2)\ ;$$

or

$$\frac{g}{v^2}\cot\phi = \frac{2\,(\tan\phi - \eta)}{1+\eta^2}. \tag{25}$$

Similarly, from equations (14) and (16)

$$\frac{g}{v^2}\ y\cot\phi = \frac{2\tan\phi_0 - \sinh^{-1}\eta}{\sqrt{1+\eta^2}}. \tag{26}$$

Taking these last equations together

$$\tan\phi = \eta + (\tan\phi_0 - \tfrac{1}{2}\sinh^{-1}\eta)\sqrt{1+\eta^2}, \tag{27}$$

which gives the relation between the tangent of depression angle and the slope η of the flight path at every point. Equation (25) shows that when y approaches zero, the limiting values of the slope angle and the depression angle are the same:

$$\tan\phi_1 = \eta_1, \tag{28}$$

so that by equation (24)

$$\tan\phi_1 = \sinh 2\tan\phi_0. \tag{29}$$

Of the constants involved, the fundamental ones are: speed v of plane, altitude y_0 at beginning of dive. The rest, x_0, ϕ_0, ϕ_1, γ_1, are derived from these.

As seen in Figure 14, there are two branches involved in the solution. Beside the normal diving path, there is a climbing path on which release at any point will also result in a hit.

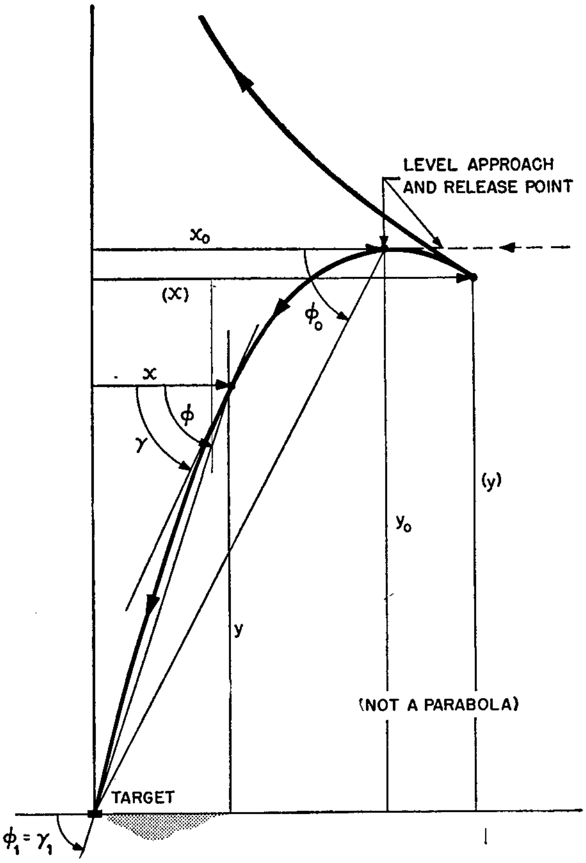


FIGURE 14. Path of unvarying releasability at constant bomber speed.

This branch approaches as a limit the zenith over the target. While a valid bombing path, it is of undetermined practicality.

7.7 THE DIVING ATTACK IN GENERAL

There are several practical reasons for wanting a method of dive bombing different from those, elsewhere mentioned, which lead to approach paths on which release may take place arbitrarily over an interval. Naturally it would be desirable on the face of it to obtain this type of approach, and angular rate methods are available to provide highly articulate aiming criteria for this purpose, since then it would not be of critical concern at what particular instant release took place. Errors in the choice

of the release instant, or in the releasing mechanism itself, would be of little consequence, provided only that the path were stably held to. However, it appears that such paths are not the easiest to fly, for while they are not strictly bomb trajectories (see above) they are paths on which the lift force essentially disappears, and an unfamiliar mode of operation of the controls must be learned and resorted to. It is recognized, of course, that this circumstance need not be fundamental, and might be altered by the proper design of airplanes or by appropriate training techniques. But another practical difficulty arises. For a large useful region of range, glide angle, and speed, the downward visual freedom of most existing airplanes is not great enough that the target is visible to the pilot at the moment of release, or, if the airplane is following a path of unvarying releasability, in the time interval over which release may occur. This is admittedly a property of airplanes, as such, and as in the previous objection does not necessarily carry permanent status. Thus it is hardly impossible to arrange for such downward visibility, since certain nonfighter airplanes, and proposed fighter types as well, do have this feature in relatively high degree. But both of these points have had at least temporary significance, and have militated against bombing tactics involving continuous releasability on the approach. We note that while not completely so restricted, such tactics would in general be associated with diving attacks.

It must here be interpolated that the property of lack of criticalness, or tolerance, of the choice of release instant would not belong exclusively to tactics in which the release path were rigorously followed. Other approaches, in which, for example, less curvature may be involved, and on which consequently a reasonable lift may still be experienced, will pass somewhat gradually through a condition of releasability, and in these of course the choice of the release instant is also, relatively to other methods of bombing, noncritical. We refer again to the bombing solution of the Army A-1 multipurpose, or Draper/Davis, sight. This system involves an approach of the above character, and should be studied by the serious student. The present writer is not qualified to represent this system in detail.

Several methods have been proposed and developed which avoid the practical difficulties above named, and which achieve accuracy in spite of losing the tolerance of release instant which follows upon the release-path philosophy. These methods involve, first, a diving approach, in general, during which the target is visible more or less straight ahead, then a suddenly imposed upward curvature of path during which the target becomes invisible to the pilot, and during which also the bomb is automatically released through a computation depending on the properties of the previously held path and on the character of the pull-up maneuver. One such method[k] was developed in the so-called DBS system, for BuOrd and later on British orders. In this system an approach was made by *proportional navigation,* to compensate for target motion in the air mass, and automatic release during the pull-up regime was effected by simultaneous computation of the accumulated pitch, the airspeed, altitude, acceleration, and the glide angle previously held. Glide and the accumulated upward turn were delivered by specially arranged gyros, and a nomographic type of computer, with mirrors and photocell, embodied the desired criteria for automatic release. The initial approach course was guided by a free gyro moving a mirror in the sight head, giving a delayed approach to a collision course in a manner referred to in Chapter 1.

A somewhat similar method which has received much recent attention and developmental effort, and which has been employed also for the gravity drop solution with other projectiles than bombs, was that originally proposed.[l] The term toss bombing came to be employed for the technique involved, although descriptively, or rather externally, the tactics referred to above, or again with the DBS, were substantially employed. That is, this term might be applied to several methods, although its connotation has been limited to those in which a special type of release computer was

[k]Devised by M. Alkan, the French fire-control expert and instrumentalist, erstwhile of Specialties, Inc.

[l]By (then) Colonel Morton of Army Ordnance.

employed. Toss bombing instruments, such as the so-called AIBR, were made to depend on the indication of an integrating accelerometer, sensitive in the direction of the airplane's vertical, which was allowed to begin integration at a point shortly prior to the pull-up, for bomb release. Thus was measured approximately the velocity impulse imparted to the bomb in a direction upward, against gravity, and normal to the line of sight to the target. Release occurred, through simultaneous computation of the duration of bomb flight, which was made, for example, by measurements on range or altitude and various speed, when the upward impulse being increasingly imparted was sufficient to compensate for the downward pull of gravity, to be accumulated during the time of flight. Inherent advantages arose from the method of integrating acceleration, since then fortuitous irregularities of flight, following initiation of the final computing phase, were automatically accounted for, and variations between flight path (measured in space) and the airplane axis were of second-order importance to the accuracy. Correction for target motion in the air mass, requiring something more nearly a collision course than the typically employed straight pursuit, was not contemplated until rather recently. It has been planned, including the toss method, with integrating accelerometer, in the PUSS system, to apply a correction for target motion.[m]

Whereas the earlier AIBR was based on a theory of horizontal approach to the target, as in plane-to-plane bombing, and was modified for the gliding approaches which were appropriate against targets on the ground, the theory as proposed for PUSS was built anew in particular recognition of this latter approach as the significant one. Continuous measurement was planned of altitude, airspeed, normal acceleration during the earlier phase, and the integral of the incremental normal acceleration during the pull-up phase. It was determined to prepare a small mechanical computer to assess the release condition in terms of these variables and automatically to initiate release of the bomb. For acceleration and the integral thereof, a pneumatic component was developed comprising a captive mass, a feedback integrator, and a valving assembly whereby the various initial conditions could be imposed as desired. Both the acceleration, exponentially averaged, and the instantaneous integral of the incremental normal acceleration were delivered as rotations, to the mechanical computer, by pressure motors responding to, or incorporating, these variables as generated by the pneumatic components. Airspeed and altitude were similarly delivered to the computer by capsule diaphragms. This toss bombing computer, called PACT, is to be combined integrally in the PUSS system described in Chapter 10. It will share inputs with other components of that system, and the mechanical-linkage computer it embodies will be physically adjoined to that of PUSS. Also theoretically developed and studied[n] is the instrumental method for use with PACT, as part of PUSS, whereby the approach in the first phase is so guided with respect to the target that motion of the latter is compensated for. This method involves a function also of the gravity-drop compensation of the computer, in elevation, and in azimuth employs a unique principle whereby the proper path for "kinematic lead" is very rapidly subsided to. Embodiment of the latter principle is made by specializing the function of the PUSS rocketry computer to give a double overcorrection in airplane heading for changes in direction of the line of sight to the target. Technically, in the jargon of lead-computing sights and deflecting gyro systems, the criterion[10] in azimuth is: $a = 1$. In elevation, the sight head is undeflected, the line of sight being along a direction normal to the computing axis of the accelerometer.[o]

[m]Proposed by H. Pollard of AMG-C, who has been responsible for the form of computation and the theory of the method as developed thus far for PUSS.

[n]By H. Pollard.

[o]For further details, and a better technical discussion, the writings of H. Pollard of AMG-C should be referred to. Some of these are given here in the bibliography.

In Pollard's work, as in Alkan's, the non-circular nature of the pull-up path, in practical circumstances, is recognized.

Chapter 8

CONTROL OF GUIDED BOMBS

8.1 REVIEW OF ACTIVITIES

ALL OUR WORK in this field was pursued either through the machinery of cooperation between NDRC Divisions 5 and 7 or directly in an advisory capacity for the former division. There is no hope for separation between the activities which occurred under each such sponsorship, since the interweaving has been most complete, so that no attempt at such separation will be made. We were first engaged on controls for guiding the RAZON bomb, following the issuance of a memo by the Director of OSRD in which he outlined computing methods which might be employed for that purpose, and in which he referred the problem in part to the fire-control experience of Division 7. (See also Volume 1 of Division 5.) The writer was made responsible for the establishment and conducting of a development program for preparation of experimental equipment in realization of these methods, which program was carried on largely between the contractors of Division 5 and those of Section 7.2 in Division 7.[a]

Aside from minor consultation and participation in conferences, the visual or optical phase only of the RAZON, and as a special case the AZON, guiding problem was treated by us. Without describing the RAZON bomb in detail, since this is an affair principally of Division 5, we may say that it is the modification of a standard 1,000- or 2,000-pound bomb in which a special tail assembly replaces the standard fins. The assembly comprises intact: radio reception for steering by rudder and elevator, gyro component for roll stabilization, movable rudder, elevator and aileron with "muscles" for the latter controls, and a flare for visual identification of the missile during the drop. Two general types of guiding were contemplated. In the first, the bomb was guided from the dropping airplane in such a manner that its trajectory terminated in the target, corrections being applied only as necessary owing to errors in dropping which would prevent its unguided trajectory from terminating in that manner. In the second proposed method, either through special maneuvers of the airplane or through a program of control applied to the bomb, the plan was to make the bomb align itself collinearly between the bomber and the target during the latter portion of the time of drop. In both cases the process was similar in one respect, in the azimuthal deflection problem, to the guiding of AZON, in which one dimension only was corrected and which was applicable particularly to long and narrow targets attacked approximately in the direction of their greatest dimension. The second method for guiding RAZON, in which it was proposed to attain temporary collinearity by visual guiding, was discarded when it became evident that sufficient maneuverability was not available in this projectile. This question in itself was a large one, involving aerodynamic design of the missile — several variants and predecessors of RAZON were also considered — and we used both numerous analytical and differential analyzer solutions in assisting with its study. With this conclusion, and subsequent experimental corroboration, attention was turned to controls for the noncollinear guiding technique. Starting from the Bush proposals and others[b] a design was pursued of an optical sight and computer, which eventually became the CRAB sight, to permit the guiding operator to carry out his mission simply by the superposition, under control, of a pair of indices. As it turned out, these indices were, in fact, images of the target and of the flare on

[a]In the former case, notably, there was Gulf Research and Development Corporation and certain contracting agencies at MIT, while for the latter The Franklin Institute was the locale of operations and supplied personnel and other developmental facilities. In the more recent stages, L. N. Schwien Engineering Corporation of Los Angeles, under contract to Division 5, was involved for several reasons, not the least being that H. A. Van Dyke, who had been project engineer at The Franklin Institute in this connection, transferred his employment there.

[b]Already advanced by J. P. Molnar at Gulf.

the bomb. Whereas a completely special computer and optical system were first under development, it became suddenly apparent to us that the same function could be embodied in a relatively simple modification of the Norden bombsight, which could then be used following the original drop of the bomb, as a normal type, to assist in the guiding process as a continuation of its normal operation. Remarkably enough, the whole operation of the bombsight was kept usefully in service for this post-release purpose of computing and presentation. The operator merely transferred his attention from the synchronizing controls to those for guiding the bomb, and from the alignment of cross hairs on the target to the alignment thereon of the bomb flare as the latter was in flight. It was thus only necessary to modify the trajectory in small and gradual degree; the paraboloidal character thereof was in general retained. Accuracy, assuming the guiding itself to be expertly and stably carried out, depended only on the time of flight setting in the Norden computer. For employment with this technique, the addition to the bombsight consisted only in the attachment of a small mirror, the so-called CRAB mirror, to the objective of the telescope of that instrument.

We have mentioned the use of the differential analyzer in this study. It was employed again for further developments as described below. In the CRAB project, a number of other aids were pressed into service, however. Aside from rather elaborate space-models in which the relationships among trajectories, bomber paths, and other geometrical objects were made evident, electronic simulators (Chapter 4) were constructed and applied to learn of the stability and effectiveness of the guiding process as such. These instruments not only gave information in highly tangible form which applied to the development of controls, including indeed those controls locally installed on the projectiles, but served later as training aids for civilian and Service operators in the field.

A subsequent activity constituted a return to the struggle for collinearity conditions toward the end of the drop. This struggle was tied up in a curious manner with the question of evasion, or of maneuvers indulged in by the bombing airplane following the release point. While such maneuvers might reduce the danger over the target area, they also imposed serious problems in guiding. Study was given to this problem, partly with the idea that a simple instrumental correction could be applied, provided of course that the evasive tactics did not prevent seeing the target at all. We had found it unlikely that the RAZON bomb, including its various modifications, and even though the airplane followed special paths in a vertical plane — as in the case of an analogous German weapon — could be made satisfactorily to follow a trajectory which would give collinearity with the target over a terminal time interval. Further with regard to evasion, it was discovered that there did, in truth, exist a natural evasive path which also assisted in meeting the criterion of terminal collinearity. For bombs only a small fraction more maneuverable than the standard RAZON this technique would probably have led to successful flare guiding on the collinearity principle as described. Although no field experiments were made, all the data for the study has been derived from differential analyzer solutions. The procedure consisted in a normal and rather sharp, or well-banked, steady turn by the bomber to one side, while the projectile is guided somewhat to the other side of the original range direction, the approach having first been made along a ground track displaced from the target on the side toward which the evasive turn is made. Thus the region near the target, where presumably antiaircraft fire might be heaviest, is avoided as completely as possible. In the guiding operations, which are added to an automatic program which curves the trajectory to one side and into a gradually increasing dive, by first curving the trajectory less, and then more than would be done by gravity alone, the manual controls can concentrate, at least toward the latter part of the drop, on holding the flare superimposed on the target in true physical collinearity therewith. For several reasons, however, this method was not tried out. One was that the ROC bomb, having reportedly about three times the maneuverability of the RAZON, came to be considered for visual flare guiding. It developed actually to have

somewhat less maneuverability, in terms of the potential normal acceleration under guiding. Since the greater maneuverability was available, special maneuvers to attain collinearity were not considered essential. From that point, no further study was given to evasion. Many bombers continued their runs anyhow, with standard bombs, and it was felt by some that the additional danger involved, even, ideally, from the point of view of the personnel, would be worth the continued straight run during the time of flight, in terms of targets successfully reduced, if such tactics made it possible to guide effectively. Such remarks apply, of course, to AZON as well.

The ROC bomb, which we shall not describe in detail either in the present brief review or in subsequent sections of this chapter, was intended as a television seeker for ground targets, to be manually controlled in steep drops. Its maneuverability was attractive for the collinear type of flare guiding, and while the television (MIMO) component was still in preparation authorization was obtained to try out this missile for that purpose. A computing sight was constructed, involving a generalization of CRAB which brought about gradually, and maintained, identity between the reflected flare image and the actual flare image toward the latter part of the time of flight. This sight was called CARP, and it was also an adjunct to the bombsight Mark 15, from which it derived automatically several vital inputs. Although its manual operation was somewhat more complex, and this in fact prevented its unmitigated success in experimental trials, this operation was in general analogous to that of CRAB.[c] As we have already said, electronic simulative methods were applied, for these flare guiding developments, for the construction of study models, humanly operable, in connection with questions of the stability and the practicality of the guiding processes. On a further application of these same methods, the study models were modified into prototypes for trainers intended for field application, which were then produced and further modified through other facilities of Division 5.

Another major application of electronic simulation has been to the control of ROC with MIMO, fundamentally a "seeking," or "homing," or "automatic interception,"[d] problem which was subsequently studied. This study was conducted by the writer.[e] The status of the study was actually that of collaboration or more properly of consultation with Division 5 and their contractors, principally Douglas Aircraft, on their developmental control problems in connection with the television bomb. Contrary to the general impression, it is not a trivial matter, given a falling bomb with remotely manipulable control surfaces, and the means for seeing ahead along the bomb's axis via a television camera, precisely to guide the bomb to any given target at ground level. Among others, there are two principal circumstances here which make the problem appreciable. One is the motion of the landscape, in the televised image, which results from pitching and yawing motions of the projectile not directly connected with the curvature of its air path. The other arises when steering is so performed that the missile heads directly or approximately toward the target. By virtue of this constraint the curvature of flight must progressively increase as the range decreases, provided the target has lateral motion of appreciable magnitude in the air mass. We note that the latter type of motion is equally occasioned by wind as by proper motion of the target on the ground or over the water. In the ROC projectile, the response of the image to pitching and yawing is reduced by the purposeful and considerable, although experimentally not yet complete, reduction of these motions by appropriate design of the projectile. This design, which involves a sym-

[c]While the instrumental part of this project was conducted at first at The Franklin Institute, as a collaborative venture between Division 7 (Section 7.2) and Division 5, H. A. Van Dyke being project engineer for the contractor, it was moved to L. N. Schwien Engineering Corporation under Division 5, at about the time testing was to begin, since Van Dyke became employed by the latter organization.

[d]By connotation, these terms have become badly confused, and their usage is widely disparate among various groups. A purer though more esoteric language has been developed by W. B. Klemperer, whose many writings on these and allied topics should by all means be consulted.

[e]With the assistance of R. M. Peters and L. Julie, the latter being responsible for the laboratory simulative equipment thus employed at Columbia University.

metrical and articulate wing structure, permitting independent alignment of the central body, is significant for future such projectiles, especially when self-propelled, but it is not the function of this report to give a technical description hereof, since such material will be found elsewhere in very complete form.[f] Both for the television application and for the flare guiding application mentioned above, ROC (or ROKH) was roll-stabilized by the same means as were AZON and RAZON, and the character of this means and of the resulting space behavior of the missiles, in the sense of rigid-body dynamics, is further discussed below since this question enters the problems of trajectory analysis and synthesis. The control artifices for the remote, human guiding of the television ROC, or MIMO-ROC, were outlined in Chapter 4. In the development and study of control dynamics and equipment for this type of guiding, and of the stability and effectiveness of such equipment under the proposed tactics of operation, we were naturally advanced considerably by having previously studied the air trajectories of this and other guided bombs. For example, we were by now familiar with the dynamic and transient properties of such trajectories, for the artificial embodiment of which both the differential analyzer and electronic simulative methods had been employed. It was thus approximately known, for the television-bomb work, which of the rationalizations, and guides to judgment, were valid in the prediction of control performance. Continued application of simulative equipment[g] allowed us to assess the several proposals which had been made for control dynamics, and to select one arrangement which appeared superior from an overall standpoint. A conclusion was also possible, by this means, on the degree of precision with which it would be required to adjust the dynamic constants of the regulatory dynamics which were recommended.

[f]We refer to NDRC Division 5, and to the Douglas contract.

[g]Our own research involved electronic models, whereas mechanical and optical models, some of fantastic intricacy and ingenuity had previously been proposed and designed. In some cases partial models of the latter variety were available for corroboration, short of full-scale tests, of the results attained by the electronic simulators in our laboratories.

One other major activity in the field of guided bombs concerned the technique of GCB, or *ground-controlled bombing,* in which a radio link allowed control, from a position on the ground relatively adjacent to the target, of a bomb dropped from a vantage point overhead.[h] The visual phase of the weapon, in which a flare was to be attached to the bomb, was principally considered by us. Again we were aided by a prior knowledge of the behavior of the projectile in flight — in this case the RAZON bomb — and of its response to control signals, by virtue of prior experience with synthetic and analytic studies made on the differential analyzer and with other aids to computation. We should not by any means imply that no trajectories were computed by numerical integration, for these processes were earlier conducted in this general connection by ourselves, and more extensively by others. This procedure, however, gave all the better appreciation of the improved facility for study and invention provided by the automatic forms of mathematical operation.

A theory of guiding was worked out, in terms of a pair of ground stations from which the falling bomb would be visible in its downward course toward the target, whereby the trajectory could be gradually modified so as to terminate, without serious warpage toward the lower end, in the target. Several alternative mechanisms were proposed which embodied such theory, and these were discussed in numerous conferences with the other personnel concerned. Certain compromises were arrived at in which the best features of widely differing proposals were reasonably well preserved in the forms finally determined upon for joint recommendation. It appeared that, with the aid of simple control equipment, and under circumstances that admitted of considerable flexibility, the technique of GCB could probably be carried out with great success, both as to the ease with which the manual phases of the process could be learned and applied, and as to the accuracy with which refractory enemy obstacles and strongholds could thus be reduced.

[h]In the study of this problem the writer, and R. M. Peters of Section 7.2 who assisted him, worked most directly with H. A. Van Dyke of L. N. Schwien Engineering Co.

8.2 ANALYSIS AND SYNTHESIS OF TRAJECTORIES

A right-handed Cartesian coordinate system is assumed in the air mass, with origin at ground level directly beneath the point of release of the bomb, and the positive x axis is taken in the same direction as that of the velocity vector of the airplane with respect to the air at the instant of release. (See Figure 1A.)

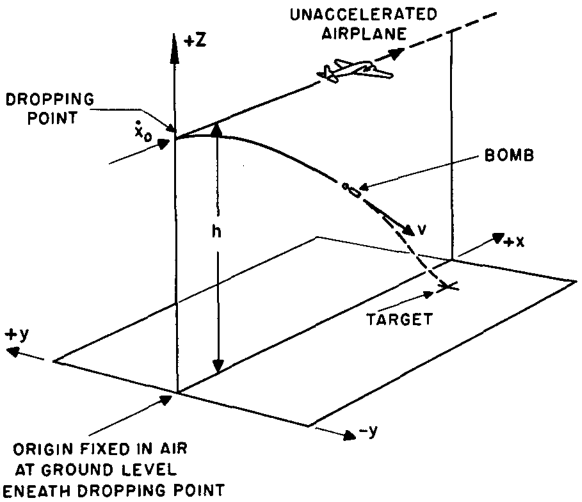


FIGURE 1A. Coordinate system and general guiding space.

The only forces acting ordinarily, on the unguided missile, are gravity and the drag force **D**, directed oppositely to its velocity vector **V** with respect to air. Deflection of the control surfaces introduces a *sideways force* **S** perpendicular to **V** but otherwise as yet undefined in direction, and a *lift force* L perpendicular to both **D** and **S**.

Then the equations of the trajectory are

$$
\begin{aligned}
m\ddot{x} &= D_x + S_x + L_x \\
my &= D_y + S_y + L_y \\
m\ddot{z} &= D_z + S_z + L_z - mg\,,
\end{aligned}
\tag{1}
$$

where m is the mass of the bomb, g is the acceleration of gravity, and the subscripts x, y, z indicate components of the forces in the directions of the coordinate axes. Assuming that the forces D, L, and S vary with the square of the velocity, these equations can be written in the form

$$
\begin{aligned}
\ddot{x} &= dv^2 \frac{D_x}{D} + sv^2 \frac{S_x}{S} + lv^2 \frac{L_x}{L} \\
\ddot{y} &= dv^2 \frac{D_y}{D} + sv^2 \frac{S_y}{S} + lv^2 \frac{L_y}{L} \\
\ddot{z} &= dv^2 \frac{D_z}{D} + sv^2 \frac{S_z}{S} + lv^2 \frac{L_z}{L} - g
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
d &= d(\delta_E,\delta_R,z) = \frac{A}{m}\frac{\rho}{2} C_D(\delta_E,\delta_R) \\
s &= s(\delta_R,z) = \frac{A}{m}\frac{\rho}{2} C_S(\delta_R) \\
l &= l(\delta_E,z) = \frac{A}{m}\frac{\rho}{2} C_L(\delta_E)\,.
\end{aligned}
\tag{3}
$$

The drag function d depends, as indicated on the two (as yet incompletely defined) components δ_E and δ_R of the CSD, (control-surface deflection), measured from the neutral position of no control. A is some area[1] proper to the bomb; ρ is the density of air at the altitude z; and C_S, C_L, and C_D are the appropriate aerodynamic coefficients. Owing to symmetry, C_S and C_L are identical functions of δ_R and δ_E respectively, while C_D depends on both δ_R and δ_E. It is assumed that C_L, and hence L, is independent of δ_R, and vice versa. In the case of both rudder and elevator control $C_D(\delta_E,\delta_R)$ is obtained from

$$
C_D(\delta_E,\delta_R) = C_D(0,\delta_R) + C_D(\delta_E,0) - C_D(0,0).
$$

Again, owing to symmetry, C_D is the same function of δ_E as of δ_R.

The gyro stabilization (see Figure 2) of the bombs in question insures that the lift force vector (scalar magnitude L) remains always parallel to the vertical plane in which the bomb was projected. The departure from this assumption is so small that L_y may be taken as always zero. Since **D**, **L**, and **S** are mutually perpen-

[1]In AZON and RAZON, the convention was to use the cross-sectional area for this quantity. In ROC it appears as the *wing* area.

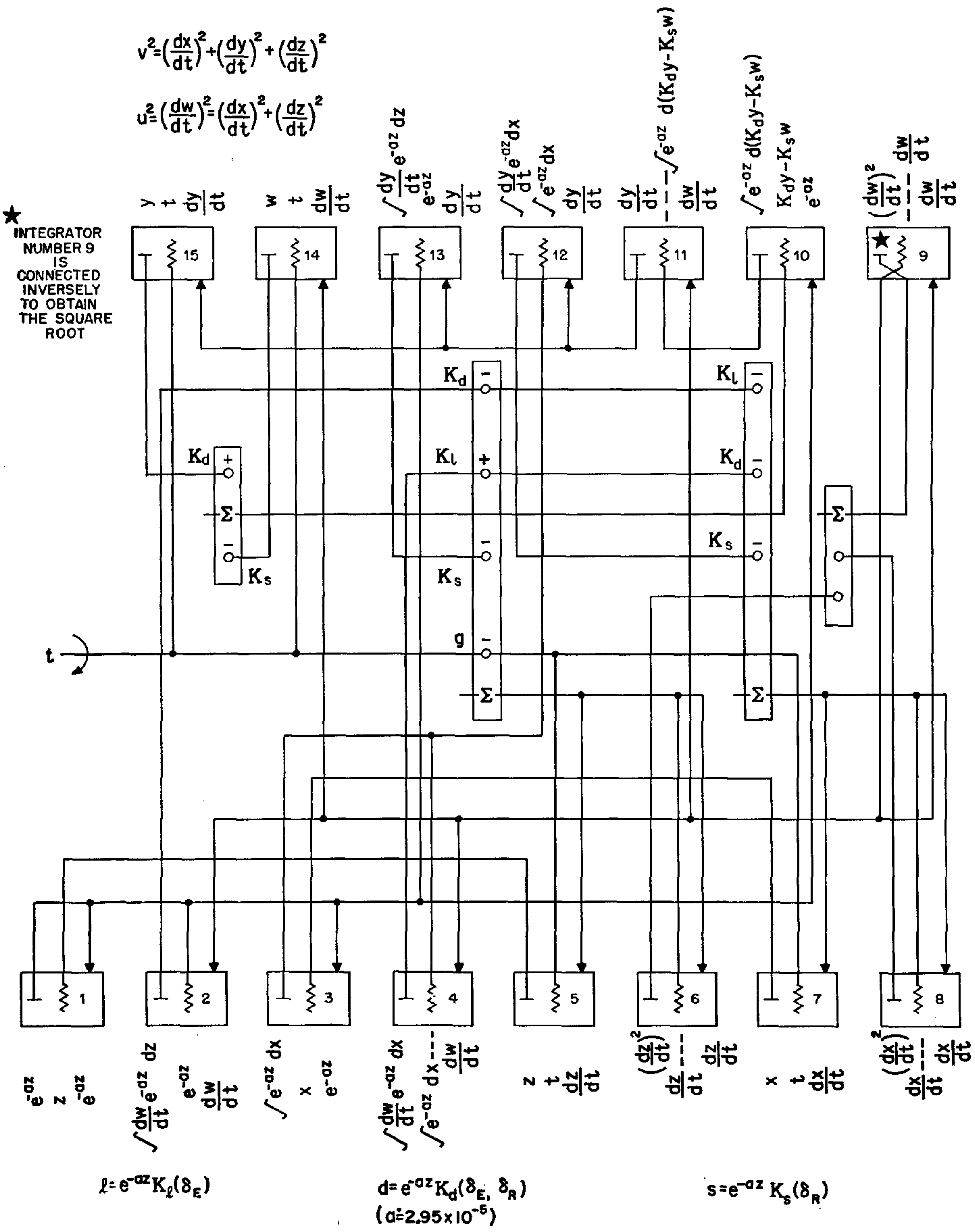


FIGURE 1B. Differential analyzer connections for guided bombs.

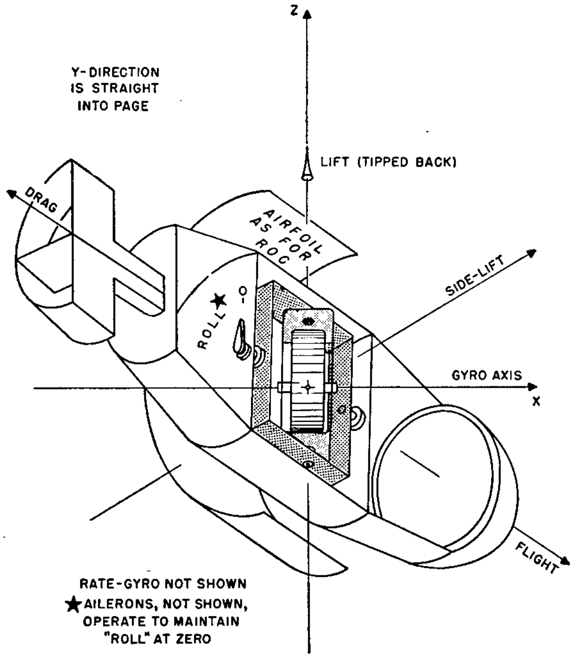


FIGURE 2. Gyro arrangement for roll stabilization.

dicular it follows that the direction cosines of **D**, **L**, and **S** are

$$\frac{D_x}{D}=\frac{-\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}},\ \frac{D_y}{D}=\frac{-\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}},$$
$$\frac{D_z}{D}=\frac{-\dot{z}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}};$$
$$\frac{L_x}{L}=\frac{-\dot{z}}{\sqrt{\dot{x}^2+\dot{z}^2}},\ \frac{L_y}{L}=0,\ \frac{L_z}{L}=\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{z}^2}}; \tag{4}$$
$$\frac{S_x}{S}=\frac{-\dot{x}\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\sqrt{\dot{x}^2+\dot{z}^2}},\ \frac{S_y}{S}=\frac{\sqrt{\dot{x}^2+\dot{z}^2}}{\sqrt{\dot{x}^2+{}^2+\dot{z}^2}},$$
$$\frac{S_z}{S}=\frac{-\dot{y}\dot{z}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\sqrt{\dot{x}^2+\dot{z}^2}}.$$

Consequently the differential equations (2) become

$$\ddot{x}=-\dot{x}\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\left(d+\frac{s\dot{y}}{\sqrt{\dot{x}^2+\dot{z}^2}}\right)-\frac{l\dot{z}}{\sqrt{\dot{x}^2+\dot{z}^2}}\cdot(\dot{x}^2+\dot{y}^2+\dot{z}^2),$$
$$\ddot{y}=\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\left(-d\dot{y}+s\sqrt{\dot{x}^2+\dot{z}^2}\right), \tag{5}$$
$$\ddot{z}=-\dot{z}\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\left(d+\frac{s\dot{y}}{\sqrt{\dot{x}^2+\dot{z}^2}}\right)+\frac{l\dot{x}}{\sqrt{\dot{x}^2+\dot{z}^2}}\cdot(\dot{x}^2+\dot{y}^2+\dot{z}^2)-g.$$

Or, setting

$$u^2=\dot{x}^2+\dot{z}^2,\quad v^2=\dot{x}^2+\dot{y}^2+\dot{z}^2,$$

we have

$$\ddot{x}=-\dot{x}v\left(d+\frac{s\dot{y}}{u}\right)-\frac{l\dot{z}v^2}{u},$$
$$\ddot{y}=v\,(-d\dot{y}+su), \tag{6}$$
$$\ddot{z}=-\dot{z}v\left(d+\frac{s\dot{y}}{u}\right)+\frac{l\dot{x}v^2}{u}-g.$$

The use of the differential analyzer at MIT to solve these equations was obtained through NDRC Division 7, and later Division 5, contracts. Appropriate constants have been used which apply to the various RAZON types. Later, solutions for the several sets of experimental ROC constants were possible. It was initially found impossible to get the complete equations on the analyzer in the case of three-dimensional control. In the case of "range-only" control, in the vertical plane purely, the exact equations could be used. For the former case, in three dimensions, the following approximate equations were found to suffice and were used.

$$\ddot{x}=-\left[\dot{x}\left(d+\frac{s\dot{y}}{\sqrt{\dot{x}^2+\dot{z}^2}}\right)+l\dot{z}\right]\sqrt{\dot{x}^2+\dot{z}^2},$$
$$\ddot{y}=\left[-d\dot{y}+s\sqrt{\dot{x}^2+\dot{z}^2}\right]\sqrt{\dot{x}^2+\dot{z}^2}, \tag{7}$$
$$\ddot{z}=-\left[\dot{z}\left(d+\frac{s\dot{y}}{\sqrt{\dot{x}^2+\dot{z}^2}}\right)-l\dot{x}\right]\sqrt{\dot{x}^2+\dot{z}^2}-g.$$

Again, these equations may be abbreviated as follows:

$$\ddot{x}=-\dot{x}u\left(d+\frac{s\dot{y}}{u}\right)-l\dot{z}u,$$
$$\ddot{y}=u(-d\ddot{y}+su), \tag{8}$$
$$\ddot{z}=-\dot{z}u\left(d+\frac{s\dot{y}}{u}\right)+l\dot{x}u-g.$$

To obtain equation (7), v^2 in the drag and sideways force terms was replaced by $v\sqrt{\dot{x}^2+\dot{z}^2}$, or by uv, but in the lift term v^2 was replaced by $\dot{x}^2+\dot{z}^2$, or by u^2. This amounts to saying that D and S vary not as v^2 but as the product of v and its projection on the vertical plane. The somewhat different treatment of the lift term was dictated by exigencies of the differential

analyzer.[j] However it was felt that this was not too unrealistic for the purposes then at hand. In fact the lift force might certainly be decreased somewhat when lateral control is also applied, and such a decrease has not been otherwise taken into consideration. The approximate equations (7) reduce to (5) when S, and hence $\dot{y}$, is zero.

The function initially assumed for density was

$$\rho = 0.959\ \rho_0 e^{-2.96 \times 10^{-5}z}$$

where $\rho_0 = 0.002378$ slug per cubic foot is the density of air at sea level. Several other such functions were later employed, in dependence on the various physical circumstances which were relevant. There is little difference in the results, however, at least as far as guiding is concerned.

The functions C_D, C_S, C_L were generally obtained from empirical data. Certain standard trajectories were obtained for the various bombs for full control deflections, applied, say, at 8, 15.5, 23, and 27 seconds after the instant of release. The initial velocities assumed were between 175 and 275 miles per hour. In general the initial altitude was taken as 15,000 feet, although some trajectories were obtained using higher and lower altitudes.

Because of the constant modification of bombs and hence of coefficients, standard trajectories were also obtained for values of C_D, C_L, C_S 20 per cent lower and higher than those in current use. From these trajectories it was possible by interpolation to determine the maneuverability, ranges, time of flight, trail, etc., of future bombs with various characteristics within this range.

In another set of solutions, instead of putting in the values of C_D and C_L corresponding to full deflection in range, the functions C_D and C_L were plotted on input tables and applied at any time during the course of solution and for any chosen deflection.

We include here, for approximate comparisons, a table of the constants of several of the bomb types for which solutions were regularly made.

[j]The machine connections of the differential analyzer for the three-dimensional solutions are shown in Figure 1A. A simpler arrangement, of course, was possible in the plane, when $y \equiv 0$, but that was a special case of the connections shown.

TABLE 1. Tentative data on projectiles.

Bomb	m Mass in lb	A Area in ft²	$\delta_{max}/\frac{d\delta}{dt}$ in sec	$C_D(0^0)$ Min.	C_D Max.	C_L, C_S Max.
RAZON A	1,000	1.865	0.7-1.0	0.406	0.978	0.981
AZON A	1,000	1.865	0.7-1.0	0.265	0.487	0.513
RAZON B	1,020	1.89	0.6	0.262	0.898	0.637
AZON B	1,020	1.89	0.6	0.262	0.580	0.637
RAZON C	2,160	2.89	0.6	0.261	0.713	0.739
AZON C	2,160	2.89	0.6	0.261	0.487	0.739
ROC A	1,700	9	1.7	0.413	0.673	0.65
ROC B	1,700	9	1.7	0.122	0.382	0.65

8.3 EARLY STUDIES ON COLLINEARITY CONTROL

It was generally agreed that the bomb, and possibly also the airplane, should for the best results be so maneuvered that over a finite time interval prior to impact the bomb, airplane, and target would be collinear (compare Figures 3 and 4). In such case all errors in dropping might ultimately be reduced to zero and high accuracy would result. It would only be necessary to provide stability in the guiding process. Consequently a detailed study has been made of this aspect of the problem, for which the use of the differential analyzer was invaluable. In the case of the RAZON bomb a point target was assumed, located normally where the unguided bomb would fall. Programmed deflections were applied in an attempt to arrive at a trajectory giving the desired collinearity during the last part of the flight. An ideal line of sight trajectory was constructed geometrically under various assumptions. For example, it was assumed that the airplane, traveling 250 miles per hour at the instant of release of the bomb, could decelerate thereafter at the rate of 2.5 miles per hour per second. For a typical RAZON, the best possible trajectory that could be obtained from the analyzer using full positive lift and then negative lift lay about half way between this ideal curve and the free-fall curve.

Similar trajectories were obtained after shifting the target from the position of a free-fall hit, that is to say after allowing for either a rangewise or lateral offset. The lateral offset seemed especially attractive, since the guiding

plane would then maneuver to the opposite side to achieve line of sight with the bomb and target.

The result of these studies has thus far been negative. From the trajectories calculations were made of where the plane would have to be at each second in the last part of the bomb's flight in order to be collinear with bomb and target, assuming the target at the point where the bomb landed, and that the plane remained in horizontal flight. In each case the same two difficulties appeared, namely:

1. The plane must be traveling during the last part of the flight at too low a speed.

2. The plane must be further back than it might be on the assumption of a reasonable deceleration. This is the most serious difficulty.

The conclusion is that with the available bombs and airplanes, the line of sight, or collinearity condition cannot be achieved without an undesirable maneuver.

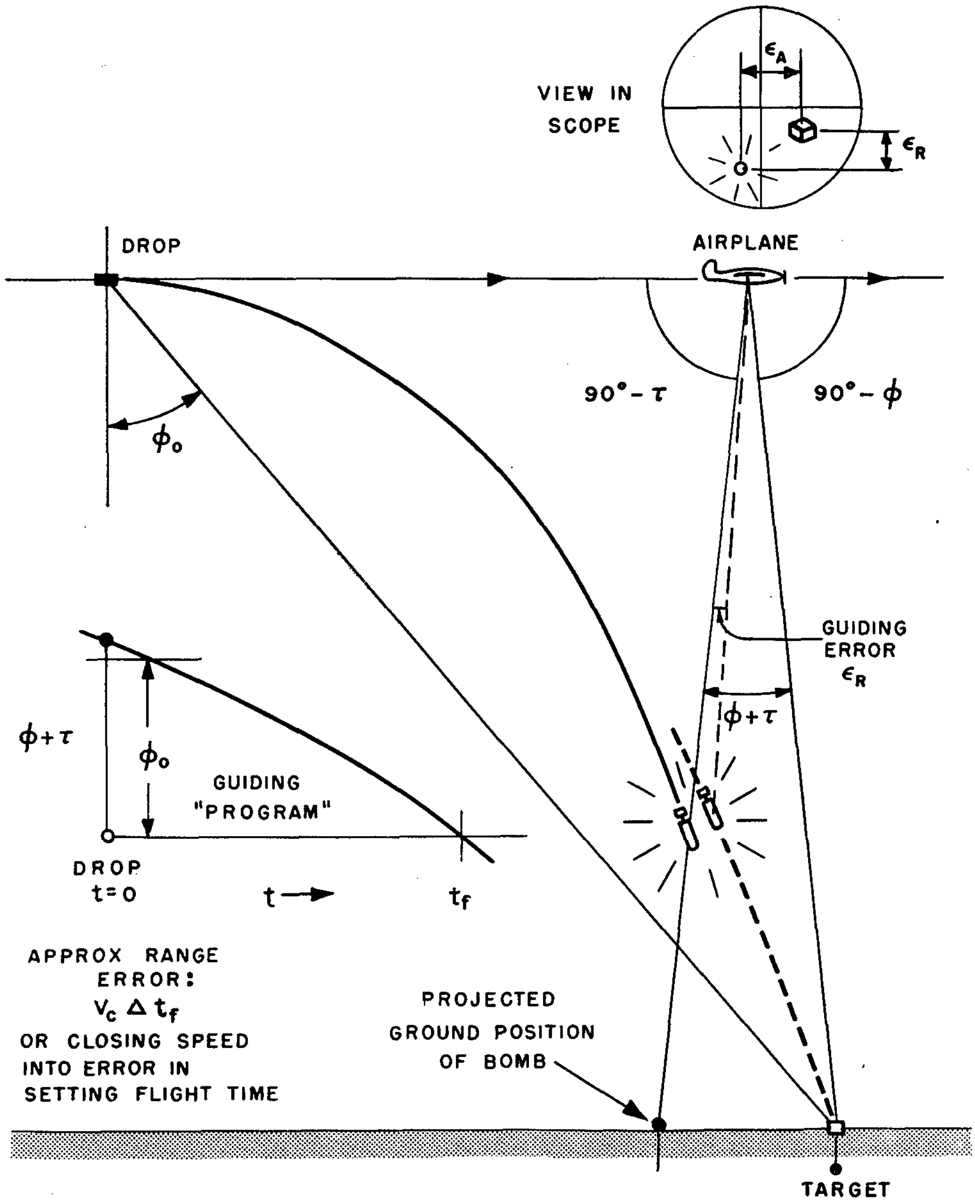


FIGURE 3. Character of trajectory with CRAB.

Some trajectories were obtained for an altitude of 28,000 feet. In these cases difficulty (1) did not arise. For a lateral offset of the target,

using rudder control and evasion to the opposite side, the plane could travel during the last part of the flight at its initial velocity of 250 miles per hour and still be collinear with bomb and target. But difficulty (2) was still present; for line of sight the plane would have to be about

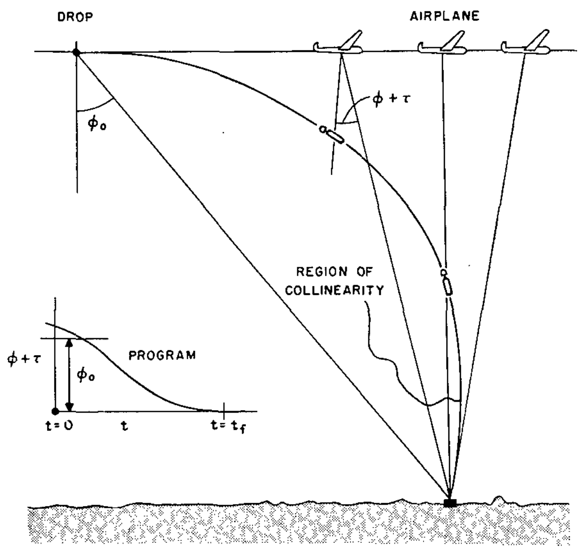


FIGURE 4. Character of trajectory with CARP.

3,000 feet back of where it would be at the given times for the given velocity. It may be noted that the necessary condition for terminal collinearity, implied in (2), namely that the tangent to the trajectory at impact contain the observer, is also barely sufficient, in the sense of (1), provided of course that the bomb hits the target.

The corresponding problem with lateral control only, as in the AZON bomb, is much simpler, where a line target instead of a point target is assumed. Then it is a question of plane of sight instead of line of sight. If the bomb is not maneuvered and the airplane continues flying in the same direction as at release, then line target, bomb, and airplane are coplanar for a precise and undisturbed drop. If the airplane deviates from its straight course, either inadvertently or deliberately as in an evasive maneuver, the bomb can be maneuvered in the same direction so as to achieve the plane of sight condition, at least terminally. The extent of the evasive maneuver is of course limited by the available maneuverability of the bomb. Analytic trajectories have been obtained for the AZON bombs using first right deflection and then left deflection, the change being effected at such a time as to make the y coordinate of the bomb zero at impact. This has been done for full rudder control to get the outer limits of the maneuver, and also for half deflection. In general it was believed that only half-control should be used for such evasion, leaving the rest for correction of dropping errors.

To return to the most crucial matter, that of collinearity in the more complete sense, we estimated that a bomb of roughly twice the lift of the "standard" RAZON would be necessary to achieve this end. More lift than this would be advisable for comfortable tolerances. The effect of large concurrent increases in the drag is not as well known, but we had some indication that this was not so important.

8.4 COMMENTS ON EVASION BY THE BOMBER

It was considered by many of the operational military personnel involved that moderate evasion is very little better than none, owing to the dispersion of antiaircraft fire. Against fighters, on the other hand, effective formation tactics again preclude violent evasive maneuvers by the bomber. Furthermore, for violent evasion following release, the stabilizing problems and the visibility requirements in computing and sighting apparatus increase alarmingly and lead to very complex and novel equipment. In particular, this is brought about by the considerable bank of the airplane which is involved if a decided turn is entered into.

An evasion may also be made for the purpose of aiding the collinearity condition. Thus the use has been made of a climb to slow down the bomber and allow the bomb relatively to advance. While this may be the only way to obtain collinearity with a weakly maneuverable bomb, it is considered undesirable as a tactical measure. For example, if an injured bomber is in the formation, the other planes naturally dislike to climb away from him. The latter consideration might not apply to an evasive maneuver, the term being used in both senses as above described, which consisted in a sustained turn at constant altitude (see Figure 5). It is inter-

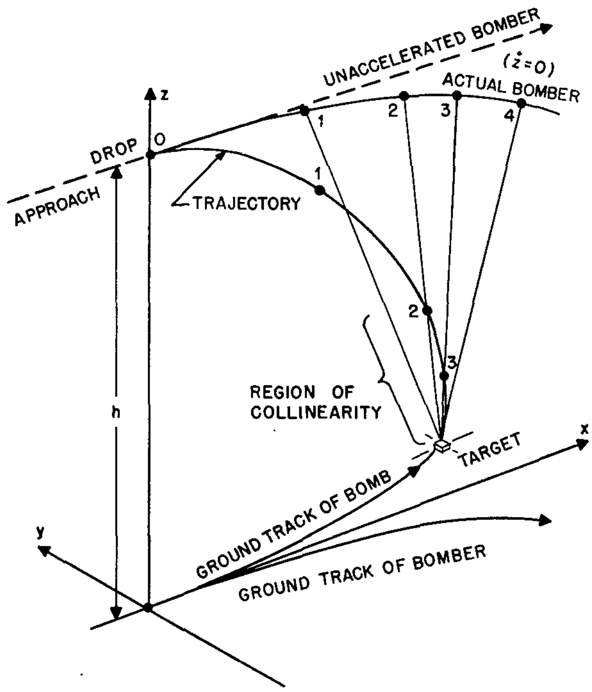


FIGURE 5. Special maneuver and guided trajectory.

esting to note that such a maneuver might make it easier to attain collinearity of bomb and target, and hence accuracy in both directions, toward the end of the time-of-flight. Such a tactic would involve dropping the bomb during a run on a point displaced laterally from the target by about one-tenth of the altitude, and entering on a turn to the other side just after dropping. A control program would turn the bomb, climb it, and then dive it toward the target. Manual control would then refine the trajectory. In such a process, the guiding would be made less confused due to the separation of the x and y directions which is involved in the special gyroscopic method of roll stabilization already mentioned. It would remain, however, in the sighting system made for this purpose, to make somewhat different provision (than heretofore considered) for separating out the influence of bomb motion in the z direction.

8.5 EVOLUTION OF THE FIRST SIGHT FOR GUIDING

The struggle for collinearity, in bombs of the type available until recently, ended in frustration. In the design of a sight for immediate applicability, we were forced to accept the earlier doctrine that a guided bomb should be corrected by operations which change its trajectory but little from that in an unguided fall (see Figure 3). It should be emphasized that, while there is certainly enough maneuverability in such bombs to compensate for target accelerations, ballistic winds, and dropping errors, these facts alone cannot assure accuracy in range when a normal approach is made. This circumstance would be very different, for example, if the normal approach were straight down from vertically overhead.

Since collinearity over an interval, as illustrated, for example, in Figure 4, appeared to be impossible we strove for instantaneous collinearity at the moment of impact. Given the proper guide to steer the bomb during flight, the range accuracy of this method depends only on the knowledge of flight time: and hence principally of bomb ballistics and altitude. It turns out that the properties of an instrument which will show how the angle between bomb and target should close up in time are also the properties of the computer, if appropriately employed, in the Norden bombsight Mark 15. This fact led to the attractive possibility of using the same group of apparatus for dropping the bomb and for guiding it after the drop. A diagram showing the operation of this sight is given in Figure 6. A normal drop is made as though the bomb were not to be guided. The operator then avoids further adjustments to the bombsight for synchronizing, but watches the target. The flare on the bomb is reflected in the target field in such a way as to lie directly on the target if the descent is proceeding according to plan. The desired trajectory is thus referred to the target itself; and if the proper time of flight is set in, and if the operator can "guide" the image of the bomb always onto the target, he will hit. It was contemplated to use this method in range only, with RAZON, permitting another operator to guide in azimuth. The latter is thus equivalent to an AZON guider. If minor evasions or accelerations are indulged in by the bomber, this will merely warp the trajectory somewhat. The trajectory will still end in the target, if the other conditions are met.

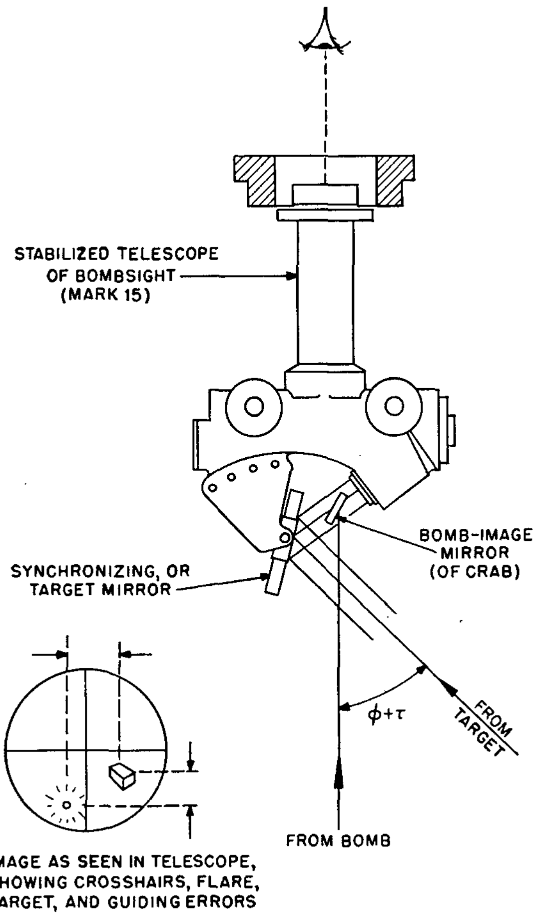


FIGURE 6. Mechanism for bomb images in CRAB and CARP.

If, due to maneuvers either by bomber or target, or due to an extremely poor synchronization by the bombardier, the target moves out of the field of the telescope, it is only necessary, and without effect on the result, to precess the vertical gyro of the bombsight back into such a position that the target may again be seen. This should normally not be necessary, however.

Modifications of the mechanism of CRAB, to lead to collinearity conditions with a more maneuverable bomb, were later developed, and resulted in a sight called CARP, for use with a flare-equipped ROC. Through the aid of this sight the bomb was to have been efficiently guided into collinearity.

8.6 ORIGIN OF THE GUIDING SIMULATOR

It is analytically difficult to work successfully on the stability problem since (1) a human being is involved, as yet of unknown differential equation, and (2) the controls themselves are subject to boundary conditions in deflections and rates: thus making stability of the system between such boundaries only a necessary, and not a sufficient, condition for complete stability. As for similar problems in other fields of endeavor, it was decided to use electronic simulative methods for this problem.

Not only is it essential to be able to guide the bomb onto the target in angle, or onto an auxiliary programmed aiming point, but it must be possible to keep it there very closely. In the laboratory a reproduction of the dynamics involved in this operation has been developed and has proved to be quite useful. An account of the apparatus and technique of this development is given elsewhere.

For the purpose of such simulation it was not considered necessary to duplicate in nearly so precise a form the overall dynamics of the bomb's motion, since secondary effects, which follow slowly after the more immediate response to a guiding impulse, are of relatively minor importance in the control dynamics.

By adjusting the time scale it is possible to include the human operator existing in conjunction with other elements of the causal loop, or to study various artificial components through oscillographic response to standard inputs.

But the principal advantage of this approach is that it permits the realization, in flexible model form, of all the discontinuities which are present in the real prototype, and which must be dealt with in the design of auxiliary equipment to attain stability under all conditions and through all regions of the variables.

A further, and not entirely accidental, property of this sort of simulative arrangement is that it may be applied as training equipment. Additional realism may be attained by the superposition of moving map images around the oscilloscope screen. It is considered that such application is a valuable addition to the provision of a laboratory "proving ground" for experimental sighting equipment. The above laboratory simulative systems, and the resulting trainers, were extended later to apply to the ROC dynamics, having initially been built

principally around RAZON. Changes in degree, only, were necessary. In fact a somewhat closer representation was obtained owing to the aerodynamic stability of the latter missile.

8.7 CONDENSED SUMMARY OF CRAB AND CARP

AZON and RAZON are guided bombs made by converting standard 1,000-pound or 2,000-pound bombs through replacement of the tail structure by a new unit containing radio, power supply, gyros, and fin-driving and aileron-driving motors. They are appropriately stabilized in roll, and may be dropped normally and then deflected moderately about a normal trajectory during a more or less straight continuation of the bomber's run. Whereas AZON may be deflected in line only, RAZON may be deflected both in line and in range.

CRAB is a sight and sighting technique for guiding RAZON. A small mirror is attached to the Mark 15 telescope in such a way that the image of the target, which normally stays in the field even after the bomb is dropped, has superimposed on it an image of the bomb flare, which may then be guided onto the target. This results, under average conditions, in the bomb following a trajectory which is perfectly normal in shape toward the end except for the guiding transients, and which may terminate in the target. CRAB may be easily installed in the bombsight and does not interfere with normal bombing. With this method, it is important to know the bomb's time of flight within good accuracy and to set in the corresponding disk speed. Tests from 15,000 feet gave a probable error of 90 feet in range and 10 feet in line.

ROC is a somewhat different missile, still in development. It features a wing structure which permits flight with zero pitch and yaw, as well as a much greater maximum lift, and hence curvature of path. Although intended for other purposes, it was also developed for an application (remote visual guiding) much like that of RAZON.

CARP, still in development, is a sight for ROC. It is more elaborate than CRAB, and differs in that the angular program between the directions to bomb and target is not that of a standard (accurate) bomb. This program converges in such a way, toward the end of the time of flight, that the angle gradually decreases to zero and remains so over an appreciable interval of time. A computer, which produces this program and drives a mirror in an auxiliary telescope, derives its information from normal bombing data which are available in the bombsight, to which CARP is attached.

Along with these sight developments, an analytic trajectory study was carried out with the differential analyzer. A large library[8] of guided trajectories, some in three dimensions, was collected, and a complete index prepared.

There was also the electronic simulator study. The purpose here was only to study the stability of the human guiding process. This has helped tremendously in the choice of guiding paraphernalia. This same technique was subsequently applied to the development of trainers for field use.

8.8 CONTROL OF RAZON FROM THE GROUND

Upon a definite military request, in the summer of 1945, cooperative conferences were arranged among NDRC Divisions 5, 7, and 14 (Radiation Laboratory at MIT), to study the question of bombing strongly held enemy positions, such as those in caves and on hills in Pacific islands, from lines of advance at the edge of occupied territory, by remote control of detectable bombs dropped onto such positions. Thus if, from two separated friendly points, the azimuth bearings of such a target were identifiable, a bomb dropping generally toward the target might be so guided as to fall on each such bearing at ground level, and consequently on the target to high precision. In this cooperative process, the bombing airplane might be appropriately guided to an appropriate dropping position by one or more of the ground stations.

An early suggestion was for the application of such radar sets as the SCR 584 as a means for complete knowledge, during the drop, of the bomb position, and indeed for the guidance of the bomber. While our own studies were on the visual aspects of the tactic, in which the bomb

was to be flare-equipped and the ground stations relatively close by, we think it appropriate to mention one solution,[k] for the combined problem of radar detection and remote guiding. It is assumed that the ground position is identifiable by some means and to high accuracy, in terms of radar coordinates, and that both bomber and bomb are uniquely identifiable in the same coordinate system. The solution was obtained as follows, with the SCR 584 coupled (say) to the M-9 antiaircraft director. Prediction would be applied in the standard manner to the bomber, as though it were a bomb, during the approach. That is, the future position of the bomb, at the end of the known time of flight thereof, and assuming a modified gravity acceleration after drop, would be continuously computed and indicated at every instant as though the bomb were dropped at that instant, by direct tracking of the bomber. Simple modification of the M-9 computer could accomplish this, and a presentation of the potential ground position of the bomb would be continuously available at the single remote station. Thus not only could the bomber be directed into position, but the signal for release could be directly provided when the predicted ground position of the bomb coincided with the target.

Following release, the radar tracking would be switched to the falling bomb. Prediction would be continued, using the same future acceleration, but now with a time of flight determined either from the remaining bomb altitude or the time duration since release. This would still provide a predicted ground intersection for the bomb, based on simplified, but adequate and continually more valid, extrapolation. Guiding would be direct and two dimensional, in plan. It would be relatively stable, and hence potentially accurate, owing to the presence of a first-derivative response in the prediction, which embodies the guiding reference. This method would certainly lead to very successful attack. Its only drawback is the weight of equipment required, some 17 tons, although this is partly compensated by the greater range from the control station which would be allowed.

It should be noted, either with the radar or the visual method of control from the ground, that the projectile need not be dropped from a bomber, but could equally well be launched, mortarwise or by other means from the ground. Launching could occur from the observation posts or from further back. For future development, it seems that this freedom is significant.

On the visual side again, probably the simplest arrangement in which control is completely applied from the ground is that in which the ground stations look toward the target along perpendicular directions, and the bomber flies directly along one such direction. The roles of such stations could be interchanged, and there might be much to be gained in such symmetry and flexibility. More generally, the bomber might attack on any track dividing the lines toward the target from the two ground stations, which indeed need not be rectangularly disposed. It is also evident that the ranges between stations and target need not be equal, and may have a large range of absolute values.

In[l] Figure 7, let A and R denote the azimuth

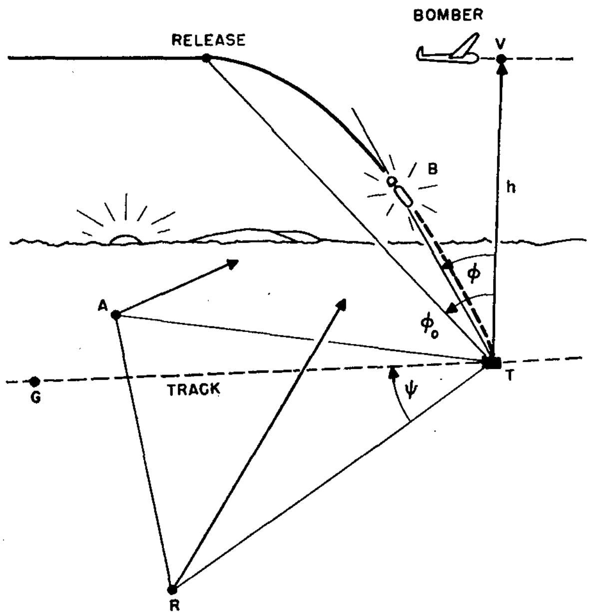


FIGURE 7. Observation scheme for ground control of RAZON.

and range control stations, respectively, and T the target. The airplane is so directed on its approach that its ground tack is a line GT between AT and RT. The question of the initia-

[k]Suggested by G. R. Stibitz of NDRC Section 7.1.

[l]The following material is adapted from an earlier report.

tion of the drop itself will be discussed below.

Let B be the position of the bomb at any instant, and ϕ the angle between BT and the vertical VT. The value of ϕ at the instant $t = 0$ is the dropping angle ϕ_0. Control of the bomb can be achieved by observation of ϕ and its projections. Since the observer at R will see not ϕ but its projection $\bar{\phi}$ on a vertical plane perpendicular to RT, it is desirable to employ for reference in control some function of ϕ which is not altered by projection.

Let ψ be the angle GTR. Then

$$\tan \bar{\phi}_0 = \tan \phi_0 \sin \psi. \tag{9}$$

If the bomb is so guided in azimuth as to remain in the plane GTV, then

$$\tan \bar{\phi}(t) = \tan \phi(t) \sin \psi. \tag{10}$$

Consequently

$$\frac{\tan \bar{\phi}(t)}{\tan \bar{\phi}_0} = \frac{\tan \phi(t)}{\tan \phi_0}. \tag{11}$$

If the bomb is allowed to deviate somewhat from the plane GTV, this relation will not hold exactly, but in general such deviations will be slight, and will not affect the guiding process.

The function $\tan \phi(t)/\tan \phi_0$ is also unaltered by variation of the airspeed of the bombing plane. A difference of 75 miles per hour in velocity produces less than 1 per cent change in $\tan (t)/\tan \phi_0$, this change being due to the increase in the horizontal component of drag at higher speeds.

It follows that $\tan \phi/\tan \phi_0$ is a function of time and initial altitude only:

$$\frac{\tan \phi(t)}{\tan \phi_0} = f(t,h). \tag{12}$$

The variation with altitude may be handled by stretching of the time scale. If 15,000 feet is taken as standard altitude, then

$$f(t,h) = f(kt, 15{,}000), \tag{13}$$

where

$$k = k(h), \tag{14}$$

or

$$f\left(\frac{t}{k}, h\right) = f(t, 15{,}000). \tag{15}$$

Consequently only one function of time, $f(t, 15{,}000)$, is needed. This could be mechanized by a cam driven at a rate dependent on altitude. If the bombsight Mark 15 should be used for tracking, the function $k(h)$ is not necessary. The cam for $f(t,h)$ may be driven by the constant speed motor through the disk whose speed is inversely proportional to the time of flight of the bomb.

The functions $f(t, 15{,}000)$ and $k(h)$ may be readily calculated from free-fall trajectories run on the differential analyzer. The bomb should be guided around the free-fall case. One operator might track the bomb with the cross hair, making such adjustments to the telescope as may be necessary to keep the bomb on the cross hair and in the field of view. A second operator might then control the bomb. From the telescope position the actual value of $\tan \bar{\phi}'(t)$ is obtained and the ratio $\tan \bar{\phi}'(t)/\tan \bar{\phi}_0'$ is formed. The primes are used to distinguish observed values from theoretical ones. This ratio is to be compared at each instant with the values given by $f(t,h)$. For a perfect drop

$$\frac{\tan \bar{\phi}'(t)}{\tan \bar{\phi}_0'} \frac{1}{f(t, h)} = 1, \tag{16}$$

$$\frac{\tan \bar{\phi}'(t)}{f(t, h)} - \tan \bar{\phi}_0' = 0. \tag{17}$$

However it is not necessary, or even desirable, that the bomb be controlled so as to keep the right-hand side of equation (17) zero. A criterion for a hit is that $\tan \bar{\phi}$ remain finite. Hence it is sufficient to keep this quantity approximately constant.

$$\frac{\tan \bar{\phi}' f(t)}{f(t,h)} - \tan \bar{\phi}_0' \doteq C. \tag{18}$$

The value of C may be indicated by a pointer on a scale (having many divisions, and also perhaps a special pattern) whose center point corresponds to zero. The operator would guide the bomb in such a way as to keep the needle as stationary as possible. The initial value of C at $t = 0$ would be approximately zero. During the drop the needle might deviate from the central position for a variety of reasons, but it should not be necessary to control the bomb so as to make the needle return to that or any other given position. A change in actual drag

coefficient C_D from the value used in calculating $f(t,h)$ may cause C to vary slightly. Change in time of flight due to application of control will cause small variation in C. Deviation of the bomb from the vertical plane GTV may also alter the perspective relationship and hence C. But all these effects are slight and would cause practically imperceptible motion of the needle.

An error in dropping, however, where the bomb, if unguided, would miss the target, would sooner or later cause a quite perceptible change in C, increasing with time, since $\tan \bar{\phi}$ in such a case becomes infinite at impact. In the case of a prospective miss of 100 feet short on the ground for $v = 250$ miles per hour and $\psi = 90$ degrees, C will increase at the rate of 15 per cent per second 9 or 10 seconds before impact, this rate rapidly increasing with time. The same statement holds for a 100-foot overshoot, except that C is decreasing in that case. This allows ample time for correction of the error. For a larger error on the ground a 15 per cent per second change in C would occur somewhat earlier in flight. The value of C at any time is nearly proportional to the prospective error, provided C is changing with time.

It is desirable that both stations A and R be similarly equipped and operated, so as to be readily interchangeable in function. Hence it is recommended that azimuth control be exactly like range control, basically recognizing the same angle ϕ. Station A will observe the projection of ϕ on a vertical plane perpendicular to AT. If the bomber flies so that its ground track lies along AT, the tangent of the projected dropping angle is zero, but the method of range control is still applicable to azimuth.

It is possible for the airplane to drop the bomb as usual with its bombsight. However, it might be desirable to direct the release from one of the ground stations, say R. R could track the airplane prior to release with the same mechanism to be used later for the bomb. The usual release condition could be used:

$$\tan \phi_0 = \frac{v_c t_f}{h} - \tau, \qquad (19)$$

where τ is angular trail in mils. However, the operator at R would observe these quantities projected on a vertical plane through the airplane and perpendicular to RT. Consequently his release condition becomes

$$\tan \bar{\phi}_0 = \frac{\bar{v}_c t_f}{h} - \tau \sin \psi, \qquad (20)$$

where $\bar{v}_c$ is the projection of v_c. If the synchronous sight is used, $\bar{v}_c/h$ would be synchronized just as v_c/h is in the normal procedure. The only difficulty is the projection of the trail τ. This is the only part of the whole process where a knowledge of the angle ψ is needed. If the airplane is guided over the line AT, then ψ is readily determined by measurement of the angles ART and RAT, as shown in Figure 7.

A mechanism for embodiment of somewhat the same principle as above described has been planned.[m] Development along such lines may continue, beyond termination of contractual relations between that organization and NDRC Division 5, under the auspices of the special weapons branch of ATSC at Wright Field.

An interesting rationalization which bears on this problem is based on vacuum flight conditions for the bomb and relates to behavior of the angle ϕ as above employed. Consider a perfect unguided drop. Whereas before release the rate of change of $\tan \phi$ was uniform, after release the rate of change of $\cot \phi$ will be uniform. We refer, of course, to the bomb itself in both cases. In fact

$$\text{Before release:} \quad \frac{d}{dt} \tan \phi = \frac{v}{h}, \qquad (21)$$

$$\text{After release:} \quad \frac{d}{dt} \overline{\cot \phi} = \frac{g}{2v}, \qquad (22)$$

where v is the horizontal component of the bomb's velocity, in target coordinates, or the closing speed of the bomber.

It has been proposed that this property be used in ground control of RAZONs, guiding the bomb in such a way as to hold $d \cot \phi/dt$ constant. Such control would cause little deviation from a free-fall trajectory and would insure a hit. It is evident that during both regimes, before and after release, a simple mechanism would suffice for tracking or for the guiding criterion, being based on the above equation.

[m]By H. A. Van Dyke, of L. H. Schwien Engineering Company, in consultations with the writer.

Horizontal and vertical "tangent-screws," for example, might here be involved. It might further be desirable to use the cotangent criterion in directing from the ground the release of the bomb.

Let a Cartesian coordinate system be set up with origin at the target T, and x axis along the ground track of the bombing airplane (cf. earlier sections and figures of this chapter). Let[n] P be the position of the airplane at any instant before release, h its altitude, v its ground speed. The ground station directing release is at R, RT making the angle ψ with the x axis, as before.

Let bars denote projections on a vertical plane perpendicular to $\overline{RT}$; for example, $\bar{\phi}$ is the projected value of ϕ as observed at the station R. Then

$$\cot \bar{\phi} = \cot \phi \csc \psi, \tag{23}$$

and

$$\frac{d \cot \bar{\phi}}{dt} = \frac{d \cot \phi}{dt} \csc \psi. \tag{24}$$

For a nonvacuum bomb at the instant of release

$$\frac{d \cot \phi}{dt} = \frac{f(C_D)}{v}, \tag{25}$$

instead of the condition given by equation (22). Here $f(C_D)$ depends only on the drag coefficient C_D, and has the dimensions length/time^2. In particular $f(0) = g/2$. By equations (24) and (25), at the release point,

$$\frac{d \cot \bar{\phi}}{dt} = \frac{f(C_D)}{v \sin \psi} = \frac{f(C_D)}{\bar{v}}. \tag{26}$$

We also know that

$$\cot \bar{\phi} = \frac{h}{\bar{x}}, \tag{27}$$

and consequently

$$\frac{d \cot \bar{\phi}}{dt} = \frac{h}{\bar{x}^2} \bar{v} \tag{28}$$

since certainly $d\bar{x}/dt = -\bar{v}$.

Consequently we obtain the simple new release criterion

$$\frac{1}{\cot \bar{\phi}} \frac{d \cot \bar{\phi}}{dt} = \sqrt{\frac{f(C_D)}{h}}, \tag{29}$$

Thus the whole operation might be carried out in terms of the cotangent, and the fact may have broader significance yet for bombing.

We note that *in vacuo* we have simply, as a release condition,

$$\frac{1}{\cot \bar{\phi}} \frac{d \cot \bar{\phi}}{dt} = \frac{1}{t_f}. \tag{30}$$

8.9 CONTROLS FOR THE TELEVISION BOMB

We have described earlier in this chapter the general arrangement for television guiding, as of ROC, where a remote operator tracks the departure of the target, given say by the variable "lead" λ, on the screen and thereby, through established control dynamics, affects the corresponding deflection δ of the appropriate control surface of the projectile. The motion μ of a tracking handle which he manipulates in this process can be conducted both to λ and to δ, so that merely by holding an index on the target, the operator automatically maintains a relationship between λ and δ, by virtue of the dynamic connections involved. A major question is where to include the control dynamics, whether between μ and λ or between μ and δ. Apparently either procedure is valid, being respectively analogous to that for a lead-computing sight (so-called) and a director, in more orthodox fields. We shall here avoid this question by considering only the result, which is embodied in the effective dynamic relation to be established between λ and δ, in each coordinate.

In Chapter 1 we have spoken of the various means for attaining a collision or interception course for a vehicle with respect to a target. Here it was not considered possible to install new gyro equipment in the bomb, but by a peculiar stratagem, mentioned elsewhere from the philosophical standpoint, and exposed in more detail below, it was found possible to obtain equivalent dynamics without additional equipment in the bomb.

The characteristic equations herein proposed were first tested on an electronic simulative structure, referred to briefly in Chapter 4 on simulation, in which the human operator could be involved. Stable subsidence to an interception approach was found to occur, without critical adjustments being necessary in the

[n]The present exposition is based on a study by R. M. Peters of Section 7.2.

auxiliary regulatory controls. This conclusion was later checked by model studies on a larger scale, thus continuing the verification of the electronic model technique in exploratory roles.

In the following analysis we refer to Figure 8, showing the bomb falling in the air mass toward the target.

It is assumed initially that the bomb has no attack angle, so that the longitudinal axis of

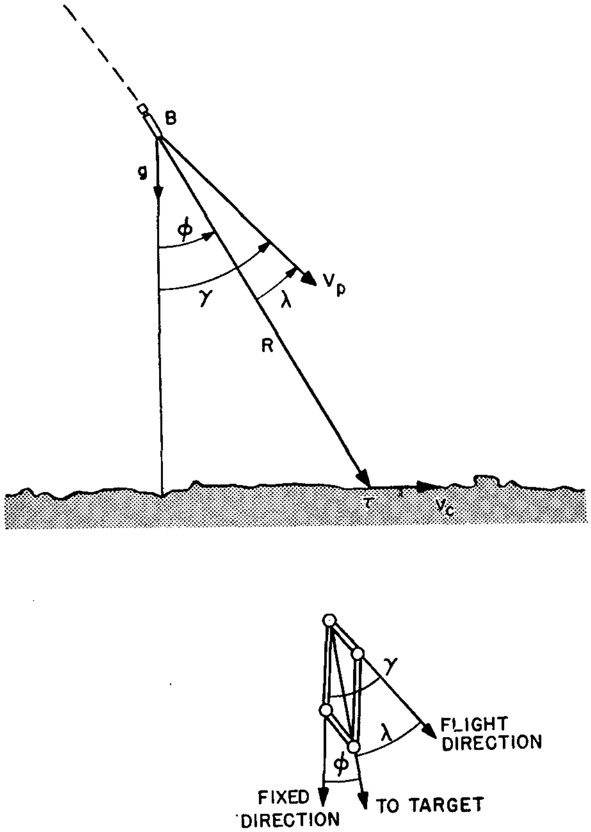


FIGURE 8. Range problem for television bomb.

the missile is in the direction of its velocity vector. The velocity of the target is v_e, and that of the projectile is v_p, both such velocities being measured with respect to the air mass. Then from the figure, assuming λ small (cf. Chapter 1),

$$\dot{\phi} = -\frac{v_p\lambda}{R} + \frac{v_e \cos\phi}{R} \,. \tag{31}$$

Suppose control is applied to the bomb in such a way as to make it turn in the air according to

$$\dot{\gamma} = \frac{N}{N-1}\dot{\lambda} + \omega\,, \tag{32}$$

where N is a constant greater than one, and ω is a constant angular rate. An important special case, incidentally, occurs when $\omega \equiv 0$. Since

$$\dot{\gamma} = \dot{\phi} + \dot{\lambda}\,, \tag{33}$$

equation (32) gives

$$\dot{\phi} = \frac{1}{N-1}\dot{\lambda} + \omega. \tag{34}$$

Eliminating $\dot{\phi}$ in equation (31) by use of equation (34)

$$\frac{1}{N-1}\dot{\lambda} = -\frac{v_p\lambda}{R} + \frac{v_e\cos\phi}{R} - \omega\,. \tag{35}$$

Now since, approximately,

$$\frac{dR}{dt} = -v_p\,, \tag{36}$$

equation (35) may be written as a differential equation in λ and R:

$$\frac{1}{N-1}d\lambda = \frac{\omega}{v_p}dR + \frac{1}{R}\left(\lambda - \frac{v_e\cos\phi}{v_p}\right)dR\,. \tag{37}$$

Let

$$\eta = \frac{v_e}{v_p}\cos\phi\,, \tag{38}$$

and

$$\lambda^* = \lambda - \eta\,. \tag{39}$$

Then equation (37) becomes

$$\frac{d\lambda^*}{N-1} = \left(\frac{\omega}{v_p} + \frac{\lambda^*}{R}\right)dR\,. \tag{40}$$

If v_p and η are assumed to be constant, equation (40) is a homogenous differential equation in λ^* and R whose solution for the case $N \neq 2$ is

$$\frac{\lambda^*}{\lambda_0^*} = \left(\frac{R}{R_0}\right)^{N-1} \frac{\dfrac{N-2}{N-1} + \dfrac{\omega R_0}{v_p\lambda_0^*}}{\dfrac{N-2}{N-1} + \dfrac{\omega R}{v_p\lambda^*}}\,, \qquad (N \neq 1, 2) \tag{41}$$

where λ_0^* and R_0 are initial values of the variables. In the special case $N = 2$, the solution is

$$\lambda^* = \frac{R}{R_0}\lambda_0^* + \frac{\omega}{v_p}R\ln\frac{R}{R_0}\,. \qquad (N = 2) \tag{42}$$

In both cases $\lambda^* \to 0$ as $R \to 0$. That is, the lead $\gamma \to \eta$, the proper value for a collision course.

If ω is zero, equation (41) becomes

$$\frac{\lambda^*}{\lambda_0^*} = \left(\frac{R}{R_0}\right)^{N-1}. \qquad (43)$$

The next question is how to control the bomb so as to satisfy equation (32). The acceleration normal to the path of the bomb is

$$v_p\dot{\gamma} = \frac{L}{m} - g\sin\gamma\,, \qquad (44)$$

where L is the lift force acting on the bomb as the result of control:

$$L = \frac{\rho}{2}v_p^2AC_L = \frac{\rho}{2}v_p^2A\frac{dC_L}{d\delta}\,\delta\,. \qquad (45)$$

Here ρ is the air density, A is the area of the wing or control surface, δ is angle of incidence of the wing. Define

$$\frac{1}{T_\gamma} = \frac{\rho v_p A}{2m}\frac{dC_L}{d\delta}. \qquad (46)$$

Then

$$\dot{\gamma} = -\frac{1}{T_\gamma}\delta - \frac{g}{v_p}\sin\gamma\,.$$

Or

$$\frac{\delta}{T_\gamma} = \dot{\gamma} + \frac{g}{v_p}\sin\gamma\,. \qquad (47)$$

From equation (32) and the equation obtained from it by integration,

$$\gamma = \frac{N}{N-1}(\lambda - \lambda_0) + \omega(t - t_0) + \gamma_0\,; \qquad (48)$$

it follows that

$$\begin{aligned}\frac{\delta}{T_\gamma} &= \frac{N}{N-1}\dot{\lambda} + \omega \\ &\quad + \frac{g}{v_p}\sin\left\{\frac{N}{N-1}(\lambda-\lambda_0) + \omega(t-t_0) + \gamma_0\right\} \\ &= \frac{N}{N-1}\dot{\lambda} + \omega \\ &\quad + \frac{g}{v_p}\frac{N}{N-1}(\lambda-\lambda_0)\cos\left\{\omega(t-t_0)+\gamma_0\right\} \\ &\quad + \frac{g}{v_p}\sin\left\{\omega(t-t_0)+\gamma_0\right\}. \end{aligned} \qquad (49)$$

This is the equation governing the control of the bomb, where v_p and T_γ are to be given as functions of t, while λ and $\dot{\lambda}$ are to be obtained from the television screen. The initial values λ_0 and γ_0 can be most easily determined at some instant before control is applied, in which case γ_0 can be obtained from the function $\gamma = \gamma(t)$ for a free fall. All these functions of time will depend on the velocity of the airplane at the instant of release of the bomb.

In case the target has a component of velocity in azimuth, the same development holds in that direction as for elevation, except that all terms in g disappear and the control equation becomes simply

$$\frac{\delta}{T_\gamma} = \frac{N}{N-1}\dot{\lambda} + \omega\,, \qquad (50)$$

where δ is rudder instead of elevator angle, and λ is the azimuth component of lead. This separation of azimuth and elevation lead and control is valid when the bomb is roll-stabilized.

It may happen that the bomb has an attack angle α due to a slight shift in the center of gravity, which, neglecting transients, is given approximately by

$$\alpha = aC_L\,. \qquad (51)$$

The factor a is positive if the center of gravity is aft of the position for zero attack angle. Then the lead observed on the scope and supplied to equation (49) or rather to a mechanism embodying this characteristic, is not the actual lead λ but a value λ' including α:

$$\lambda' = \lambda + \alpha\,. \qquad (52)$$

The quantity $dC_L/d\delta$ is also altered by the presence of an attack angle. If the new value is denoted by $dC_L'/d\delta$ then[a]

$$\frac{dC_L'}{d\delta} = \frac{1}{1 - 3.43a}\frac{dC_L}{d\delta}, \qquad (53)$$

for δ and α measured in radians. Equation (51) may be written

$$\alpha = a\frac{dC_L'}{d\delta}\delta\,. \qquad (54)$$

Replacing λ by λ' in equation (49) gives

$$\begin{aligned}&\left\{\frac{1}{T_\gamma} - \frac{g}{v_p}\frac{N}{N-1}a\frac{dC_L'}{d\delta}\cos\left[\omega(t-t_0)+\gamma_0\right]\right\}\delta \\ &\qquad - \frac{N}{N-1}a\frac{dC_L'}{d\delta}\dot{\delta} \\ &= \frac{N}{N-1}\dot{\lambda} + \omega\frac{g}{v_p}\frac{N}{N-1}(\lambda-\lambda_0)\cos\left[\omega(t-t_0)+\lambda_0\right] \\ &\qquad + \frac{g}{v_p}\sin\left[\omega(t-t_0)+\gamma_0\right]. \end{aligned} \qquad (55)$$

It has been assumed that δ_0 is zero.

[a]This function was furnished by W. B. Klemperer of the Douglas Aircraft Co.

Equation (55) is not altered in form by the presence of an attack angle, except for the change in T_γ due to the change in $dC_L/d\delta$.

For the ROC projectile, currently (1944-45),

$$\frac{1}{T_\gamma} = \frac{0.244\rho V_p}{1 - 3.43a}$$

and

$$\frac{dC_L'}{d\delta} = \frac{2.85}{1 - 3.43a}$$

with

$$|\delta| \leq 0.227 \text{ radians}, \quad |a| \leq 0.0506 \text{ (approx.)}.$$

The quantities ρ and v_p are functions of time which will depend on the initial altitude and velocity, and also on the control applied to the bomb. They have been computed approximately from differential analyzer data for ROC with full brakes ($C_D = 0.413$ for no control), initial altitude 15,000 feet, initial velocity 250 miles per hour, assuming a moderate amount of control and a time of flight of 40 seconds. These variable parameters are tabulated against the duration of the drop in Table 2. The angle γ_0 at time t_0 is to be determined from the function $\gamma(t)$ computed for a free fall.

TABLE 2. Estimated functions for the ROC-MIMO projectile.

$h = 15{,}000$ feet
$v = 250$ mi/hr $= 366.7$ ft/sec

t(sec)	ρ(slugs/ft^3)	v_p(ft/sec)	γ_0(free fall) (degrees)
0	.001460	367	90.00
1	.001462	361	84.95
2	.001465	359	79.87
3	.001470	359	74.86
4	.001476	362	69.97
5	.001483	367	65.30
6	.001490	375	60.87
7	.001500	384	56.71
8	.001510	394	52.83
9	.001520	406	49.25
10	.001530	414	45.95
11	.001540	426	42.92
12	.001550	436	40.13
13	.001570	446	37.57
14	.001585	457	35.22
15	.001605	467	33.06
16	.001625	477	31.10
17	.001645	487	29.26
18	.001665	496	27.59
19	.001685	505	25.90
20	.001710	514	24.56
21	.001730	522	23.20
22	.001755	530	21.95
23	.001775	538	20.77
24	.001800	545	19.68
25	.001827	553	18.66
26	.001855	559	17.17
27	.001885	565	16.79
28	.001910	571	15.94
29	.001938	576	15.13
30	.001970	581	14.35
31	.001997	585	13.66
32	.002026	589	12.95
33	.002060	593	12.33
34	.002090	595	11.72
35	.002120	597	11.14
36	.002150	598	10.59
37	.002185	599	
38	.002215	600	
39	.002245	600	
40	.002280	600	

8.10 SINBAD'S PARADOX

In connection with dynamic means whereby an interception course can be stably attained by a homing missile, there is currently a good deal of misunderstanding.[p] Since gyro equipment could not be added to the bomb, it was difficult to see how one might use the means[q] of *proportional navigation,* as described in Chapter 1, wherein one made the vehicle turn in the medium n times as fast as did the line of sight. In the projection shown in Figure 8, for example, this desirable criterion would amount, in a special case, to

$$\dot{\gamma} = n\dot{\phi}\,. \tag{56}$$

Our proposal for attaining equivalent, although somewhat generalized dynamics, as described above, was to make

$$\dot{\gamma} = \frac{n}{n-1}\dot{\lambda} \quad (n > 1)\,, \tag{57}$$

this being equivalent to equation (56), since

[p]For the particular case of ROC-MIMO, which nevertheless has definite and general significances, the present writer dealt, on this whole question, at some length and very pleasurably with W. B. Klemperer of Douglas Aircraft. The latter organization had the ROC projectile, in its various phases, under direct development. WBK had been working for some time past on the problem of guiding and seeking, and had disseminated much knowledge and many original theoretical and practical structures for attack upon the problem.

[q]The reader should compare, for example, the report: *Some Aspects of the Design of Homing Aero-Missiles,* by H. L. Dryden, to Division 5 from National Bureau of Standards.

identically and by definition

$$\gamma = \lambda + \phi \ . \qquad (58)$$

Consider the significance, stepwise, of equation (57) as it stands, as a guiding criterion. In terms of finite alterations, we have

$$\Delta\gamma = N\Delta\lambda \ , \qquad (59)$$

where N is a positive numerical quantity. Considering this criterion in terms of Figure 8, for example, assume that during the approach the target were to slow down, so that λ increased. It is evident that to reattain, or to come further toward, a collision course, the bomb must now be more steeply dived. But what does equation (59) instruct us to do? It appears to indicate an *increase* in γ, giving a shallower dive. Evidently this manner of control is unstable. And the instability seems permanent. There is, however, a paradox here, which gave rise to protracted arguments, in the writer's case, and which deserves the title of the present section. Paradoxically, this criterion is perfectly stable, as developed in the previous section. The trouble is with our logic. In the first place, the stepwise argument, as given, is invalid — since it ignores equation (58). It is evident, further, that equation (57) is completely equivalent to (56), itself corresponding to known mechanisms, as shown, for example, by the inset in Figure 8. Generalizations to space are immediate, though not trivial, when the manner of roll-stabilization is specified.

Note that in general $\dot{\gamma}$, in two components, may be provided by absolute angular rate meters (see Chapter 3) installed in the projectile, whereas $\dot{\lambda}$ is available from the local detection of relative target direction. Thus equation (57) may be directly mechanized, whereas explicitly equation (56) would require local free-gyro references. We have assumed, in this simplified exposition, that pitch and yaw, relative to the medium, are zero. Otherwise a more involved analysis, as earlier outlined, is required. Finally, we remark that in MIMO-ROC, the effect of proportional navigation, using the term as here generalized and liberalized, was attained experimentally with no additional equipment in the projectile.

8.11 GUIDED ROCKETS AND THE FUTURE

The writer will here permit himself a few brief remarks in a field not otherwise dealt with in this report, which however treats both of rockets and guided bombs.

It appears that there are several lessons which should be perpetuated in that broader field. One is that there is no substitute for the collision course, or at least for courses lying much nearer to the collision, or interception type than to the pursuit type. We refer here exclusively to guidance from the vantage point of the projectile, rather than the remote method, for example by collinearity, which tends to interception, anyhow, at infinite observing range. Another lesson is that by proper control dynamics, giving subsidence in agreement with existing theory to intercepting conditions, the case of a target moving in the medium reduces essentially to that of a target stationary therein.

We add a few further opinions. (1) Absolute angular rate meters, of high sensitivity and performance, will probably replace free gyros for control references, owing to their lack of limits and to the compactness to which they may be reduced practically. (2) Ambitious projects in guided missiles, where complex controls and interlocks are involved, will be greatly advanced through the extensive application of the methods of electronic simulation, in the study stage, for the determination of parameters for stability. Owing to the inherent and sometimes hidden nonlinearities of such systems, analytic methods are almost utterly impotent there except for certain necessary conditions under control circumstances. Through such models or simulators, however, nonlinearities may be readily and naturally recognized, and speculative dynamic components may be proved or disproved or altered at a moment's notice. Rocket propulsion should hold no analytic terrors for simulators, and aerodynamic data may be directly incorporated, once determined, by independent testing of components and reduced, say, to graphical form. We note in passing the appropriateness of the ROC type of projectile for this purpose.

Chapter 9

AIMING OF ROCKETS FROM AIRPLANES

9.1 SUMMARY OF PROJECTS AND WORK

When the 3."5 forward-fired aircraft rocket [AR] came into prominence as an antisubmarine measure in naval operations, it was natural, since we had already dealt with airborne aiming controls for antisubmarine bombing and were fairly well saturated with the larger problems of this type of warfare, that we became involved in the development of rocket sights for this application. Apparently the question of a sight for rockets had been given attention, during the earlier development of that missile, only by a few, although it is understood that requests for liaison had been made by fire control groups in NDRC before the writer's engagement in 1942. In any case no such sights were available, except reflecting sights with fixed settings, and no development projects were in process for aiming controls when our rocket activities began in 1943. At least this was the case to the best of the writer's knowledge and information.

We were approached, in November of 1943,[a] with a request to construct a computing sight for the aiming of rockets against sea targets: chiefly German submarines. At that time it was principally the 3."5 AR, as already mentioned, with solid head, with the old JP propellant, and launched from the now obsolescent rails, that was used, but even this weapon, patterned somewhat after the corresponding British RP, and later to be vastly improved, was appropriate, owing partly to its excellent underwater trajectory and partly to the deteriorating effect it had on enemy morale, as an antisubmarine measure at a time when the menace was still growing. The airplanes in which we were then concerned with installations were the Ventura and the Avenger, and the aiming was to be carried out by the pilot, substantially alone. We had used the radio altimeter, prior to this time, for automatic setting of aiming computers, and it was suggested[b] that this instrument be made the basis for a rocket sight. With a fast and consistent measurement of altitude over the water, and a reasonably good measurement or estimation of the glide angle toward the target, a computation of the trajectory, and hence of the superelevation allowance for gravity drop, could be continuously made. This represented an approximation, of course, even aside from the kinematical correction for target motion in the air mass and the aerodynamics of launching, since airspeed and temperature should also be accounted for. However, on the basis of the meager information then available on the projectile, a computing system was blocked out and an experimental model was flown in less than two months from the original request. The results were irregular but promising.

Better information was soon acquired on the behavior of the rocket, partly by visits of the writer and others to California Institute of Technology [CIT] in Pasadena, and a development program for a sight (originally this project, and its initial product, was called VERB), later called RASP, was arranged at a considerable level of priority. A special deflectable sight head was prepared, from a "retiflector" which had fixed optics, by moving the normally stationary reticle in two dimensions by remotely controlled d-c servos. It was thus possible to deflect the line of sight, for automatic aiming, by voltages developed for azimuthal and elevational corrections in a computer placed elsewhere. Inputs to the computer included automatic radio altitude, automatic glide from a special gyro horizon, airspeed in either manual or automatic form, temperature, and a vectorially computed measurement of the target speed in the air mass. The computation was in terms of sums of products of functions of these inputs, and was determined on a semi-empirical basis. Attenuators in a-c circuits were physically applied for this purpose, fed by servos and autosyn transmission for the automatic

[a]By Commander G. R. Fiss of the Antisubmarine Development Detachment at NAS Quonset, under Captain A. B. Vosseller.

[b]By Commander Fiss.

inputs. Three distinct developmental models were prepared, each being tested at some length both in the laboratory and in the air. Accuracy of fire, roughly independent of range up to 2,000 yards, improved until dispersions of about 5 to 7 mils were experienced regularly. Each model of RASP was progressively more compact and easier to operate. Request for large-scale production was made by the Atlantic Fleet, but before the machinery for this was arranged the urgency had decreased to a far more comfortable degree for all concerned.

While the RASP development was still in progress, a separate development was begun under our auspices, principally carried on at The Franklin Institute, and now recognized under Navy Project NO-216 for rocket-sight research, which aimed at a major simplification of the computing and aiming system. By this time it was desired also to instrument single-seater airplanes, mostly fighters, for effective rocket fire. The problems here differed considerably, and for several reasons. For one thing the tactics were far more violent, so that certain components, such as gyro horizons in available form, could not be used for automatic inputs. Similarly, much steeper glides were possible, and the computation in this region was different, requiring other instrument technique. The problem of space was far more stringent, and in particular only highly specialized sighting equipment could be put before the pilot. In this connection, the availability of the Marks 21 and 23 gyro gunsights, and later the Army K-15, was to be taken advantage of for the new project, which acquired the somewhat ludicrous name of GRASP. Such sights[c] as the gunsight Mark 21 provided, for GRASP, only the means for deflecting a line of sight developed by a separate computer. The basic idea for GRASP,[d] was to compute, during a direct gliding attack, the altitude at which a given range would be attained from the target, applying for this purpose a measurement of the glide angle: actually obtained from the "vertical" component of gravity, properly interpreted and presented. The range itself, at and beyond this critical range, or continuously as the attack progressed, was to be indicated in terms of the airspeed and time approximately as the definite integral of the range rate *added* (note the sign) to the critical range. Then with range, glide angle, airspeed, and temperature as inputs, an automatic computation of the gravity-drop and aerodynamic connections would be made and applied to the sight head. Thus the pilot, on the approach, merely noted when his altimeter agreed with the indication of the critical-altitude display, pushed a button, and aimed by tracking the target in his automatically deflected sight. Kinematic lead was not computed, although an accidental correction entered for part of this error, and it was planned ultimately, if it proved essential, to use the lead-computing properties of the gyro gunsight being employed as a deflected sight head. We note that an advantage of the GRASP proposal for fighters which carried rockets and alternatively fulfilled their normal function, was that a rapid change-over was possible from gunnery aiming to rocketry aiming, since the same basic sight was employed for both, only electrical switching being necessary for the adaptation. This circumstance is of interest in connection with the PUSS project, discussed in Chapter 10, on integrated controls. A very elaborate program of testing was conducted on the GRASP equipment, which became extremely compact and progressively simpler to use. Tests ranged from laboratory procedures through flight tests of components, to full-scale firing tests at NOTS, Inyokern. A very high accuracy was finally attained, relative both to the dispersion of the weapon and to competitive equipment, of which latter there came to be many examples. V-J day terminated the latest tests abruptly, and it is unknown whether continuation of this project, principally an interim development, will be considered to be as warranted. The second model of GRASP became aircraft rocket sight Mark 2.

[c]Actually, they were not used operationally by the Navy to any considerable extent, whereas the Army sight, derived from the Navy prototypes, saw a good deal of combat action. For RASP, the Mark 1 sight unit, which adapted the gunsight Mark 8 to automatic elevational deflection, was later employed in experimental models, and subsequently this sight unit became available in Navy fighters, so that it might thus have been used for GRASP. Several paradoxes are contained in these circumstances, which were, nevertheless, unpredictable to us during development.

[d]Proposed by E. C. Cooper to the writer at The Franklin Institute.

A smaller project was that for development of the miniature rocket sight known as PARS. This project was initiated at The Franklin Institute.[e] Inputs were motion of a damped and spring-restrained mass (to measure the vertical acceleration), deflections of capsule diaphragms for barometric altitude and indicated airspeed, the temperature being manually set in. Small linkage resolved these inputs and yielded a mechanical motion as output, measuring the total allowance in elevation, which was both indicated on a scale for visual reference and servo-duplicated at the sight head. Although flight tests were carried out, of models of this instrument, and considerable Service interest was in evidence, no time was available finally for definitive firing tests. The equipment called PARS was designated Aircraft Rocket Sight Mark 3, and also Computer Mark 36, by BuOrd. This very interesting development, which was supported partly to test a computational method for employment in more ambitious projects (see for example, in this connection, PUSS) will probably be reported on in more detail by those men[f] who cooperated in the theory and design.

Finally, the PUSS project itself, for the development of integrated aiming controls for the usage of pilots, contained a computation for rocketry as its central solution and reason for being. While this project is treated elsewhere, we point out here that the rocketry solution was considered the most complex of the problems to be solved by this equipment, and the apparatus for that solution was most readily adaptable to the companion problems of gunnery and bombing which were to be included as capabilities of the system. Briefly, a new angular rate method was developed, for computation of both gravity correction and kinematic lead. The inputs were: angular rates in space of the thrust axis of the airplane, airspeed, altitude, and glide angle. Although the several variants of the method admitted different sets of input data, as with radar range explicitly provided, the method as such was characterized by relative insensitivity to the "classical" inputs, including the range itself, being more heavily dependent on the measurement (or provision) of absolute angular rates for its ultimate accuracy of aim. The measurements of such quantities are elsewhere discussed, and we note only that a major question is what particular method, among those experimentally available, should be exploited. The technique of rocketry proposed for PUSS is somewhat analogous, although far more general in capability and concept, to several other methods which have arisen in different quarters. Thus the fundamental property whereby a lead diminishing with range is generatable by means of a downward-rotating line of sight, measured in space, is characteristic of certain independent proposals by the writer.[g]

9.2 NATURE OF THE WEAPON

The forward-fired, finned, aircraft rocket, exemplified by the Navy 5."0 HVAR for example, has many attractions as a tactical weapon, particularly for lighter offensive airplanes such as those of the carrier-borne variety. Its characteristics relevant to such application include: the penetration and explosive power delivered in comparison with the weight of firing equipment and in comparison with the shock of projection; the small dispersion, or unpredictability of trajectory, which fortunately becomes still smaller as the launching speed increases; and the proved effect on the morale of the attacked. We speak principally of targets at ground level.

In leaving the airplane, the rocket-round accelerates along its trajectory at something like 50 times gravity for just about a second, during burning of the propellant, after which it has attained a total airspeed of about 1,500 feet per second and subsequently flies somewhat as a standard unpropelled projectile. In its

[e]By the cooperative promotion of H. Whitney of AMG-C and the present writer, following a proposal by the former for a compact all-mechanical computer to solve for the gravity, aerodynamic, and (partially and approximately) the kinematic corrections, for airborne rockets.

[f]H. Whitney and others of AMG-C.

[g]Original although not necessarily novel in a more comprehensive history of the Draper/Davis method of glide bombing, of E. P. Cooper's proposal for a rocket sight in which an additive deflection is given to a gyro gunsight employed with a long characteristic time, and of the somewhat similar British method, so called, for rocket fire with the gyro gunsight.

initial phase of flight following the launching, the round necessarily follows an air path rather close to that of the airplane, since its accumulated relative speed is small. Being finned, it is erected in the air stream and subsequently is urged on by its propellant substantially in this same direction, which is thus more nearly that of the flight path of the airplane — at launching — than that of the initial heading, either of the round or of the airplane. Thus the rocket behaves more like a fast bomb than like a bullet or shell, and this fact is of primary importance in the matter of aiming.

During flight, the shape of the air path is somewhat different from that of either a bomb or bullet, although it lies approximately halfway between, all three projectiles being assumed released at the same speed and glide. A special type of trajectory is followed during the first 500 yards or so of flight through the air, that is to say during burning, which is characterized by an oscillation of small amplitude and a space period of some 100 yards, and in which the trajectory is bent downward more than in the immediate period following burning. For most purposes a rather satisfactory analogy may be used between the rocket trajectory and that of more orthodox projectiles, even *in vacuo*. Thus a good approximation is afforded by fitting parabolas to the trajectories of these rounds, corresponding in a variety of ways to equivalent trajectories *in vacuo*. For example, it suffices, to good accuracy, to say that the round behaves like a projectile fired *in vacuo* along a parallel path which originates behind and above the actual firing position, and at a different initial velocity, which incidentally must be a function at least of airspeed and temperature. Again it has been found that the rocket trajectory may be replaced by a parabolic path corresponding to that of a hypothetical projectile launched at a definable angle from the actual flight path or the initial air-velocity vector of the actual round. Mathematical studies[h] have shown how the parabolic constants may be determined for optimum fitting to the rocket trajectories. Such constants, of course, must be expressed, for any given rocket type, as functions of the circumstantial variables of the firing, or of temperature, airspeed, and glide angle. It is found very nearly true that the glide angle, the "negative elevation" or *declination* of the initial velocity vector, affects the trajectory in much the same way as for a shell or a bomb. That is, such qualitative effects as the *rigidity of the trajectory,* at shallow angles, are here also evident. Furthermore, the degree of gravity-bending is also roughly proportional to the horizontal range, being zero for an attack out of the zenith and maximum for one "on the deck."

The longitudinal acceleration of the rocket is substantially unaffected except by temperature, but this latter variable thus affects the whole character of the problem and becomes a new variable to cope with if high aiming accuracy is sought after. All such effects appear to depend uniquely on an interior temperature of the round, namely that of the propellant grain. This governing temperature, although responsive via a definable dynamic lagging-function to the air temperature, is dependent also on the initial temperature determined by the conditions of storage; this latter dependence decaying gradually during the time since the round was placed into service. The problem is not simplified by the variation of air temperature with altitude, since while rocket attacks are typically made at the lower strata the airplane may recently have been at higher altitudes. As to time constants, proper to the absorption and dissipation of thermal conditions in the rocket round, these are, so to speak, of the order of half an hour or so. One solution which has been proposed, and at least partially developed at CIT, for the provision of this substantially unmeasurable variable as an input to rocket sights, was that of an externally mounted thermal model of the rocket. The thermally significant part of this dummy round would presumably be stored with the standard rounds with which it would be used. On the whole, however, designers of rocket sights have either incorporated the temperature influence by means of a manual setting or have ignored that variable altogether out of an altogether understandable sentiment on the subject.

As to time of flight as a function of range

[h]Made by R. M. Peters of Section 7.2 and by personnel of AMG-C of AMP.

along the trajectory, the rocket lies, again, approximately halfway between the bomb and the bullet. For aiming purposes, particularly with regard to kinematic lead, this makes a more difficult problem than that of the bullet, on the one hand, whereas on the other hand one cannot apply the special properties which the bomb possesses in this regard owing to its preservation of the forward velocity of the vehicle. More precisely, the time of flight of the rocket, as a function of range, is given approximately by

$$r = v_a t_f + G t_f^2$$

during the initial burning interval, where v_a is the airspeed of the launching craft and $G = G(\Theta)$ is the rocket's *proper acceleration*, as a function of temperature. In the subsequent phase, when burning has ceased, a more natural deceleration takes place. This character of the rocket, with regard to the relationship between range and time for the trajectory, by virtue of which it accelerates and then decelerates, has special meaning in the problems of aiming control.

9.3 THE RASP ROCKET SIGHT

The project for the development and test of RASP has been briefly described in the first section of this chapter. There are also very complete reports, on the several models of this rocket sight which were designed.[i] In the present section we may supply some interpretive comments from the point of view of other and similar developments. Certain component developments are worthy of comment since they are of more general applicability than to the RASP system alone.

It should be pointed out that this development, which was undertaken before receipt of the formal Navy requests for Projects NO-216 and NO-265, was largely promoted and sponsored by the staff of the Antisubmarine Development Detachment at NAS, Quonset. Assistance in the procurement of research facilities, including the provision of airplanes and other military equipment and auxiliaries for testing purposes, was given by that agency in considerable abundance. In particular, officers were permanently assigned to the project, even to the extent of setting up residence in the laboratories at The Franklin Institute.[j] Following acceptance and establishment by NDRC of the above named projects, we considered that the RASP development constituted part of the work they embraced, although new directions were then indicated as well. In particular, and this viewpoint was early advanced, the educational and indoctrinary value of this development was emphasized, since during the initial phases there had been almost no experience anywhere on the problems peculiar to rocket sighting — particularly the automatic variety thereof. Thus at least the primary RASP equipment was expressly designated as having the character of *study models,* the teachings of which would apply more broadly than to the project as then contemplated. Initially it was felt to be unimportant how complex the system was, as long as it could be flown and made to function predictably, for then at least the interrelations among the variables of the problem could be subjected to quantitative study under practical conditions. It was naturally not overlooked that such study models might themselves be reduced, as indeed they later came to be, to more generally useful form for operational purposes. However, the chief contributions of this development project were to our knowledge of the practical phases of the sighting problem, and such knowledge was subsequently applied in other developments.

A diagram of the flow of information in the RASP system, from the inputs through the various computing components to the outputs, is given in Figure 1. All experimental models did not include every component and connection thus shown; the diagram in this sense is somewhat generalized. More precise information on the details of such models will be found in the contractor's reports listed in the selected bibliography hereto attached.[5,6]

For the altitude input an FM altimeter was

[i] Compiled principally by U. C. S. Dilks, the engineer directly responsible for this project at The Franklin Institute.

[j] The promotional efforts and participation of Captains Vosseller and Turner may be mentioned in this connection, as may those also of Commanders Fiss, Edgar, Millard, Swensen, and others.

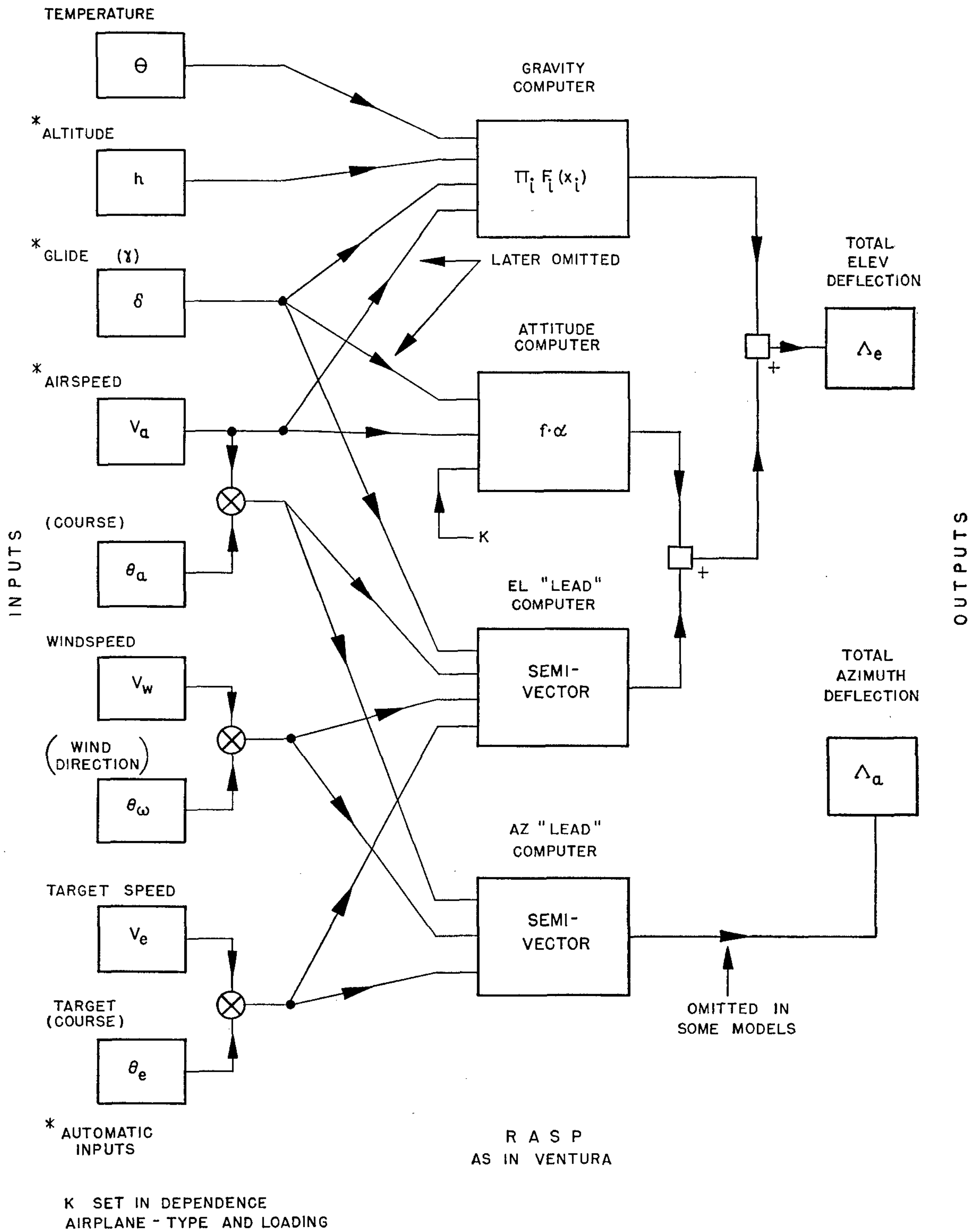


FIGURE 1. Influence diagram for RASP rocket sight.

predominantly employed. No difficulties arose on this account from irregularities of terrain, the testing and intended operational areas being over water. Some doubt was indicated at first as to the accuracy of measurement when the airplane flew in steeper glides, but that apparent source of difficulty disappeared, perhaps, as was suggested, owing to subsequent alterations in the antenna designs. Several different types of servomechanism were worked out and employed for this input, none of which required appreciable interference with, or modification of, the radio altimeter as such. All such methods involved d-c servos, and resulted in shaft rotations in the computer component in proportion to the altitude.

For the glide angle (γ) input, or rather for that of the depression δ of the longitudinal axis of the airplane from the horizontal, an appropriate correction being involved for the approximations thus introduced, a modified directional gyro which was arranged to provide rotation in space about a lateral axis, and was equipped with an automatic follower, was initially prepared as an experimental component. This input was somewhat critical, at the shallow glide angles then contemplated, owing to the sensitivity of the range, and hence of the gravity drop, to the glide when computation was effected in terms of the latter variable and altitude. A specially equipped gyro horizon of Sperry origin, became experimentally available however for this application, and was used in preference to the above glide-giving component. This horizon had autosyn-detection of the gimbal rotation, and provided to the computer a varying voltage as a function of the glide angle. Later several other methods for the determination of glide were tried out conjecturally in this project, owing partly to the limited aerobatics permitted by the gyro horizon just referred to. Such limitations, however, did not apply as crucially to the bomber-types, which were in mind for GRASP, as to the fighters which were later specified.

The airspeed, measured in terms of the difference between the dynamic and static air pressures as in a standard airspeed indicator, was automatically fed in by means of a pneumatic follower, although in preliminary models this variable was manually incorporated by an operator who merely imitated the reading, in the position of a knob, of the standard panel instrument. Setting of this variable in advance, through prediction of the airspeed which would be indicated when the firing point would be attained, was found to be unreliable; and the luxury of another operator, in addition to the pilot, who might make this setting continually under operational conditions, was too much to expect. A manually set temperature input was employed, however, since automatic means, for duplicating the necessary propellant temperature of the rocket, were not yet available.

As to motion of the target in the air mass, considered as the vector sum of its motion in the water and the local wind velocity, this motion was corrected for in RASP, as the kinematic lead thereby required, by separate determination of the magnitudes and directions of target speed and wind speed. Vector computation, mechanical and electrical, was applied for this purpose. The directions of wind and target motion having once been set in, as by estimation, on a mock compass scale, these were "stabilized" against subsequent turning of the airplane by means of a final setting of own-course just prior to the attack when the latter course was finally determined. A simple mechanical arrangement was employed for this stabilization which had been previously applied in a mechanical type of ground speed computer. Considerable controversy raged over the ability to determine the relevant target motion in this manner. At worst, however, the magnitudes of wind and target speed could always be set to zero, giving no kinematic correction as had been originally proposed by Navy agencies anyhow, and as in most other sights which have been in development. Certainly many operators would be able to determine these navigational quantities to high accuracy and to set from into the computers, but admittedly it would be preferable, as in later developments, to correct by more nearly implicit methods for the motion of the target in the air mass.

The method of computation applied in RASP was by attenuators in a-c circuits. To this end the several contributory terms of the total aim-

ing deflection were expressed as products of functions of the single and separate input variables. The possibility of such expression is of course not automatic; it had to be discovered and demonstrated. Remarkably enough the approximations involved were shown, with little trouble, to be sufficiently good.

The sighting component, or sight head, employed in the several RASP systems took on a variety of forms. In the first experimental model an old *retiflector*, so-called, was redesigned so that its fixed reticle could be moved over the focal plane to cause the emergent beam to move bodily in two angular dimensions. A system of sliding mechanical ways was involved, and a small d-c servo assembly was attached so that the deflections, in azimuth and elevation referred to airplane coordinates, might be ordered by output voltages in the remotely located computing components. This arrangement did good service at a time when few such remotely manipulable sight heads were available. The reflecting collimator represented a saving in space, as compared to lenses, although its position at the top of the final transparent reflector was found objectionable by some, who felt that the sight could thus not be called one of the "open" variety. No production status was enjoyed by the retiflectors at that time, and in order to deal with equipment more readily available in quantity, this instrument was later replaced as a component of RASP by a Mark 30 sight head which we modified in somewhat similar manner by moving the final reflector. Later yet, a Mark 8 sight head was employed, to which was attached, for the elevational deflection, a specialties sight unit Mark 1, this unit being remotely driven by a voltage in the computer in a precisely similar manner to that applied before.

Three distinct models of RASP were prepared, each with several variants in detail, in the course of this project. The first and third such models, or RASPS I and III as they were called, were much more extensively tested than was RASP II. Each succeeding model was progressively more compact and more completely engineered. Since it was not considered sensible to adapt the RASP system to the single-seater types which were the principal rocket carriers by the time Model III was completed, there being then the GRASP project well in progress intended specifically for such application, it was determined that there was no requirement for further development on RASP. The personnel and project facilities were applied henceforth to the PUSS project.

9.4 THE GRASP ROCKET SIGHT

For fighters or other light single-seaters, the requirements on rocket sighting equipment were more stringent in terms of size and weight; the freedom from disturbance, under maneuvers, in the component equipment; and in the complexity of manual operations which are necessary for tactical employment.[k] The basic principle of GRASP lay in the method whereby the range to the target was recognized, as a variable of the aiming problem, in the computation of the gravity correction, and, potentially at least, of the correction for target motion. Assuming that the approach to the target occurred in its final stages as a direct glide, it was noticed that the arrival at any given critical range from the target could be signalized by agreement between the altimeter reading and a function of the glide angle. The deflections having been previously established, for this critical or initial range and the glide angle and airspeed, etc., in effect, they could be kept "up to date" for shorter ranges by a chronometric input-motion initiated when the critical range was encountered. In preliminary models the said initiation was to be manually made, by the pilot when he noted coincidence of a pair of indicators, whereas in later models an automatic initiation was embodied which simultaneously signaled to the operator the closing of the range to that degree and the entry into the continuous-firing regime.

Two principal developmental models were involved, called GRASP I and GRASP II. Extremely complete reports on the course of the research itself, as well as on the elaborate testing program which was undertaken and on

[k] As one of a number of proposals made to meet these requirements, one made by E. C. Cooper at The Franklin Institute, for the system which became GRASP, was accepted and was embodied in a development program in which Cooper, among other duties, was project engineer.

the final equipment, were regularly issued by the contractor. Reference to certain of these reports will be found in the bibliography of this chapter.[7,8] From the beginning the Navy Project NO-216, under which we formally operated, covered the development of the GRASP system. GRASP II was finally designated aircraft rocket sight Mark 2, when taken together with the gunsight Mark 21 or gunsight Mark 23 which it employed as sight head, and the GRASP II computer component was designated computer Mark 35. While other output components than the said gunsight might equally well have been applied as means whereby the pilot's aiming indicia were to be automatically deflected, it was the plan of the GRASP project to use the gyro gunsight in this manner, as it was also the expressed desire of the Service request which preceded the project designation. The purpose of this combination was simply to permit a semi-universal sighting arrangement, at least for gun and rockets, with rapid changeover capabilities between the two functions. As before remarked, this gunsight did not receive wide application in Navy airplanes, as it happened, whereas a similar Army gunsight was widely installed and applied. A modification of GRASP II was consequently prepared, in the latter part of the development, under Army Project AC-121, but at the close of World War II had not yet been tested under Service auspices.

Some remarks on the components of GRASP, and on their systematic arrangement, are probably here in order. A "flow diagram," Figure 2, is here included to illustrate the handling of the data by the computing system. The temperature and the airspeed were manual inputs, although it had variously been planned that airspeed might be automatically provided, by means now well known, or again that airspeed might adequately be correlated with the steepness of glide — and in consequence automatically incorporated therewith. It was opined that the presence of a manual setting attracted the pilot's attention to the rigors of making a prediction as to his airspeed when firing was to occur, and that his impression of how precisely this variable must be set for accurate aiming was exaggerated.

The altitude, as we have implied, was not set in continuously but was recognized in terms of the instant when this variable agreed with the glide input in determination of the initial range. In the first model such recognition was by the operator, whereas in the second model it was automatic and involved only a contact arm on the "slow" indicator arm of an auxiliary barometric instrument. This component, installed in a small (approximately 4x4x8 inches) single computer box, was provided with adjustments for target altitude and the barometric pressure at sea level. Establishment of electrical contact, at the initial or critical range, set in motion the chronometric sequence of the computation. The latter was delivered mechanically by automatically wound clockwork, which returned to the initial position and latched itself when the operation was completed.

The most elaborate component of the GRASP system was that which provided the input of glide angle, or the airplane pitch as an approximation thereto. This comprised a *captured mass*, or *captured pendulum* accelerometer, so aligned in the airplane that the total (gravity plus inertia) acceleration, in the direction of the airplane's vertical axis, is thereby measured. A servomotor, actuated by special damped and overriding contacts connected to the pendular structure, rotated a spring in a degree sufficient continuously to hold that structure essentially stationary in a framework, the rotation of the spring then being taken as a measure of the acceleration. Although the acceleration thus measured was only proportional to the cosine of the pitch or glide in straight flight, the effect of curvature of the airplane's path was not fatal since (1) it was in the nature of a prediction of future glide angle, so to speak, and (2) it tended to a partial compensation,[1] in the elevation direction for motion of the target in the air mass. To guard against fortuitous accelerations, having no real significance for glide, a limiting speed was incorporated in the servomotor, and to avoid extremes such as incurred when the airplane entered the dive a cutout was included which

[1]Noticed first by H. Whitney of AMG-C, after whom it was sometimes called the "Whitney effect," of which in fact there were several others.

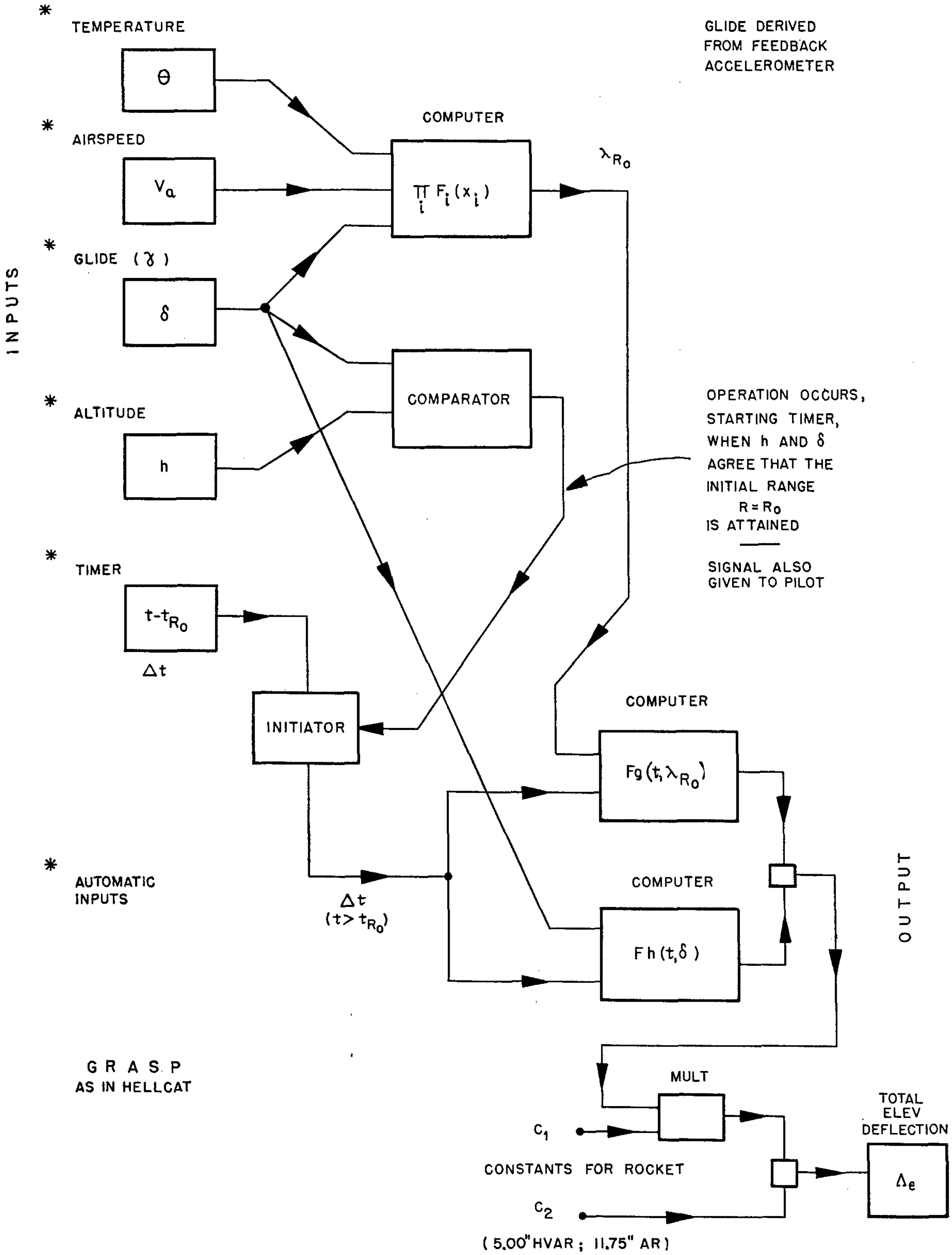


FIGURE 2. Influence diagram for GRASP rocket sight.

was operated automatically under excessive accelerations.

Computation of the several functional relations in GRASP was accomplished by logarithmic cams. The output, as an aiming deflection in elevation, was developed by rotation of a resistor in the trail coil circuit of the gyro gunsight Mark 23. Trimming resistors, of fixed but adjustable values, were involved in switching circuits to allow the use of different rockets, for which, incidentally, it was found that the total deflections were sufficiently expressible, rather surprisingly, as functions of one another directly. Employing a large current in the range coil, the reticle could be deflected over the whole range, it was discovered, by alteration of the trail coil current. This worked, in fact, for both coordinates, and the relationship between angular deflection and the ratio of trail coil current to the then existing range coil current was found to be adequately linear. Further, a method was worked out[m] whereby this ratio could be set by linear manipulation of a single low-power resistor placed in special bridge circuit. This technique was valuable in maintaining the desired compactness of the computer, of which the total size has already been referred to above. For installation, such as those made for trials in the F6F, it was only necessary to remove the standard cable connector of the computing gunsight and insert a two-way adaptor on a new cable leading to the GRASP computer.

9.5 THE PARS ROCKET SIGHT

We shall not give an exhaustive description of this development, although it has been one to which great promise was attached. The historical circumstances of the project were already referred to briefly in this chapter. It was planned that in this system not only an extremely simple and compact computer should be involved, with very little else, but that at the same time the solution carried out thereby should be most complete and automatic. The computation was all mechanical, involving small linkages of a type later embodied in PUSS, although in one model the output motion affected setting of a light potentiometer, as the variable according to which remote transmission to the sight could be accomplished. Physically, the first experimental model of the PARS computer was constructed entirely within the case of a standard barometric altimeter, of which the motion of the aneroid capsule provided one mechanical input to the miniature computing linkage. Airspeed entered the computation similarly (see Figure 3), a dynamic-pressure diaphragm being placed in the same case, the ambient pressure in the latter being the static or barometric pressure as needed both for altimetric and airspeed measurements. The temperature variable was manually set into the instrument, although at one time it had been considered likely that an adequate correction for temperature could be accomplished by inclusion, in the linkage, of a small bimetallic "motor" which would respond with some delay to the external air temperature via the static-pressure line.

The accelerometer component, whereby the steepness of glide or dive was detectable by the PARS computer, deserves special mention. It constituted a spring-constrained mass, on a pivoted arm which lay normally and approximately in the plane of the wings of the airplane, and which moved out of a normal position in response to the total acceleration in the airplane's "apparent vertical." The free arc-motion of the constrained mass, which was damped by a pneumatic carbon-chromium dashpot, formed one direct input to the computing linkage. A number of experimental models of this component were prepared and tested, both in the laboratory and on flight tests arranged solely for this purpose, under conditions which allowed comparison of the acceleration record of this component with that derived from the readings of other instruments used for calibrational functions.

In an early form of the PARS instrument, it was contemplated to reduce the system to its ultimate and basic instrumental simplicity by displaying, on the instrument panel only, a mock reticle-pattern representing that included in a fixed infinity sight, and on the mock pattern a moving point corresponding to the correct position, in the real reticle pattern, at which the target should be taken for effective

[m] By E. Cooper.

fire. Thus the output deflection, which might normally be transmitted to a manipulable sight head as the aiming deflection, would here merely move a visible indicator on the face of the computer, the latter to be incorporated as a standard panel instrument. Owing to the many voices raised in objection to this plan, it was later resolved (perhaps wrongly) to employ the now more orthodox technique, and to "servo" the output of the computer, resistively recognized, to one of the available sight units. Only an elevational deflection was thus concerned, there being no azimuthal computation in PARS. While an inherent computation occurred in elevation for target motion, at least over a central band of glide angles, owing to the accelerometric sensitivity to curvature of path via the glide measurement, no similar correction was evident for the azimuth component. It was considered by many, however, that the indeterminacy of skid, which affected almost one-to-one the azimuth aiming errors with any existing types of sight, made it unworthy to add much complexity to the computer for sake of that dimension. This question of skid has been controversial indeed, and was only beginning to come under an articulate quantitive study, by the agency of a testing program, toward the close of hostilities. We shall not here add our own prejudices to those in evidence elsewhere. Works on the subject by AMG-C and AMG-N, of AMP, are recommended to the interested reader. If the skid problem cannot be simply resolved, either by aerodynamics or by instrumental means in computation, then this certainly is one of the largest objections to the finned rocket as a weapon, which behaves in this connection almost oppositely to that of a bullet or shell. Our own view of this problem is a generally optimistic

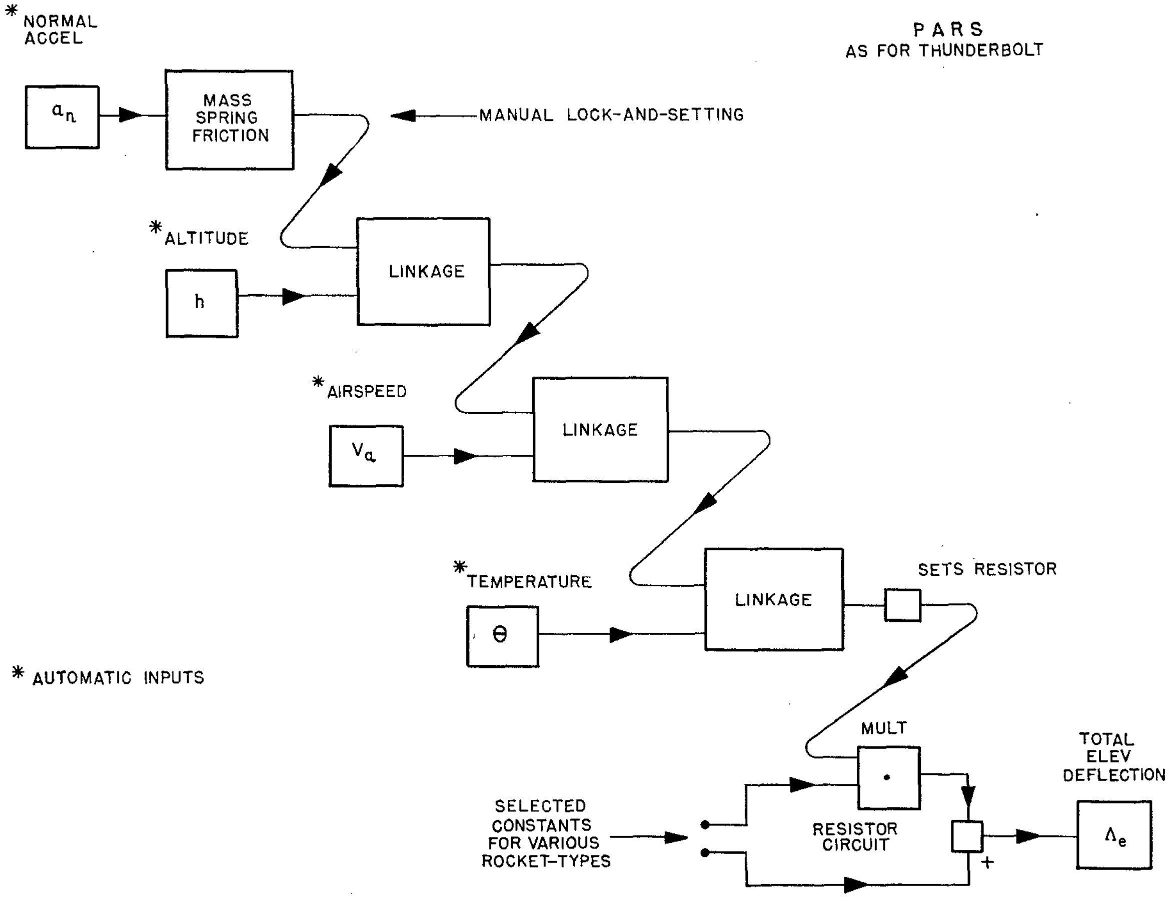


FIGURE 3. Influence diagram for PARS rocket sight.

one, but admittedly the next step is to arrange to have reliable data at hand. We appear to have digressed somewhat from PARS, to which we now momentarily return.

There was not time, during the project thus far, for extensive or definitive firing tests on the PARS rocket sight, which incidentally became officially aircraft rocket sight Mark 3, and simultaneously computer Mark 36. Mechanical failures, arising from vibratory conditions not adequately foreseen and guarded against, were experienced on several earlier flight tests. PARS remains a highly ambitious attempt to compress a fire-control system to insignificant dimensions, and the technique is certainly of future value. Thus for example the lessons there acquired at some pains are well learned if they are applied in such broader and more futuristic programs as that of the continuing PUSS project.

9.6 PUSS AND SOME COMMENTS ON THE FUTURE

We have referred freely to the PUSS project at numerous other points, and Chapter 10 is devoted exclusively to that topic. PUSS, however, is fundamentally an aiming-control system for rocketry, at any rate that was the primary purpose for which the computing method was proposed and developed. While also a bombsight and a gunsight, as occasion may demand, the most general and most difficult aiming problem solved by this system is that for the rocket, and in this sense it is believed that the project is unique.

For future development the following courses are suggested: Without losing sight of the universality toward which this system was directed, it would be wisest to concentrate first on the rocketry phase, and to press the development to a testing status for that weapon alone. As the one type of rocket on the basis of which the assessment should be made, we should choose the 5.″00 HVAR in preference to the 11.″75 AR, owing to the better state of present knowledge of the characteristics of the former. As to subsequent development and testing of the system as a gunsight, considered as an adaptation of its function in rocketry, consideration should be given to lower projectile velocities and longer ranges, and consequently to much longer times of flight, in view of the probable coming into importance of low-velocity, high-caliber shells, which may, indeed, replace the present rocket types for airborne fire control. Here we are thinking, for example, of plane-to-plane combat, where the rocket now presents an aiming problem of very great magnitude. For bombing, on the other hand, thought should be spent, if it is at all on PUSS, in the direction of a bombing tactic in which a firing course, or rather a release course, is followed. With the greater downward vision and possible aeronautic alterations in airplanes, the practical advantages of toss bombing may disappear. Furthermore, not only is such a manner of bombing one which may be controlled by a system which is a special case of the present PUSS rocketry computer, but it may be made inherently more accurate by being, as are the other PUSS computations, less sensitive to all the inputs other than angular rate.

There will be a great advantage in a universal aiming control based on a uniform method for all types of projectile, since then the same basic components may be most fully utilized for all applications, thus leading to smaller, lighter, and simpler instrumentalities. Furthermore, it would thus be simplest to adapt the system, once developed, to the newer missiles as they appear. Aside from possible radar inputs, for blind operation and for ranging data, there are not many other basic items of primary data. It appears worth while in this vein to arrange for supplying all such inputs, comprising dynamic and static pressures, accelerations in all directions and of both primary types, and the angular rates of the vehicle in space, in a compact universal input assembly. Whatever firing or computing methods are employed, they will be most efficiently based, at least statistically and ideally, on the maximum number of available inputs, so incorporated in the sighting method and in the computers that the overall errors are minimized. In any case it will be easier in the future to omit a given input than to include it by modification. This may seem a naïve philosophy, but it is based

on a certain amount of experience, nevertheless.

It is recommended further that a study be made of the usefulness of higher-order linear operators in the dynamic computer of PUSS, for all weapons being considered. This is an almost entirely unexplored field, and may contain many treasures, in terms of reduced tracking time, stability, and accuracy. It may also be found that such problems as skid are resolvable implicitly when the more general dynamic forms are resorted to. In this study, the electronic simulative techniques will probably prove invaluable, as they have in other and analogous fields of effort. The pilot's tracking simulator, now in its initial form, is to be placed in operation at The Franklin Institute very soon, and should provide a nucleus of the simulative apparatus whereby the more recondite dynamic questions may fruitfully be studied. To name an example of the instrumental possibilities for future appraisal, by this and other means, we mention the general plan of recognizing, as inputs to such computing systems as that of PUSS, the motions during flight of the control surfaces. Especially with the greater symmetry which may be attained in jet-propelled flight, these motions may be found highly significant when applied by retroactive processes in the dynamics of advanced aiming controls.

Chapter 10

INTEGRATED EQUIPMENT FOR THE PILOT

10.1 HISTORICAL REVIEW OF THE DEVELOPMENT

As WORLD WAR II progressed, several trends in the application of warplanes became evident. One of these involved the increasing use of the lighter planes, such as fighters, for such purposes as bombing and rocketry. There was of course the dive bomber, as a classical type, but whereas the German Air Force had used this type strategically, in a sense, the American version was only in the Navy and was employed tactically against ocean targets. Strategic bombing by fighters was actually employed by the Navy in the Pacific war, witness for example the bombing of Tokyo from carriers. Part of the trend, both in Army and Navy, and we need speak only of tactical warfare, was more particularly in the more diversified applications of the single-pilot types. Consider the activities of the Hellcat and the Corsair, in the Navy, and of the Mustang, Lightning, and Thunderbolt, in the Army. For strategic support, almost in every type of attack, these planes became nearly predominant. They operated as regular fighters, both for escort and interception, or simply, and especially toward the end, merely patrolling for quarry of any type. They carried bombs to strategic ranges, releasing them in gliding and diving attacks. They were rocket carriers, bearing as many as eight rounds apiece, from the 2.″25 SCAR (for practice) to the 11.″75 AR. Such airplanes were impressively potent and versatile. Against ground targets especially, whether for support or on independent forage, they could strafe, with gun and cannon, drop bombs, and launch rockets from ranges measured in thousands of yards. The desire for, and evident benefits of, such flexibility of function is of course apparent in the airplane types which have appeared. A notable early example was in the TBF Avenger, which although not too impressive as a fighter, could carry torpedoes, bombs, and rockets.

A number of us, noticing this trend in a broad sense, and having other reasons to wish for a more generalized approach to the instrumental problem of aiming controls, argued for the development of systems which would apply optionally to the several weapons coming into use by the same vehicle.[a] An important premise, in support of the possibility of such a widely applicable single system, was the evident similarity among the fundamental solutions to which the fire-control problems of the various projectiles were amenable. Thus typically absolute angular rate was increasingly useful in this field, for several types of inherent computations, and progress had been made and was promised in new components for the measurement and delivery of this variable. In the case of other variables too, these appeared as inputs in more than one of the numerous proposals being made for bombing and later for rocketry. An obvious saving was thus possible through combining input components in a universal system. Further, it was considered essential that a single, universally applicable sight head be made available, to which the outputs of the various computations could be selectively conducted, and for which the very difficult installation problems, on each form of airplane, could be engineered once and for all. Not the least advantage contained in the flexibility of this general mode of design — and this is emphasized particularly in retrospect, it was not at first insisted upon or indeed as vividly realized — is the freedom whereby future developmental changes might be incorporated. None can foresee either the newer circumstances which will obtain in the future, as the novel instrumental and theoretical techniques which never cease to emerge. To design in expectation of such change is the wisest course; to freeze as many components as possible and to leave their interrelations more arbitrary was one of the precepts of Project PUSS, and remains a valid criterion

[a]The present writer remembers conversations on this topic, at an early state, with Commander E. S. Gwathmey (then) of BuOrd.

today, especially in multipurpose aiming controls. Where inputs, computers, and outputs are too innately combined, such alterations, to fit new weapons or tactics or instrumental theories and techniques, are a more serious task.

In any sight for fighters an inevitable problem is that of reducing the amount of added equipment placed in front of the pilot, not that space and weight restrictions are liberal at other locations in the airplane, but the space around the instrument panel comes highest of all. This was part of the reason for wanting a pilot's universal sight head, which with its servo equipment could be remotely operated by equipment at other points, and which would be designed down to the minimum size and inconvenience compatible with effective operation. With modern servos there should be no trouble in attaining, by this means, all the smoothness and precision and other requirements of performance which could be desired. There is much more to be said on the question of the sight head, but such dissertation can be better relegated to sections, in this chapter and elsewhere, on specific parts of the project. We refer here altogether to visual operations rather than to blind attacks. In rocket sights and in the PUSS project radio detection was considered, by us, only as a potential means for measuring altitude, and possibly range, as inputs. In any case the designer of fire controls has found himself faced with many problems which were posed by the constructional peculiarities of airplanes, and particularly of fighter airplanes. Whether or not some of these peculiarities were essential is another question, but one of the measures which would permit circumvention of such problems was the proposed universal sight head, to be perfected as such and to be deflected automatically, referring to the relative direction of the line of sight thereby established, by remote components of whatever nature.

As a group we brought to the problem of planning and building a pilot's universal sighting system, referred to henceforth as PUSS, experience in the development of computing gunsights, rocketsights, and bombsights, which was to serve as valuable background. We had worked also on the problems of reducing the size and weight of computers, and of designs which did not require high-precision manufacture. The projects, then still in progress, for the rocketsights RASP, GRASP, and PARS were considered as contributory, not only since similar problems in design were there being met, but because any of the techniques employed therein for solution of the rocketry aiming problem were valid candidates for the rocketry phase of PUSS. The newer proposal for the central rocket solution, based predominantly on absolute angular rate, was relatively untried, but its conjectural development was thought to have a promise worth the gamble, since the more orthodox methods of earlier projects could be resorted to rapidly if the gamble turned out poorly. For plane-to-plane gunnery it was planned to use the well-known first-order principle involving angular rate and range, although it was hoped that higher-order methods might provide a better compromise between precision and stability of tracking, on the one hand, and that certain rangeless methods, on the other hand, would ultimately be applicable. Mentioned briefly in Chapter 1, such methods are based even more crucially on angular rate criteria. It was considered, anyhow, that a simpler instrumental problem was involved, and that apparatus which might solve completely the problem for rockets could be specialized rather readily to bullets or cannon shell for air-to-air combat. For strafing as a gunnery operation, the need for computing sights was not given prominence, although with rangeless methods, again, and these appeared particularly applicable here, the question was held open. How to handle the bombing solution was a major question, since the logical use of the angular rate principle for gravity-drop correction was unusable in typical airplanes at significant range, the superelevation allowance being greater than the available downward vision. For this reason, and owing also to certain psychological preferences which appeared to be evident, and further to some problems in the stability of flight with existing airplanes, we chose to consider toss bombing as the method for PUSS. While the gravity drop is not allowed for by the methods of absolute rotation in the preferred procedure for toss bombing, such methods do appear for the target motion in

this problem, and it was planned to capitalize on the presence of gyro reference equipment incorporated in the PUSS system for the other functions.

The initial experiments on a unified PUSS computer involved a free gyro as basic component. By various precessing arrangements such a gyro was to be alternatively caused to rotate about a horizontal axis in space, for gravity corrections, or to "pursue" an index for synthesis of a disturbed sight. It was recognized that both such modes of operation might be followed simultaneously. It was soon found precisely how to generalize the classical computing sight to give the more general solution for rocketry, with all major corrections. There are three: the gravity drop, target motion, and aerodynamic error. Although these may be considered as independent angular corrections, when they are to be developed by angular rate methods their functional independence disappears and separate and additive consideration is impossible from the instrumental standpoint. The theory involved the addition, in the equation of a lead-computing sight, of terms independent of the rapidly changing variables, or in other words of the lead itself and the angular rates in space of the line of sight and of the thrust axis. A full-scale program for reduction of this method to practice has now been in progress for about one year, and has followed several parallel paths in terms of the philosophies of apparatus. Fully captured gyro systems have gradually replaced the partially captured systems which were earlier contemplated, a number of experimental designs of both types have been undertaken. Several choices are still evident among the other inputs, it being possible for example to measure glide either from the rate of change of altitude, altitude itself, and airspeed, or from the indication of a stable gyro horizon.

Further on the captured-gyro angular rate components, there is still a choice as to whether null methods or purely mechanical constraint and sensitive detection should ultimately be employed. Both types of instrument have been built, and the final selection depends on the nature of other components. In a larger connection, both electric and pneumatic techniques in instrument design have been continued together, and neither has as yet proved demonstrably inferior. Mechanization of the static functions in the system, as contrasted with the dynamic differential equations which must be embodied, have been made by means of small linkage-computers (see Chapter 5), and this type of computing component is planned for both the electric and the pneumatic version of PUSS.

Most of the above development, for which Navy Project NO-265 had been assigned, was conducted at The Franklin Institute.[b] In the meantime, the electronic flight simulator (see Chapter 4) was being prepared at Columbia, and had incorporated the principal dynamics of the PUSS computer as well as the characteristics of flight which would be invariant for all such computing methods which might be proposed. It was hoped that this simulator would soon become available for tracking studies along with the instrumental phases of development, but developmental work on the simulator, aside from the consolidation of existing components and the preparation of a descriptive report, ceased by October 1945. However, it is planned to move this equipment to Philadelphia where the remainder of the PUSS project will continue under a direct Navy contract. Although at the present only rudimentary flying operations have been successfully simulated, there is no question but what the more intricate questions of stability, such as that of the rapidity of subsidence to a solution under manual operation, can be submitted to experiments upon this apparatus in the laboratory. A means will thus be available for exploratory alterations without the expense and duration of full-scale testing.

10.2 REMARKS ON THE HUMAN PROBLEM

In the development of generalized aiming controls for the pilot, particularly where he is the lone occupant of the airplane, it must be well recognized what burdens can and what burdens cannot be placed on such an individual as participant or impresario in the ceremonies

[b]The project engineer concerned was J. A. Bevan. M. Golomb was a prominent contributor there, on mathematical, experimental, and general design phases.

of practical utilization. The development engineer cannot dispense with this realistic background to his problem, although his thinking should not thereby be limited; he should talk with fighter pilots extensively and intensively, hearing although not necessarily believing all they have or can be persuaded to say. He should thus try to determine the common denominators among the sometimes widely disparate opinions of such experienced pilots, and to find out, not so much what they want but what they need.

Note that the fighter pilot is a high type of individual, the select survivor of a series of rigors in training and combat. Such a man is at the height of his prowess, in the matters of undeliberating brain and nerve. He is old, for his kind, at the age of 25. But he is hard beset. For not only must he be the master, in modern combat, of several specialized weapons of attack, but also he must be navigator and tactician as well, and an expert at recognition and deception. As captain and crew of his craft, the modern fighter pilot must be self-sufficient.

It will readily be evident that the fighter pilot will not greatly exert himself to obtain computing apparatus which adds to the complexity of his already gadget-ridden existence, unless his survival is otherwise and compensationally made more likely. This view, of course, is by no means universal, even among fighter pilots themselves. However, it is common enough, and not without certain justification.

Specifications for aiming controls, such as proposed in Project PUSS, are affected by such considerations as those above. While these have been written out more completely elsewhere (Section Re8c BuOrd), by the present writer and others, we mention some highlights here since they may apply more broadly than to this single project: No manipulations should be required of the pilot, other than those of flying and firing, during the attack — with whatever weapon. Only rudimentary instrumental manipulations should be required at any time prior to the attack. The pilot should further be free to fly in any manner he chooses until he steadies out on the attack itself. This is for evasive purposes, and for selection of a propitious moment or location for pushing the attack to its crisis. Any approach which requires an inflexible routine, or pattern of elements, is doomed in advance. The sighting or computing system should not require any intellectual gymnastics by the pilot at any phase of the operations. To expect him to "go into the literature" via tabular data, or to make even the most trivial calculations by slide rule or nomogrammic computers, is to impose a burden which may be intolerable. All adjustments should be localized and arranged mnemonically and unmistakably, and there should never be any question of what to do, or whether any given operation, once made, was the proper one.

10.3 THE PILOT'S UNIVERSAL SIGHT HEAD

This important component of a universal sighting system, relatively speaking, can be considered somewhat independently of the others, especially if only a simple reticle as aiming index thereby is to be displayed, movable automatically on a two-dimensional field of solid angle. Such is not the case, of course, when multiple reticles are involved, as in the proposed Texas Sight or in certain other proposed constellation displays where the relative motion of a *pattern* with the target is to be made zero. Again it is not the case when a ranging ring must be used, as in the classical lead-computing sights. It has been the policy in PUSS, however, to hope either for automatic ranging, as with ARO radar or *Pterodactyl*, or for aiming methods in which the range does not explicitly appear (see Chapter 1). Another complication is the expressed and understandable desire for the availability of a fixed sight, which incidentally is a sort of "universal fire-control system" in itself, for standby or other purposes. This matter further divides itself into the cases in which a fixed reticle is to be maintained simultaneously visible with the moving reticle, although perhaps optionally illumined, and the cases where it is adequate to operate a control which "freezes" the moving reticle in its boresighted or fixed zero position. Ideally it should be possible to do all these things, and also to set a fixed deflection other than the zero one by a calibrated adjustment. In the physical ar-

rangements whereby a fixed reticle may be optionally available, this is best accomplished by a mechanical connection capable of very rapid operation, since such a standby sighting facility may be desired owing to failure of servos, etc., and it is questionable to expect an electrical adjustment then to be workable. Rebuttals to this, in which it is claimed that under such failures the reticle lamp will be lost also, are only partially valid. This quick and foolproof "throw over" criterion, in any case, has been upheld thus far for the PUSS system. Within PUSS, the pilot's universal sight head has been known as PUSH.

In the PUSH developments to date, as a subdivision of Project PUSS, the principal approach has been to manipulate a plane mirror in a collimating sight. Very different problems are encountered, depending on whether the (single) mirror is placed in the divergent beam from the reticle or in the subsequent beam from the collimator, the latter being either a lens assembly or a shaped mirror. In the former case the mirror-driving component must compensate for the angular aberrations of the lens, but here the mirror itself may be fairly compact. In the latter case, with the mirror outside in the beam which is focused at infinity, an exact type of geometry[c] may be employed. In both cases the finally collimated beam is reflected back to the pilot from a "transparent" reflector, which may be specially introduced or may be the windscreen of the airplane itself. The illumination problems involved here are considerable,[d] to cover the range of ambient intensity which is encountered, unless either a sun screen can be placed *in front of* the entire system, or a combination of lower transmission and greater reflectivity may be attained for the final reflector, and simultaneously a correspondingly lower transmission for the whole canopy. Furthermore, the problem of eye relief is nearly insuperable with the orthodox systems. A poor compromise must be accepted between the blocking of forward vision and the width of the emitted beam, in typical instances. One way out is the avoidance of collimating lenses through application of reflex collimation, as for example in the Bowen Sight (Mount Wilson). Another is to bring the final reflector, then having small dimensions, near to the pilot's eye, and perhaps to allow it to move about with his head, being maintained in direction by a stabilizing system. Probably the best measure of all is the use of a forward "periscope" or one-to-one telescope of wide angular field (again Mount Wilson) and of large aperture and eye relief. This solves many problems at one blow, but is highly unorthodox. It would be possible to arrange for all the pilot's forward vision to pass through this telescope, which might allow, say plus or minus 25 degrees of angular vision in every direction, might be bulletproof, and would permit *better* streamlining through the elimination of the flat windscreen. Such a telescope has two focal planes, so that reticle motions and other desired indicia, focused at infinity, such as the whole instrument panel around the edge of the field, could readily be incorporated. Opaque reticles, automatically solving the illumination problem against bright backgrounds, could be illuminated laterally by known methods for dim operational conditions. Ultimately such a telescope might well be a foot in diameter, extending from the pilot at the center of gravity to the nose of the plane. The weight imposition, owing to the large glass or plastic lenses, would not be serious to the imaginative planner, since this object would literally be worth its weight in gold. Think, for example, how easy it would be to include an image of the radar scope in the field! This whole possibility is not altogether original[e] with the writer, but is an extension of much older proposals. Its consideration for future development is most heartily recommended.

To return to the rather more nearly earthbound topic of past developments on PUSH, a program of mirror-linkage design was carried on both at The Franklin Institute and at The Bristol Company, although not on a tremendous

[c]See the application of the Orthopentax as shown in Chapter 5.

[d]Although, as we may note here, the attractive suggestion has been made that an intermittent gaseous discharge might provide a more intense illumination of higher optical efficiency, that is to say with smaller dissipation of heat, this question is still somewhat controversial. Additional equipment would be required, but this might add only fractionally to the computing system already contemplated.

[e]See, for example, T. Dunham, Jr. of Section 16.1.

scale. Experimental designs were prepared, for the (single) mirror placed both "inside" and "outside" the collimating lens. For the available optics, the former method was ultimately given up, owing to the compromises which were necessary in the ability to meet specifications, as to focus, field, and precision, which had previously been agreed upon. Furthermore, this method was explored by Navy at Specialties, Inc., who found a rather good compromise and are preparing an initial model for experimental application to PUSS; the cooperation is officially via BuOrd. A number of mirror linkages were developed under our own direction, principally for the second method, which gave great accuracy over a field of plus or minus 15 degrees. Models had been installed, with large collimators, in cockpit replicas at Philadelphia, and the design of housings of practical types was about to be considered when the project began to close. Presumably such development can be continued under Navy auspices if it is desired. The interested and authorized reader should study also the sight head designs of the Draper-Davis A-1 sight in the Army. For future development, however, we urge that a much longer view be taken of this whole problem, and that the conjectural methods mentioned above should be considered along with newer proposals which most certainly will be received.

A very interesting possibility is in generalization of the fly's eye collimator, so called, which is described in contractor's reports by Eastman Kodak and The Franklin Institute, to give a deflectable reticle-image. (See the report of Division 16.)

Whatever the nature of the sight head, if it is of the universal type here referred to, remote followers or servomechanisms must be employed to deflect the line of sight in obedience to the outputs of computers placed at some distant points. The reference variable at the sight head may be a resistor or capacitor, say, in the case of electric servos. For d-c servos the resistor is very convenient, and has been employed by us and by Specialties, Inc., in experimental models. The capacitor follower, however, for a-c circuits, permits extreme fineness of setting, and consequent smoothness, where this is desirable and where high-frequency methods are elsewhere applied. Transmission systems of more nearly standard type may also be applied, however, thinking now of small selsyns and autosyns, especially where the computers produce mechanical rotations directly. Pneumatic transmissions have also been considered, and would certainly be practical when this medium is employed in other parts of the system. One advantage is that no follower sequence is required. Pressure in a closed system may be carried to a remote point for deflection purposes with considerable guarantee of accuracy, speed, and lack of hysteresis, provided that the appropriate newer techniques are applied.

10.4 THE QUESTION OF ROLL STABILIZATION

The general method, used by us in the PUSS system, of splitting the computation and manipulation into two components corresponding separately to azimuth and elevation, these coordinates being referred to from the pilot's point of view, has several consequences in dynamic performance which compare unfavorably with the circumstances of a single-gyro deflected sight. It is evident, thus, that when the airplane is rolled instantaneously, this having obviously no effect[f] on the trajectory of the projectile, the sight line will initially be moved in space in a direction and speed depending on the lead angle, measured from the axis of roll, and on the rolling maneuver as such. While this occurrence is only transient and will disappear in the steadier conditions which should presumably precede the firing instant, the response of the line of sight in this manner to roll may enter the tracking dynamics and influence the stability thereof. One possibility is that it may affect skid, although one does not know whether beneficially or harmfully, and such questions are important enough to be studied before flight tests are made. Flight tests on analogous structures give reason to believe that this problem

[f]Aside, for example from centrifugal effects owing to the displacement of the projectile from the roll axis. These are probably small in practice. We have postulated only a hypothetical circumstance, not directly related to actual events.

may not be serious, however, but we should not like to let a large development program depend on such tenuous data. One usage to which it has been hoped that the electronic tracking simulator might be put was to just such questions of dynamic stability as this, where the human response is involved. The reader is referred to Chapter 2 for a discussion, on a more general plane, of the analytic and synthetic possibilities in connection with this type of question.

In any case we now know that this roll transient, whether adverse or otherwise, can be compensated for quite completely by relatively simple means. We say relatively since it is quite evident that a straightforward although cumbersome measure for its elimination would be to roll stabilize the whole sight head, for example, or the whole system, allowing no rotation in space about the roll axis. It is only necessary to employ an angular rate meter measuring the roll rate, or an angular accelerometer measuring its acceleration, and form the appropriate[g] dynamic connection to prevent transient turning of the line of sight. It is now felt that this problem, if it indeed arises in serious proportions, will not long remain unsolved in a practical sense.

On a somewhat higher plane it may be reiterated here that all the functions of free gyros and stabilization equipment may be duplicated by a system fundamentally including a set of three absolute angular rate meters (see Chapter 3) fixedly disposed in the vehicle.

10.5 COMPONENTS AND SYSTEMS

Between the various inputs and the output aiming deflections, the computing portion of PUSS may be roughly segregated into a dynamic computer and a static computer, each possessing several subsidiary components, and each being joined functionally to the other at a number of points. The dynamic computer embodies an operational connection, between one component ω of angular rate and a corresponding component Λ of angular sight deflection of the form

$$\Lambda = B + \frac{1}{1 + kTp} \cdot T\,(\omega + A)\,, \tag{1}$$

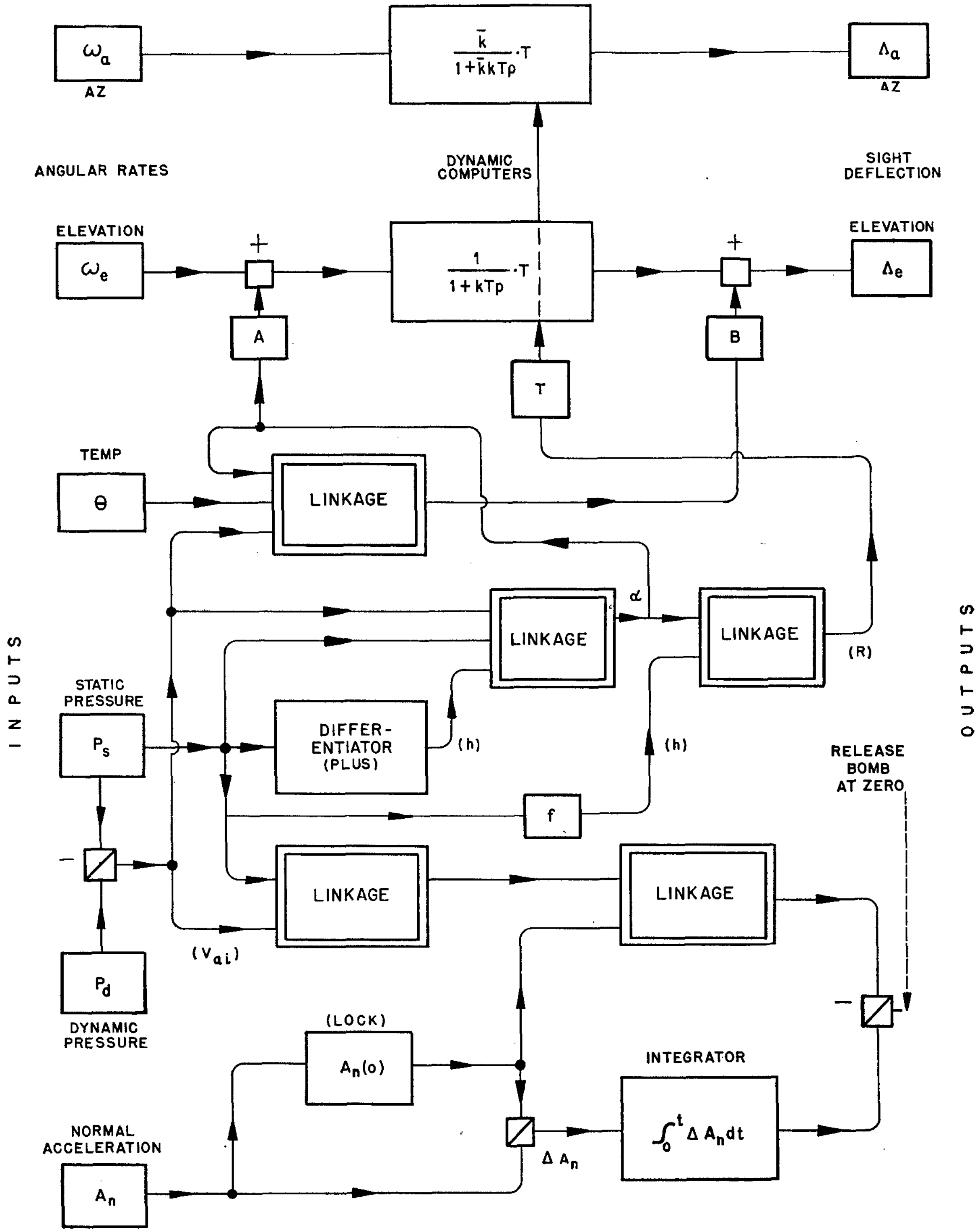
in which A, B, and T are outputs of the static computers, and are slowly varying with time, at least in comparison with ω and Λ. If one prefers it, the differential equation equivalent to the above operational equation is

$$k\frac{d\Lambda}{dt} + \frac{\Lambda}{T} = \omega + A + \frac{B}{T} + \overbrace{k\frac{dB}{dt}}^{\text{small}}\,. \tag{2}$$

When A and B are made zero, this characterization in either form will be recognized as that for a standard disturbed gunsight. Again if the time-parameter T is made infinite, the equivalent of a moving sight which is coupled to a free gyro is obtained, whereby, for example, a collision-course approach might be governed as explained in another place (see Chapter 1 and Chapter 8). It is evident that the characteristic of the dynamic computer is essentially that of a *first-order lag,* as it may logically be called, with an adjustable lagging parameter coupled also to a sensitivity adjustment, and with additive terms imposed on the input and output. There are many means for mechanization of such a characteristic. Those chosen thus far for the PUSS system have been resistance-capacitance filters, one synthetically produced in the electric or electronic version, and one obtained directly by pneumatic components in the pneumatic version.

Figure 1 shows one schematic arrangement of the computation as employed in PUSS, which has had numerous variants, however, depending on precisely which are applied of the various alternative input components which have been in development. No attempt is made in this figure to show in any detail the switching arrangements, for example, by means of which the operator may select at will the type of weapon for which solution is intended. These important technical features have been very completely described[2,4] in reports to which reference is also made below, and it must here be pointed out that this entire development is still in progress, although it is now a closed

[g]Suggested as a technique by the present writer, and given analytic form and justification by M. Golomb at The Franklin Institute.



PUSS
INCLUDING PACT

FIGURE 1. Schematic diagram of PUSS system; AFCS Mark 4; Project NO-265.

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issue as far as the direct responsibility of NDRC Section 7.2, as such, is concerned.

To name the automatic inputs of PUSS in one of its most likely forms, we may list basically: static and dynamic air pressures, vertical acceleration, temperature, and the two absolute angular rates of the vehicle. Certain implicit inputs relate to the characteristic of the airplane itself, and are made on installation. Only one manual input must be made during operation, however, aside from the choice of weapon, and assuming of course that appropriate automatic means is worked out to synthesize the temperature of the rocket propellant, and that one is target altitude. Even this is unnecessary if, with a different basic set of inputs, automatic radar range or radar (FM or pulsed) altitude is provided as is being considered. To continue with the inputs named, the angular rate inputs would be handled in dynamic computers of the type for which equation (1) is definitive, one channel[h] each for azimuth and elevation. In the azimuth channel it may be assumed that the "secular" parameters A and B are absent, and further the time-parameter T, although simultaneously set into the dynamic computer of both azimuth and elevation channels, is multiplied by a constant ratio in the one as compared with the other. This is a stratagem whereby the correction for gravity can be effectively made in the proper direction, in the presence of uniform *bank,* in spite of its explicit treatment in the elevation component alone.[i] It remains to describe, for this same set of inputs, how the parameters A, B, and T are computed, and then to describe the interconnected computer (PACT) for toss bombing.

As intermediate steps in the computation, certain implicit variables are first obtained. Thus glide angle is obtained in terms of the static pressure and the dynamic pressure, in the process of which a differentiation of a function of static pressure is performed, it being evident for example that the glide angle γ may be derived in terms of true airspeed, itself a function of indicated airspeed V_{ai} and altitude h, and the rate of change of altitude, the altitude itself being (again) expressible in terms of the static pressure. It is probably not necessary to mention that indicated airspeed is measurable in terms of the difference between the dynamic and the static pressures. Range R to the target, of course on a relatively straight approach, is calculable, as an intermediate or implicit variable of the computation, directly from altitude and glide angle. Then the angular rate parameter A, and the lead parameter B are themselves obtainable from the quantities now available. The parameter A is derived as $A(\gamma)$, or as a function of glide alone; while B is given by $B(\gamma, V_{ai}, \Theta)$, and as a function of glide, indicated airspeed, and temperature is the most complex item. The time-parameter T is computed as $T(R)$, a function of range alone. These latter computations take place principally through multiple linkages, which were designed directly from graphical representations of the form the respective functions must have for accurate results of fire. In Chapter 5 this general philosophy was more fully treated. The computation for rockets and bullets, in one variant of PUSS, has now been traced through. We shall return again to a consideration of various questions with regard to certain individual components. But first we shall explain the operation of the PACT toss bombing computer which is involved, thus far, only as a component of the pneumatic version of PUSS. It was felt that an electric mechanization of the toss bombing (or toss rocketry) functions would too closely duplicate other BuOrd activity, notably that at Bureau of Standards.

The inputs for PACT are the dynamic and static pressures and the vertical acceleration as measured by a captive-mass accelerometer. Again a manual input of target altitude must be made, but this input is common both to PACT and the rest of PUSS, as are the static and dynamic pressures. The static pressure, as a function of altitude, is utilized through the motion of an aneroid capsule, together with the motion of a differential meter responding, as a function of indicated airspeed, to the difference between the static and dynamic pressures, to develop an auxiliary variable as the output of a

[h] Note, however, the remarks in an above section on the possibilities of roll stabilization. See also the bibliography.

[i] This technique is believed novel in PUSS, and was conjured up by M. Golomb, mathematician to the project.

portion of the PACT computing linkage. In a pneumatic force-balance accelerometer, a pressure corresponding to the vertical acceleration, adjustably lagged, is made to provide a mechanical motion, along with the auxiliary variable above mentioned, continuously to compute the velocity a bomb must have, in the direction upwardly normal to the direction toward the target, sufficiently to counteract gravity. On the pull-up, in the toss bombing tactic, action of a pneumatic integrator is initiated which computes, via the integral of the *increase* in total vertical acceleration of the airplane, the corresponding normal velocity being thereby imparted to the bomb. When these two computed variables are equal, as indicated by coincidence of output linkage rotations, the projectile is automatically released. In PUSS, the inputs and computing components of PACT are, at least in the preliminary model, physically interspersed and interwoven with those of the remainder of the system. All this has taken place according to Pollard's theory of toss bombing, as also has the following arrangement for special settings in the dynamic computer of PUSS during the operation of PACT. In toss bombing operations in order to follow an initial approach which allows for target motion, the elevation sight deflection is made zero,[j] the time parameter T is set at maximum, or approximately at 20 seconds, and the parameters A and B are set to zero. Simultaneously the "fixed" parameter k, the constant of coupling as shown in Figure 1, is set at the value 2 rather than about 1.2, as it is for the other aiming functions.

The heart of the PUSS system proper, namely the dynamic computing component, is fed principally by the directly measured absolute angular rates. In early forms of PUSS, the functions of the present gyro inputs and dynamic computers were accomplished by other means, and such other means were the subject of much investigation. Proposed systems ranged all the way from the now standard dome-type eddy-current gyro systems to those with separate precessing motors similar to gunsights developed, as has recently come to light, by Zeiss for the GAF. A variety of special linkages and servo systems were considered for such systems, all of which involved only a single gyro. We have discoursed, in Chapter 3, on the advantages claimed, particularly in flexibility for new functions, for the present technique of operating on a fundamental direct measurement of angular rate by whatever dynamic characteristic is appropriate to the occasion. In particular the way to higher performance is thus opened through the application of higher-order dynamics. We have discussed also, in Chapter 3, the captive gyro developments which have been carried out for the angular rate inputs to PUSS, including the pneumatic and capacitative one-dimensional units, and the two-dimensional current servo case. Certain phases of this research are still under way, but progress and future promise is evident on all the types named.

In the measurement of glide, the pneumatic version of PUSS depends at present on a measurement of the rate of change of barometric pressure, which together with that pressure itself, and indicated airspeed, gives the glide angle, and consequently also the range, by a solution equivalent to that of the obvious triangle. Some research has been conducted on a refined type of differentiator, involving feedback, which is still in development. The standard rate-of-climb instrument is somewhat inadequate for this task. An analogous solution to the problem of the provision of glide, is now planned for the first electric version of PUSS. Here an existing component, employed in the so-called glide bombing attachment [GBA], for the bombsight Mark 15, has been modified for this purpose. By following the motion of an aneroid diaphragm system, having a static pressure connection, through a mechanical integrator disk, both altitude and its rate of change are developed as shaft rotations. Hence, with airspeed, glide may be derived again as above. A competitive method for measuring the glide, in this case δ, as has been employed in several other systems, is by detection of one of

[j]Or rather, the sight reticle is aligned, in elevation, with a definable fixed axis in the airplane, along which also the vertical accelerometer does not detect. Allowance for target motion in elevation is a consequence, discussed elsewhere in this report, and also in the writings of H. Pollard (of AMG-C), of the influence of curvature of the flight path on the vertical acceleration itself. This latter phenomenon is similar to an analogous one in rocketry computers. In both cases the method works only near certain average glide angles.

the gimbal deflections in a gyro horizon. One objection to most such instruments, however, is that they will "spill" at certain extreme bank or dive angles. This is not the case, or at least very rarely so, with the new Sperry altitude gyro, nor, it is understood, with a Pioneer horizon now in development. Here, however, the measurement of gimbal rotation is made difficult since it is the *inner* gimbal which is involved. A method of capacitor measurement can solve this problem, it has been decided, and may be appropriate if such measuring methods are elsewhere employed. An experimental assembly is now in preparation, involving a pair of radial plates, rotating relatively with the desired (but mechanically remote) gimbal motion, in series with a larger and constant capacitor acting as slip ring. No friction load need thus be added, and known methods of capacitance measurement may be directly applicable. We may mention briefly still other methods which have been considered for the effective and continuous determination of the glide. In PUSS, the accelerometer method, giving the cosine of glide (δ) under conditions of straight flight, is not appropriate on the one hand, since the error due to target motion and path curvature does not necessarily correct for the target motion, as in other cases, but on the other hand such errors might here be removed since angular rate of the airplane about its lateral axis is continually measured as an input. A question of stability in tracking still arises, however, and it has been less troublesome to apply other techniques. Still another possibility, instrumentally unexplored, is to employ the longitudinal component of total acceleration, as a measurement of glide, since it is unresponsive to curvature of path. Correction for the rate of change of true airspeed would there be required.

Certain altogether different approaches to the PUSS problem have been proposed, and as yet only partially explored.[k]

[k]There is the proposal of R. O. Yavne (see bibliography) to mechanize the firing solution with a system equivalent to a free-gyro sight, however embodied, in which the coupling constants (such as the above) are varied with range, for example, or time. It is not impossible that such an arrangement might ultimately prove to be superior, and exploration of such alternatives should continue as long as there is judged to be a need for PUSS as such.

From a more practical standpoint, we may mention the question of installation. In this connection it has been proposed by BuOrd that the PUSS computer, whether pneumatic or electric, be contained in a wing tank with remote connections to the pilot-operated controls and to the sight head (see above). At least for experimental work, and possibly further, this may have certain nonignorable advantages. Note that such radar systems to be employed as potential inputs to PUSS, in particular for the range itself, will be somewhat similarly installed in any case. It is evident that the removal and reinstallation of computers, for calibration or for several experimental purposes would thereby be greatly aided. Further, this would allow a separate and local pilot installation, for dynamic and static pressures, which permits engineering of these inputs to suit special needs better than ordinarily do the standard equipments available in airplanes for flight purposes. It is now planned to make such installation, with the preliminary models of PUSS at least. Certainly even the smallest droppable auxiliary gas tank should provide adequate space.

This is perhaps an appropriate place to mention that the current experimental PUSS model, soon to be prepared for tests, has been officially designated aircraft fire control system [AFCS] Mark 4.

10.6 COMMENTS ON PNEUMATIC COMPONENTS

In Chapter 3 of the present report, we have discussed the properties of captive gyros, in particular of the pneumatic variety. The pneumatically captured gyros, employed experimentally for PUSS and for other[l] systems, have all been of the one-dimensional type, measuring absolute angular rate of a framework about one axis fixed therein. These developments have been the work principally of NDRC Section 7.3. In the pneumatic version of PUSS, two such captive gyros are contemplated for the angular rate inputs, and a third — forming, thus, a symmetrical set — might be included for more

[l]For example in BARB, as embodied in bombsights Mark 23 and 27, and for aircraft camera stabilization.

general, or more universal, types of aiming control. Components are now available for testing, both separately and in combination with the other components of the system.

As we have said, pneumatic force-balance components are being applied for the determination of rate of climb and for the integration of incremental accelerations, the latter variables themselves being determined by similar means. For such applications the pneumatic medium leads, at least potentially, to very smooth and rapid operation, and to lightness and compactness of design. For dynamic computations, connected systems of rigid tanks and capillary tubes are usable in a manner similar to electrical resistance and capacitance, and analogous feedback techniques apply where nonpassive systems must be synthesized. In such cases, however, certain restrictions must be observed to obtain predictable or linear performance, for example. Typically, the time parameters of such networks are generally proportional to the absolute pressure in their component elements, so that either compensation must be included for such effects or the variations in pressure (as in orthodox acoustic theory, for example) must be kept small in comparison with the absolute level. Furthermore condensation and leakage must naturally be rigidly guarded against, although techniques are known which may insure safety from such deleterious influences. (See Chapters 3 and 4 of Volume 1, Division 7.)

The purposeful variation of parameters, in dynamic computers of the pneumatic type, may be accomplished by several means. If continuously made (during normal operation, that is) the obvious mechanical method of varying pneumatic capacitance by changing the volume of tanks seldom gives the desired result since a "forcing function" is imposed on the pressures in the system, continually altering thus the initial conditions, so to speak. Adjustment of pneumatic resistance is more appropriate, since the storage of energy is not involved there, and the techniques of so doing are centrally important in this art. Needle valves are prone to many failings, from a practical as well as theoretical standpoint. Step adjustments of capillary tubing lead typically to much equipment and to concern over the continuity of overall operation. Under the express urges of the present writer, an ingenious and appropriate component for the variation of pneumatic resistance was devised for this application.[m] The solution, now experimentally embodied in components for pneumatic PUSS, involved a stack of thin punched disks, with alternating types of hole, whereby a labyrinth resembling capillary resistance was adjustable in effective length as imposed between the terminal connections. An easy mechanical adjustment was permitted. For details we refer again to reports of Section 7.3.

For shifting operations, whereby the pneumatic computer and allied equipment could be adapted rapidly and at will to the different functions desired of them, small solenoid-operated valves of special type were developed. The entire pneumatic assembly is to be contained in a supercharged space, possibly later with complete recirculation of air (or whatever gas is employed). Regulated supply pressures, by means of pumps and feedback pressure-regulators, have been designed for and are expected to perform valiantly. The only possible objection to the pneumatic version, of PUSS or of similar equipment, appears to be the unfamiliarity of maintenance procedures. This is a controversial point, however, and appears less important anyhow from the longer-term viewpoint.

10.7 THE CAPACITOR IN INSTRUMENTAL TECHNIQUES

The application of continuous measurement of electrical capacitance in components of aiming-control systems has been referred to at several points. For the measurement of small mechanical motion, either for repetition in other physical variables or for the detection of error in a follow-up, it has numerous advantages. It can compete with photoelectric methods as concerns the absence of "detent," or of interference with the physical system under detection. But further and perhaps more significantly, precision measurement may be car-

[m]By J. F. Taplin of Section 7.3 (and Lawrance Aeronautical Corporation).

ried out without the need for following agencies placed locally where the primary measured variable occurs. Thus, very small capacitors may be located in otherwise almost inaccessible spots, and their capacitance, in response to the primary variation with which they are associated, may be reproduced remotely in any desired tangible form. The possibility of employing capacitative "slip rings," in series with the measuring capacitor, as already referred to, permits electrical extraction of such measurements from mechanically complex systems, where otherwise direct conduction, with its attendant problems, would be required.

To derive any reasonable function of a rotation, capacitatively, it is necessary only to shape the plates — or the dielectric — of the measuring capacitor in appropriate fashion. Employing the electric laws of series and parallel capacitance, numerous elementary mathematical operations, such as addition, may be performed from mechanical motions in a computer. For these applications, of course, a stable oscillator must be provided. Frequencies of one megacycle appear to give satisfactory results in typical instances, whereas much higher and much lower frequencies have been successfully applied. It is naturally most efficient to resort to capacitor techniques where many of the operations in a system may thus be carried out, and where electronic channels are performing other functions, as well, of computation or manipulation. The combination, however, of such techniques with mechanical ones is significant, where for example the final motion of complex but miniature linkage computers may be extracted without load on the inputs or bearings, and where the size of the computing unit itself is not thereby appreciably increased.

Information on the practicality of electrical capacitative methods for such applications should soon be available, since at least two such components for PUSS are now in development. Following rigorous tests in the laboratory, flight testing under conditions properly arranged should simulate those of the final application at hand. Since such methods have been used in industrial and other systems for a long period, there is little reason for suspecting that they are inapplicable here.

10.8 PATHS FOR CONTINUING RESEARCH

One modification of PUSS not already mentioned is that which has been suggested from several sources, and consists in what amounts substantially to presetting the gravity drop and other nonkinematic parts of the aiming correction by a computation different from that executed by the dynamic process which is fundamental to the principle of the current PUSS method. We refer chiefly to the solution for rocketry. Such a modification might certainly lead to a more rapid subsidence to the firing conditions, and in any case would provide at least an approximate aiming correction if there were no time even for the brief tracking interval which the PUSS method demands. It may be possible, at the longer ranges, to arrange for such presetting automatically by redesign of the linkage computers which are now contemplated.

It goes without saying that many of the conjectural schemes for aiming control which have been mentioned throughout this report may well deserve further study, and it would hardly be surprising if some of them proved revolutionary in the increased powers of fire control which they might allow. For the future, certainly, there is more time for such theoretical or basic study, and the newer techniques may arise either out of mathematical or instrumental advances in thinking, or again purely empirically from quantitative study of the physical sections of the problem. It would naturally be folly to work without recognition of the probable forms of future weapons, but the recommendation of the present writer would be to treat projectiles from a rather generalized point of view, so that rather than being outmoded, the theories and systems to be developed may simply be adapted by the fixing of parameters in the control and computational components.

One may look somewhat beyond the immediate objectives of PUSS, and without, indeed, losing sight of them, consider some of the logical extensions which should ultimately be attainable. We mention these although thereby we actually pass into somewhat different branches of ordnance (see Chapter 8). More or less automatically, PUSS will be associated with

the jet-propelled airplane types, and this has certain significance to our final point.

In somewhat earlier discussions[n] it was contemplated what might ensue if the fighter plane were to be equipped with an automatic pilot of really high performance. As a generalization of PUSS, suppose that the pilot, in attacking a target wherever located, were simply to employ a stabilized sighting system, carried in his fighter, to track the target through the manipulation of controls which operate the line of sight through optimum dynamics, and do not primarily have to do with flying his vehicle. With the direction to the target thus well established, and the special properties of this direction continuously determinable, a computing component would develop, in continuous manner also, the direction of flight necessary for successful fire. This latter direction, by instructions from the computing system to the high-performance automatic pilot, would be undertaken by the airplane. Thus we have what amounts to a *director* for fighters. While there are many details to be filled in, there is nothing really insuperable about the problems of such a system.

Further, it is evident that with the pilot's task reduced to that of tracking the target, he may be replaced by an automatic follower which "locks on" the direction to the target by the available radiated or reflected data therefrom. The vehicle, without human passengers, is now unlimited as to violence of maneuver, for example. As one important special case, which incidentally may be resorted to if desired with PUSS in its present form, the vehicle itself as projectile may be steered on an interception course with the target. Here, then, we enter into the field of guided missiles. The techniques of the two fields are not widely separate, in fact, even at present. Each has much to learn from the other, in apparatus and in developmental procedures; and for the newer weapons — if such there must be — can best proceed in the fullest kind of cooperation and understanding.

[n]With Commander E. S. Gwathmey of BuOrd.

PART II

AERIAL TORPEDO DIRECTORS

PREFATORY COMMENTS

AIRCRAFT TORPEDO DIRECTOR developments under Section 7.2 of NDRC were carried out at The Franklin Institute as part of Contract OEMsr-330. G. A. Philbrick was responsible for the earlier work, which he has described in Part I. Since the spring of 1943 these developments have been under the cognizance of the present writer, but with much helpful collaboration from Mr. Philbrick. The transition from one supervisor to the other was somewhat gradual, with no sharp dividing line. Consequently, it has not been practical for Mr. Philbrick and the present writer to prepare two reports, each covering distinct phases of the development work. Mr. Philbrick has described a part of the aircraft torpedo director work, and this report is intended to complete his descriptions and to add descriptions of the remainder of the work in this field.

For complete detailed descriptions of the various directors developed, the reader is referred to The Franklin Institute reports listed in the Bibliography.

Several ideas that never reached the development stage might warrant a word or two. Some thought was given to a so-called rule-of-thumb method of aiming torpedoes.[31] This consists in the pilot aiming a predetermined number of apparent ship lengths ahead of the target. The trouble is that analysis shows that for even moderate accuracy the predetermined number referred to must come out of a fairly large and complicated table which the pilot must memorize. Opinions differed widely as to the value of the method.

Further work on the apparent length method was done in applying it to the technique of tossing torpedoes.[44] The suggestion was to toss a torpedo like a toss bomb, but aimed to hit the water about 200 yards (the arming distance) in front of the target. A modified toss bombsight was used for this purpose, and the Navy asked Section 7.2 for an analysis on what lead angles to use. Opinions differed on the value of using an expensive torpedo for a toss bomb. Tests indicated that sufficient accuracy to make it worth while was not obtained.

At one time the Bureau of Ordnance suggested making a torpedo director for dropping gyro-angling torpedoes. Section 7.2 did not have much enthusiasm for this development and did not undertake it. If an attack is made by aiming the plane at the target (either with zero lead or on a collision course), relying on the gyro angle setting to provide the lead by causing the torpedo to turn in the water, a no-deflection shot is given to the antiaircraft gunner on the target ship. If the gyro angle setting on the torpedo is used to permit the pilot to attack so as to minimize the danger from AA fire, a large lead angle must be employed, and that increases the difficulty in sighting through the director optics. It appeared that a zero gyro angle (or straight) shot represented the best compromise between the two difficulties, and it certainly resulted in a simpler director. Eventually the Navy abandoned the idea.

One development that might be called one of the odds and ends was the computer for a torpedo director trainer.[46] This was to calculate and show the error in hitting while using the standard trainer for the torpedo director Mark 30. The computer was 90 per cent complete when stopped at the end of the war.

A. L. RUIZ

Chapter 11

COURSE STABILIZATION

In the solution of the aircraft torpedo attack problem it is necessary to know the target course and speed. These quantities could be measured if a suitable tracking mechanism were installed in the attacking plane to determine target bearing and range over a period of time and to feed that information into a computer. The amount of mechanism involved, however, would be fairly large and the use of an additional operator would be required. In order, therefore, to avoid such undue complication and weight, aircraft torpedo directors have been built on the assumption that the course and speed could be estimated with sufficient accuracy.

Normally, instead of target course, it is much easier to estimate target angle (or "angle on the bow"). In effect, this is the orientation of the target with respect to the line of sight, and is, therefore, the angle seen by the pilot as he looks at the target. From the target angle, own-plane course, and the relative target bearing, target course may be computed (Figure 1). But it is not necessary to make this computation since the target angle can be used in solving the attack problem (Figures 2 and 3).

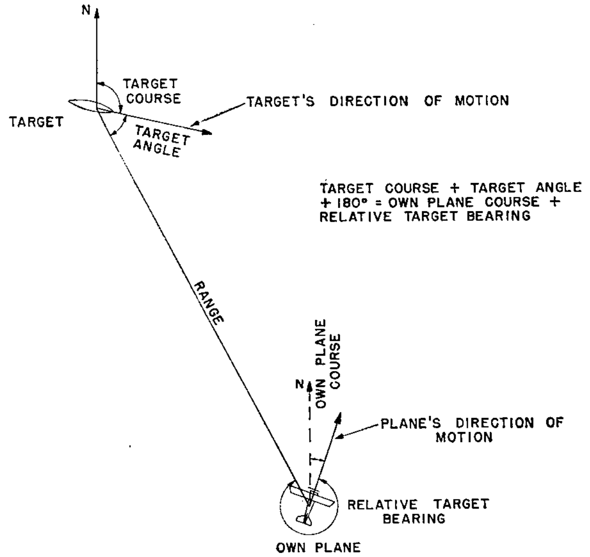

FIGURE 1. Plan view of target and own (attacking) plane.

Inasmuch as the target angle is a function of the target course, own-plane course, and relative target bearing, it is apparent that the target angle changes as the attack progresses

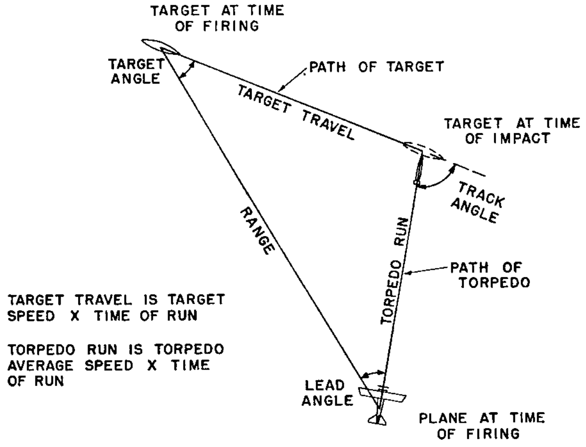

FIGURE 2. Plan view of torpedo firing problem.

and the plane approaches the target (Figure 4). Thus, if the pilot estimates and sets target angle early in the approach his setting will be wrong at the time of attack. Hence the pilot must make a last-minute adjustment and estimate of the target angle before releasing his torpedo. With the numerous other things that he has to do at the moment of release this is an additional burden of which it would be well to relieve him.

A method has been suggested[a] for relieving the pilot of this last-minute estimate of target angle. The torpedo attack problem can be equally well solved by using track angle instead of target angle (Figures 2 and 3). From Figure 4 it is apparent that track angle is the supplement of the angle between the target course and own-plane course. To determine this angle directly, a miniature ship model is located on the end of a shaft convenient to the pilot. The pilot may turn this ship model until it lies parallel to the target's direction of motion as it

[a]By Lieutenant Freeman of the Bureau of Aeronautics.

appears to him. Momentarily then, the angle between this ship model and the plane's fore-and-aft axis would be the track angle if the plane were on a proper course for release. To keep this angle correct after it is once set, the ship model is clutched in to a directional gyroscope so that regardless of the plane's changes of direction the ship model will remain parallel to its original setting. Thus, as long as the target does not change course the torpedo director has available a continuous setting of the difference between target's course and plane's course. At any time that the plane is turned so as to be on a correct torpedo firing course by putting the director's cross wire on the target, this difference represents the cor-

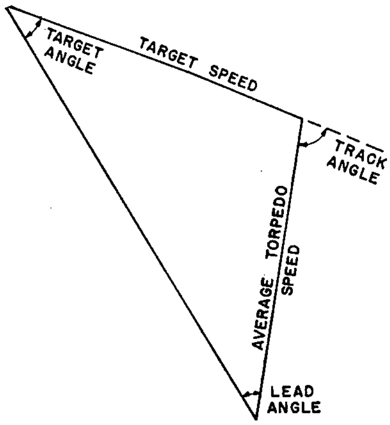

FIGURE 3. Plan view of torpedo firing problem; same as Figure 2 with all sides of triangle divided by time of run.

rect track angle. Consequently, the pilot may make his target course setting early in the approach and is relieved of the necessity of making a last minute adjustment before release.

Use of a directional gyro and the principle of course stabilization was first made in the torpedo director Mark 32.[39] As a result of tests on this director both the Army and the Navy requested the modification of a torpedo director Mark 30 to add a directional gyro and a ship model for stabilized target course.[36] Tests on

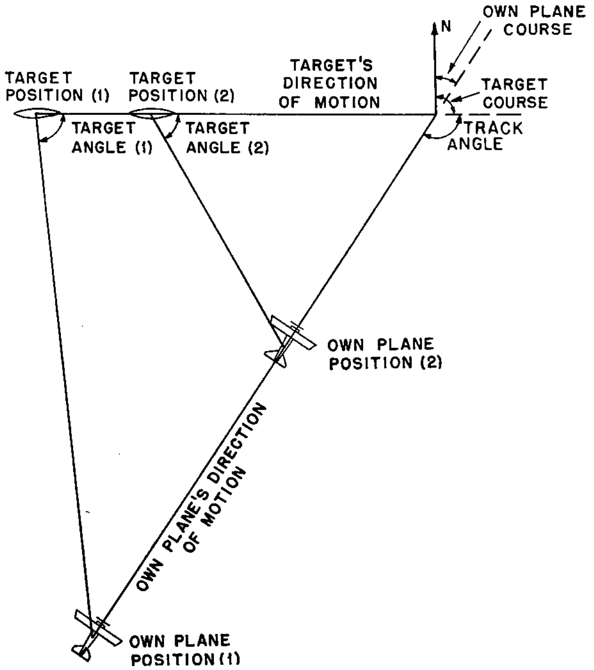

FIGURE 4. Illustrating changes in target angle.

both of these at Fort Lauderdale and at Eglin Field were very gratifying. All pilots who used the director were well pleased with the course stabilization feature. As a consequence of the Eglin Field tests the Army requested the development of a new torpedo director utilizing a different set of optics. This was completed in June, 1944 and was adopted as standard by the Army and designated the torpedo director B-3.[41] A similar director with different optics combined with a low-level bombsight was built for the Navy at about the same time.[42]

Chapter 12

PRESENT-RANGE TYPE TORPEDO DIRECTORS

The torpedo directors Mark 30 and 32 (and all earlier ones) were built to use an estimate of torpedo run as an input. The reasons for this were twofold: the solution for lead angle as a function of present range and other variables is implicit and transcendental; no simple mechanization had ever been found. Consequently it was simple to rationalize and convince oneself that it was just as easy to estimate torpedo run as present range.

But the increasing use of radar in torpedo planes spurred on the search for a simple

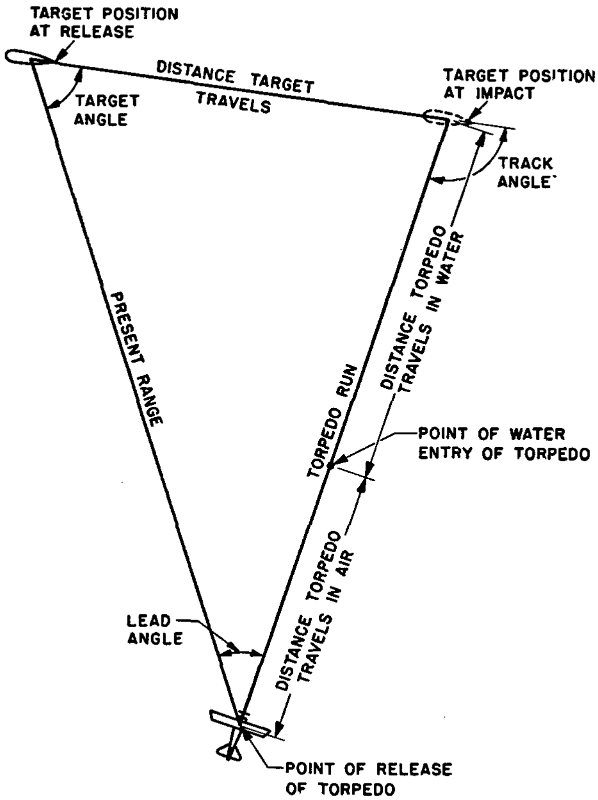

Figure 1. Present-range solution, first step.

present-range type of computer. At last the answer was found: it is described in Chapter 6.

The trick consisted in locating a point on the torpedo path from which a fictitious torpedo, having a constant speed equal to the water speed of the real torpedo, could be launched in water at the same time the real torpedo is released, and reach the target simultaneously with the real torpedo. Using this point it is

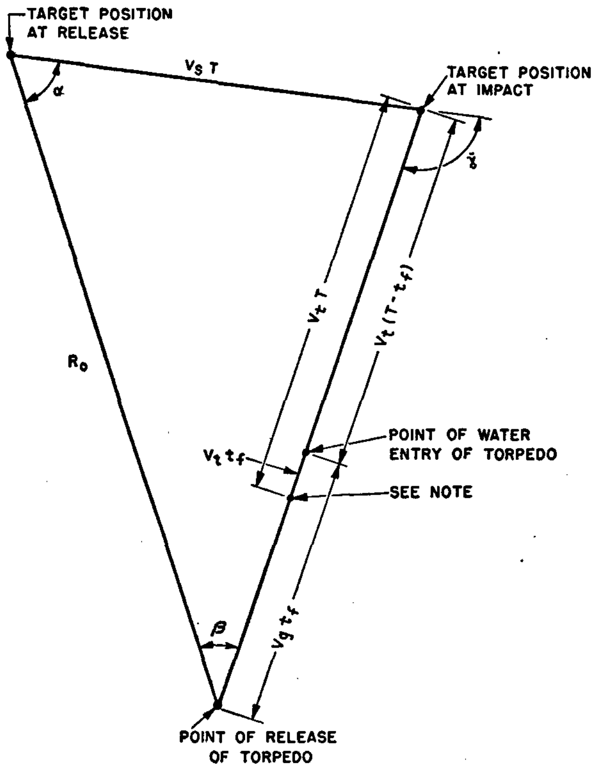

Figure 2. Present-range solution, second step.

Note. Point from which a fictitious torpedo with a constant speed equal to that of the real torpedo starts in water at the time the real torpedo is released, and reaches the point of impact simultaneously with the real torpedo.

possible to break the problem down into two parts: the first is a computation for the lead angle for the fictitious torpedo, and involves only the target speed, track angle, and torpedo water speed; the second is a correction computed from own-plane speed, present range, and time-of-fall (obtained from altitude by assuming a fall in vacuo $t_f = \sqrt{2H/g}$).

The detailed stages in the solution are illustrated in Figures 1 to 6 inclusive. Figure 6 is a schematic diagram of the linkage built. The unit

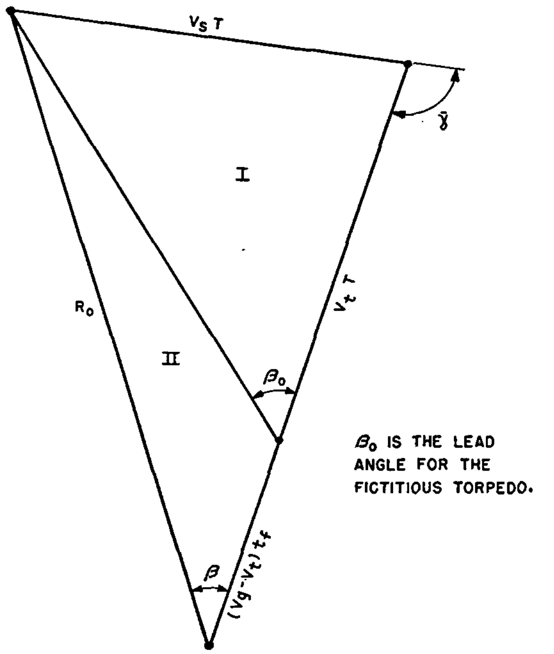

FIGURE 3. Present-range solution, third step.

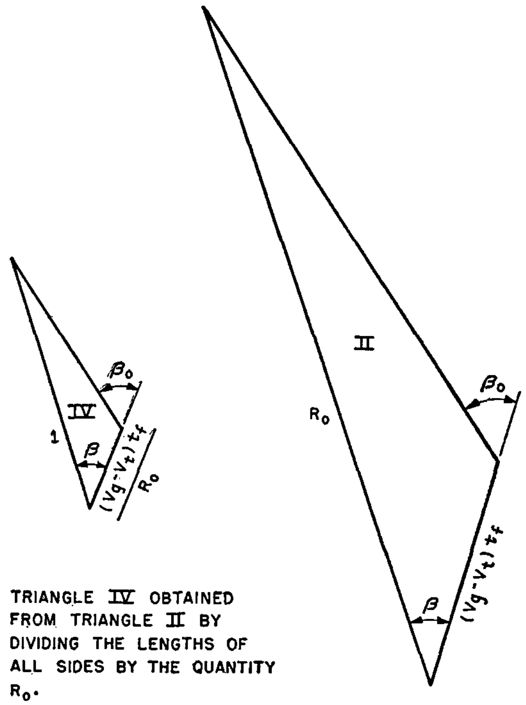

FIGURE 5. Present-range solution, fifth step.

length in triangle III is the base line, at one end of which is pivoted the link A (turned through the track angle by the pilot's setting and the directional gyro), and at the other is the link B. The length of A is made equal to V_s/V_t by a cam, and the ends of A and B slide together. In triangle IV the link C is fixed in length and equal to unity. One end of C slides in link B, and the other is moved along the base line by a series of logarithmic dials and an exponential cam a distance equal to $(V_g - V_t)\, t_f/R$. The lead angle β is found in triangle IV and applied to the optical system.

Two present-range types of director were built by modifying torpedo directors Mark 30, and were completed just as the war ended.[43]

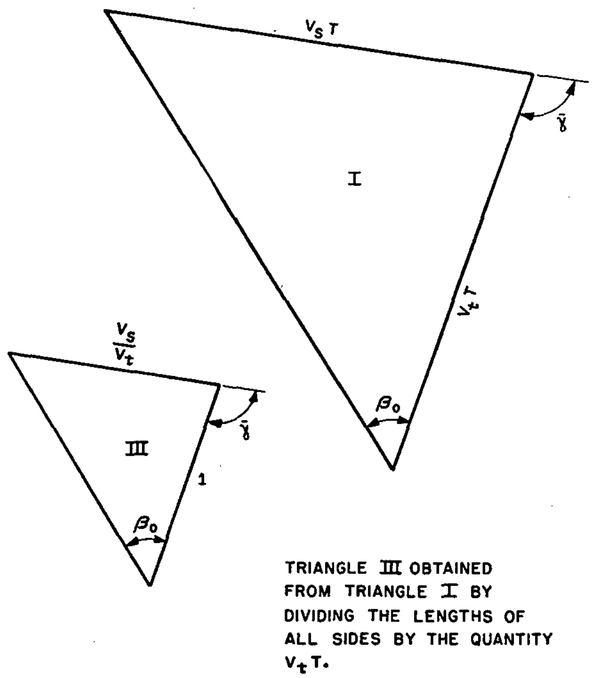

FIGURE 4. Present-range solution, fourth step.

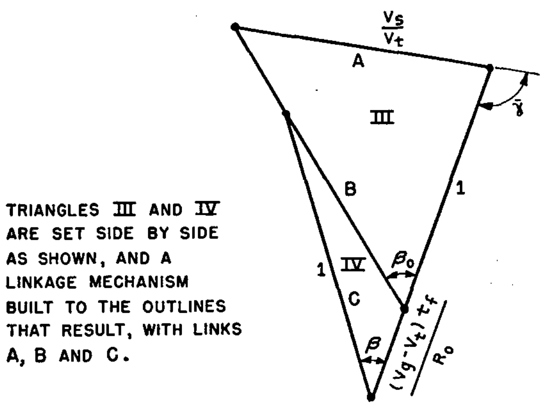

FIGURE 6. Present-range solution, sixth step.

Chapter 13

TORPEDO DIRECTORS FOR USE AGAINST EVADING TARGETS

All torpedo directors to date (and in fact all gun directors) have been designed for use against targets moving in a straight line with constant velocity. The arguments in favor of the constant velocity assumption were twofold:

1. Many targets were not dangerous unless moving in a straight line (e.g., a bomber had to make a straight-line approach).
2. Since no control over the missile was possible after launching, a straight-line target motion represented the most probable condition for prediction.

These arguments were considerably weaker in the case of a torpedo-plane attack than for many other missiles. Upon observing the approach of a number of torpedo planes, the target was likely to start turning in an effort to avoid being hit. With modern antiaircraft systems this turning would not materially lessen the ability to put out a strong deterring fire against the torpedo planes. The attacking pilots were thus forced to estimate the correction to be applied to their straight-line type of director in order effectively to fire torpedoes at a turning target, and this correction could often be very large (Figure 1). In order to relieve them of the burden of making this estimate the development of a maneuvering type of torpedo director was undertaken.

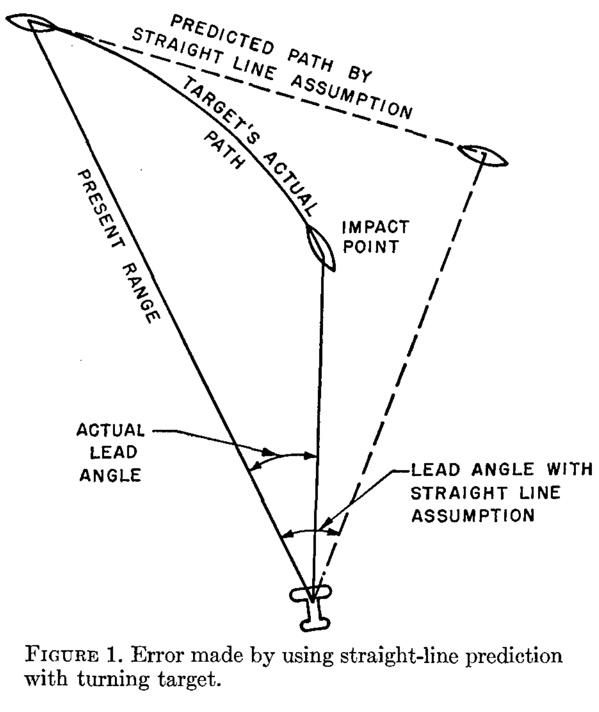

Figure 1. Error made by using straight-line prediction with turning target.

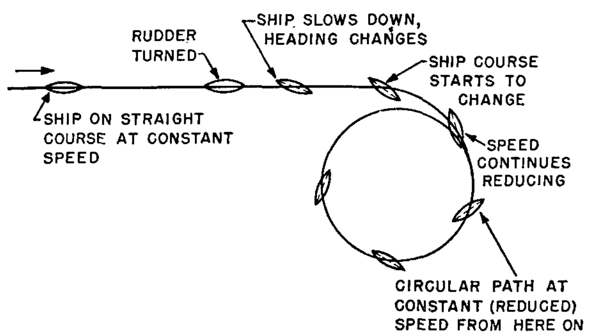

Figure 2. Kinematics of turning ship.

The kinematics of a turning ship have been described in detail in various Navy publications. Qualitatively the sequence of events following the order to execute a turn may be described briefly as follows (Figure 2):

When the rudder is turned, the bow of the ship gradually begins to turn in the same direction. The ship, however, due to its momentum, continues to move in a straight line for an appreciable length of time, say 20 to 30 seconds depending on the type of ship. At the end of this time interval it will be noticed that the ship has begun to change its course or direction of motion. The actual path of the ship then departs more and more from a straight line until the ship has turned approximately 90 degrees, when its path of motion becomes a circle. During all this time the ship's speed has gradually reduced reaching a stable value after about a 90 degree turn. From that time on the ship continues to move in a circle at a constant velocity with its fore-and-aft axis pointed inward from the tangent to the circle. The diameter of the circle, the terminal speed of the ship, the amount of deviation of the ship head-

ing from its direction of motion and the characteristics of the transition to circular motion, all depend on the type of ship, its initial speed and the amount the rudder is turned.

Early attempts at making a torpedo director effective against a maneuvering target consisted of arbitrarily reducing the target speed input to the director and altering by a fixed amount the target angle (Figure 3). The

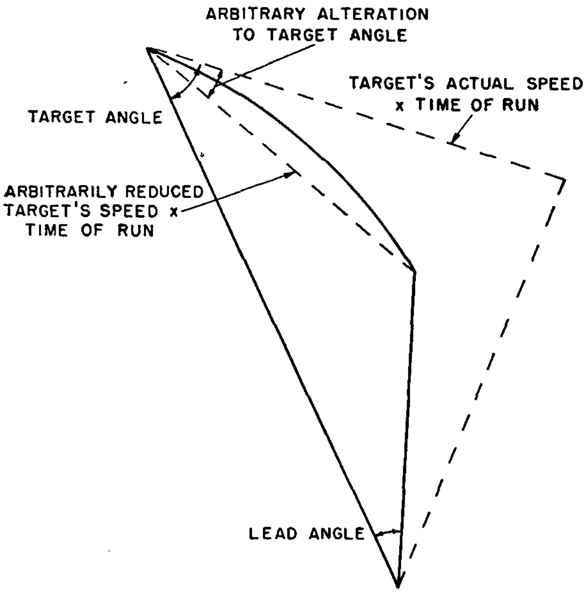

FIGURE 3. Illustrating early method for using straight-line prediction on turning target.

amount of speed reduction and target angle alteration were chosen for a specific set of standard conditions. Any departure from these conditions during the attack caused errors which sometimes were more than the total lead angle.

It was considered that a much better solution was needed if the resulting director was to be of practical use. The Statistical Research Group [SRG] at Columbia University, operating under contract to the Applied Mathematics Panel, was asked to analyze the theoretical aspects of the problem. The first attempt at a solution was to consider a squadron attack. Taking the turning characteristics of the USS *Washington* and assuming that all variables were known exactly, and that there was no dispersion in the missile, the SRG calculated the required number of torpedoes and their lead angles to insure at least one hit regardless of what the target should do after the launching of the torpedoes. It was contemplated that each attacking plane would be assigned a number which designated its position in the squadron and that this number would be one of the variables set into the torpedo director. Each attacking plane then would have a different lead angle depending upon its position in the squadron.

While the early work indicated that this method could be used, it was felt after consultation with the Bureau of Ordnance that reliance should not be placed on coordinated attacks of this sort, since there was too much opportunity for matériel and personnel failures. Instead it was felt that a director should be constructed that would permit the successful launching of a torpedo against a maneuvering target in such a way as to give the optimum chances of hitting for each plane in the attack. With this revised point of view a new analysis was undertaken by the SRG. The first step was to systematize the procedure for calculating lead angles in attacks on turning ships.[47] Since the equations of motion of the ship were unknown, this lead-angle calculation was of necessity done by a semi-empirical method, but a procedure was worked out whereby the number of steps was reduced to a minimum and tabulated so that comptometer operators could grind out the answers.

The next step was to pick the most representative types of maneuvering that a target might be expected to use. It was agreed with the Bureau of Ordnance that all work would be carried out using the turning characteristics of certain classes of American warships. It was felt that with the inadequate existing knowledge of the characteristics of enemy ships, this would probably be reasonably representative of the actual conditions, considering that in battle the number of factors that would be totally or partially unknown would more than outweigh any errors that might be made because of this assumption. Since the purpose of going into a turn would be for the target to do its utmost to avoid being hit, it was reasonable to assume that the turns to be considered would be at essentially maximum rudder angle. Consequently all calculations were made for a rudder angle of 30 degrees.

Inasmuch as the characteristics of a ship just entering a turn are considerably different from those of a ship well into a turn, all turns were divided into two broad classes — developed and undeveloped. It was arbitrarily assumed that developed turns would be all those in which the ship's heading had turned 10 degrees or more from its original direction, and undeveloped turns would be all others. One further assumption was necessary. A ship observed doing 20 knots in a developed turn might have started its turn at a speed of 25 knots, or might have started its turn at a speed of 30 knots, depending on how long it had been turning. There was no way to distinguish between the two cases without knowing in detail the past history of the target motion. Therefore, lead angles were calculated on the basis of an equal probability of the target having started its turn at either of the two speeds. (Check calculations were made on the basis of other probabilities such as 3/4 and 1/4 or 2/3 and 1/3 instead of 1/2 and 1/2. The results were found to be not far different.)

Having decided on the above assumptions and procedures, the SRG then proceeded to compute a series of tables of lead angles for various attack conditions and target speeds, for all target angles, and for three classes of ships:[48] namely, a battleship of the *Washington* class, a carrier of the *Yorktown* class and a light cruiser of the *Philadelphia* class. These lead angles were calculated so as to give the highest probability of hitting for the given conditions. With these tables it was possible to begin the work of mechanization.

The problem of mechanization was rendered more difficult by the fact that no formulas existed. It was necessary to devise a mechanization which would fit with sufficient accuracy the tables of data computed by SRG. It was considered more important to have an accurate solution for the case of a developed turn than for an undeveloped turn. Therefore, the efforts were concentrated on the former. Many approaches to a solution were possible. The following were the important ones that were considered:

1. To build a brand new mechanism which would solve the lead angles for curved target paths. This method would mean abandoning the results of all the development work that had been done for torpedo directors for straight-line target motions. Inasmuch as the final director should, as a special case, compute lead angles for straight-line motions, it was felt desirable to use this type of director as a foundation. Consequently, although work was done on this method, principally by expanding the lead angle in a Fourier series, it stopped when it soon became apparent that the results were not leading to an obviously simple mechanization.

2. Another line of attack was to assume that a director existed that would accurately solve for the straight-line lead angle and to add to that another mechanism giving the correction for a curved path. This might have worked out satisfactorily if sufficient time and personnel had been available to continue the work to completion. However, the third approach showed more promise and it was decided to concentrate on it.

3. Mechanisms already existed for solving the lead angle problem for a straight-line course, as stated above. The question was asked, is it possible to determine fictitious values of input variables to such a mechanism so the output lead angle may be correct for a curved-target course rather than for a straight course. If this could be done then the resulting torpedo director would consist of a straight-line type of computer with an auxiliary computer that would add corrections to certain of the inputs when it was observed that the target was actually turning. Examination of the tables of data soon showed that mathematically speaking it was always possible to determine a fictitious value of target angle in such a way that, when used instead of the actual target angle in a straight-line type of computer, the resulting output would be the correct lead angle for a curved course (Figure 4). It was[45] also found that it was not possible under all circumstances to find a fictitious target speed that would accomplish the same results. However, an appropriately chosen fictitious speed together with a fictitious target angle could be used. (A special case of this last method was that described above as the first attempt to produce a curved-type fire director, that is, to reduce arbitrarily the target speed by a fixed amount and alter the

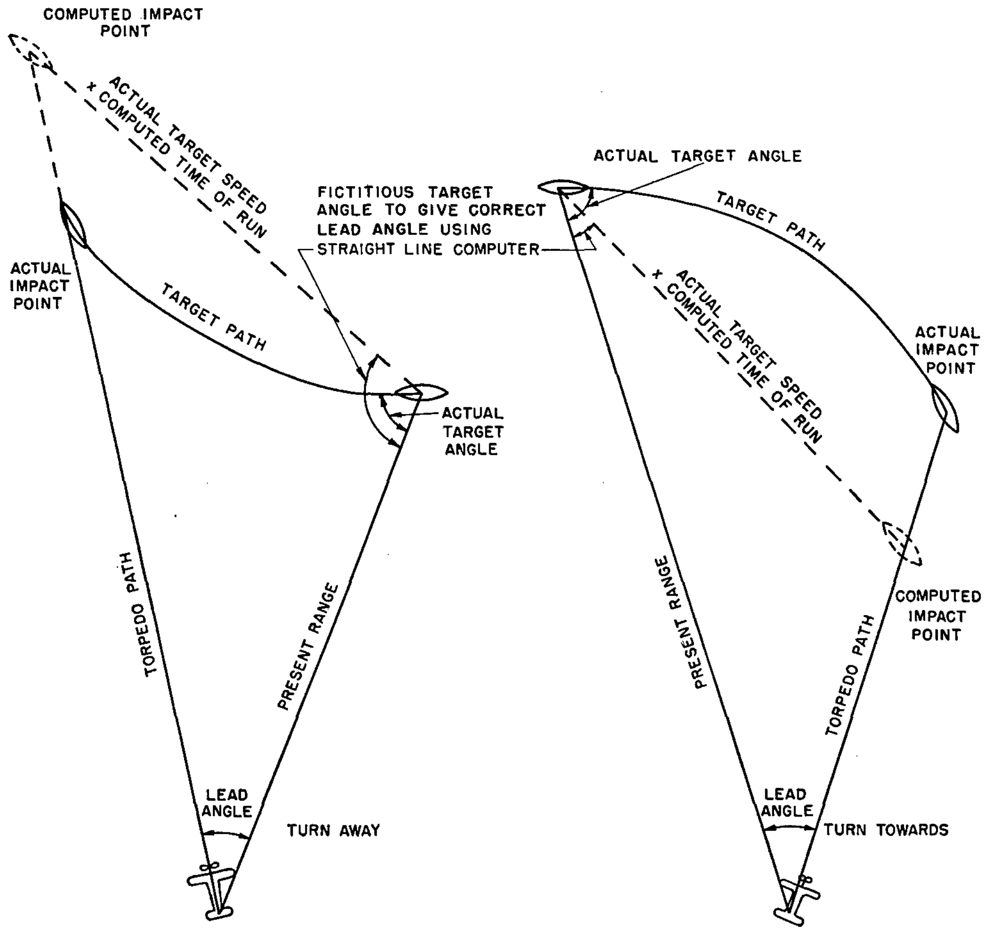

FIGURE 4. Use of fictitious target angle with straight-line computer to give lead angle for turning ships.

target angle by a fixed amount. However, in the special case the results were valid for only one set of input conditions, whereas this development contemplated giving an accurate director for all input conditions.) Consequently, work was carried on for the first and third of these possibilities.

The early lines of attack consisted in expansion in Fourier series. Considerable effort was put into this to bring the coefficients into tractable forms, but the results obtained were not simple when sufficient terms were used to give what was felt to be a reasonable accuracy. Consequently, this expansion method was abandoned in favor of empirical curve fitting.

One rather accurate solution was obtained utilizing both a fictitious target speed and a fictitious target angle. But it was apparent that to build a director for such a solution would result in an exceedingly complicated mechanism with many problems to solve before it could be made to work satisfactorily. So nothing further was done on this.

The major effort was then concentrated on finding a suitable mechanism to give a fictitious target angle to be used in a straight-line director. The variables involved were as follows:

τ = target angle
V_s = target speed
V_t = fixed torpedo speed
R = present range
h = altitude
V_a = ground speed of launching plane
S = type of target ship
τ' = fictitious target angle as the output

(Other variables were eliminated as a result of the assumptions and simplifications made as described earlier.)

It was found that the launching plane's altitude and ground speed could be combined in a

single variable which represented the difference between the horizontal projection of the air travel of the torpedo in yards and the water travel of that torpedo for a time equal to the time-of-fall. (This was labeled A, and was equal to $\sqrt{2h/g}\ (V_a - V_t)$.

Two empirical formulas were derived using these variables to give a fictitious target angle. The first of these was the sum of arbitrary functions of certain linear combinations of the input parameters, as follows.

$$\tau' = \tau - x + 197.1 - 0.099R + 0.101(A - 291) + F_3(V_s) + F_4(S),$$

where x is defined by the equation

$$F_1(x) = F_2(\tau) + 200 - 0.1\,(R - A + 291);$$

and all the F's are empirical functions.

The second was the product of an arbitrary function of the sum of some of the variables and an arbitrary function of the sum of the target angle and others of the variables, as follows:

$$\tau' = 0.00047 \cdot [4900 - R + 1.5A - 60V_s + F_2(S)]\, F_1(Q)$$

where Q is defined by the equation

$$Q = 67.3 + \tau + 0.035(A - R) - V_s + F_3(S)\ ;$$

and all the F's are empirical functions (not the same as those in the first formula). Both of these could be mechanized fairly simply. Since the second, however, gave appreciably less error in the overall result, it was decided to use this formula as the basis for the final mechanization.[45]

Work was just starting on the actual construction of this director when it was terminated as a result of the end of the war. As stated earlier the efforts were concentrated on an accurate solution for developed turns. It had been hoped to find a simple addition to the mechanism which would give the answers for an undeveloped turn. This has not been carried through to completion. However, it is not felt to be worth while to spend too much time to find a solution or to permit much additional complication to take care of this case. There are too many unknown factors entering into the problem and too many guesses that the attacking pilot has to make to warrant excessive accuracy at the price of excessive complication.

Had the director been completed its form and method of operation would have been substantially as follows.

In front of the pilot there would be an optical system, perhaps similar to that of the torpedo director Mark 30, through which the pilot would observe the target and which would lay off the proper lead angle. This lead angle would be received from a remote computer by means of a servomechanism. Adjacent to the pilot would be control knobs for the input variables. Before making the approach the pilot would set his anticipated ground speed and altitude of attack. He would also adjust the turn knob to indicate a straight-line course. On sighting the target, the pilot would enter his estimate of target speed and the target class. He then would set target angle so that the small model adjacent to the optical system would line itself up parallel to the actual target. This model would then be clutched to a directional gyro so that once set it would retain its direction in space regardless of the plane's maneuvers, and thus always remain parallel to the target until the target should change course. As soon as it was observed that the target was in a turn rather than on a straight course, the turn control would be set to indicate a clockwise or counterclockwise turn.

The pilot would then grasp the target angle rate-knob (which would be mounted concentric with the target angle knob) and adjust it so that the ship model would turn at a rate equal to the target's turn. Thus the target angle would be continuously changing at the right speed and would keep set properly, allowing the pilot to concentrate on last-minute details of getting into proper position to launch his torpedo.

If the director received range automatically from a radar, the pilot might drop the torpedo at any time he desired. If the range were set manually, the pilot would have set the desired dropping range early in the attack and would launch his torpedo when his plane reached that range.

As soon as the torpedo was launched, the pilot would turn sufficiently from his course so as to avoid as far as possible the target's antiaircraft fire and get his plane sufficiently far from the target to get him away from the blast of his torpedo as it hits.

PART III

AERIAL GUNNERY

PREFATORY COMMENTS

THIS REPORT gives a summary of the field of aerial gunnery, and particularly that part of the field with which Section 7.2 of NDRC was primarily concerned. During World War II, by far the largest part of the development work in this field was on flexible gun systems for bomber defense so that very little is said in this report about fixed gun systems. This is not intended to imply that fixed gun systems are less important than flexible gun systems. On the contrary, the success of fighter-bomber airplanes has made the development of adequate fire-control equipment for such airplanes extremely important, and in particular, equipment that can be used with several weapons such as guns, rockets, and bombs.

The objective of this report is to acquaint the reader with the important aerial gunnery equipment used or under development during World War II and the various problems encountered in the development and evaluation of this equipment. The best way to have met this objective would have been to make this report a self-contained text book on aerial gunnery. This was impossible for many reasons, the principal one being lack of time. The method chosen was to give a descriptive account of the problems encountered and the work done and to provide the reader with a carefully selected bibliography. This bibliography contains only a small fraction of the technical reports available on the subject but will be sufficient to provide the interested development engineer with the background on which to build future work. There are two other volumes of the Summary Technical Reports of NDRC containing material on aerial gunnery. The first is that on *Military Airborne Radar Systems*[1] prepared by the Radiation Laboratory of Massachusetts Institute of Technology, and the other is on *Analytical Studies in Aerial Warfare* prepared by the Applied Mathematics Panel of NDRC.[2] In addition there is an excellent summary of the work in aerial gunnery which was done by the Applied Mathematics Group of Columbia University[3] and a bibliography of reports prepared by that group.[4] All of this material should be given careful consideration in any future fire-control work.

The contributions of Section 7.2 of NDRC to the field of aerial gunnery consist mainly of the coordination of the work of the Navy, the Army Air Force and the various civilian organizations concerned, and the development of methods and equipment for assessing aerial gunnery systems. In both cases the initiative for the work came largely from the section. There was such a variety of gunnery equipment under development in the Services and in industry that the section undertook very little of this work. The one major equipment lack was in the field of assessment, and the section put its major equipment development effort into correcting this situation. The coordination work in aerial gunnery done by the section was a substantial part of its effort in aerial gunnery and was brought about principally because there was no effective planning and coordinating agency in the Army Air Force for this purpose. The two full time members of the section primarily concerned with aerial gunnery were the writer, who was a member of the section from September 1941 until he resigned to join the office of the Secretary of War in May 1945 and Dr. H. C. Wolfe, who was with the section from June 1944 until its termination early in 1946. The work in aerial gunnery was started by the section's chairman, Dr. S. H. Caldwell, when the Fire Control Section of NDRC (now Division 7) was first organized in 1940. He continued his able leadership in this work in spite of the increasing pressure of other section responsibilities. It should also be noted that even though the writer and Dr. Wolfe were the only full-time members of the section devoting their time to aerial gunnery, all the other members of the section contributed to the work in varying degree and the work would have been much less successful without their assistance.

The development work of the section in aerial gunnery was carried on almost entirely by six contractors. Reference is made to their work throughout the body of this report. The fol-

lowing is a list of these contracts with the corresponding service directive numbers and an indication of the type of work.

1. OSRD Contract OEMsr-330; Directive NO-268.
   Contractor: The Franklin Institute, Philadelphia, Pa.
   Work: Tracking Studies, Own Speed Gunsight.
2. OSRD Contract OEMsr-732; Directives AC-47, AC-128, NA114.
   Contractor: University of Texas, Austin, Texas.
   Work: Development and construction of testing machine for flexible gunnery systems and tests on such systems.
3. OSRD Contract OEMsr-991; Directive NO-152.
   Contractor: Jam Handy Organization, Detroit, Michigan.
   Work: Development of own speed type of assessor and gunsight.
4. OSRD Contract OEMsr-992; no directives.
   Contractor: General Electric Company, Schenectady, N. Y.
   Work: Development of improved Central Station Computer.
5. OSRD Contract OEMsr-1237; Directives AC-36, NO-265.
   Contractor: Columbia University, New York, N. Y.
   Work: Development and use of electronic equipment for simulating Airborne Fire Control Systems.
6. OSRD Contract OEMsr-1276; Directives AC-119, NA-161.
   Contractor: Northwestern University, Evanston, Illinois.
   Work: Development of equipment and methods for the assessment of aerial gunnery equipment and the installation and operation of such equipment for the Navy.

In addition to these contractors, Section 7.2 supervised the work of several other contractors doing work for NDRC under the general direction of other divisions. The most important of these are the General Electric Company, the Sperry Gyroscope Company, and the Fairchild Camera and Instrument Company, all of whom were developing computers or other equipment for airborne central station fire-control systems.

The section acted as a consulting group to the Navy and the Army Air Force on airborne fire-control matters and advised many members of both Services on special problems. One of the best examples of this work is the participation of the section in the Joint Army-Navy-NDRC airborne fire-control committee. This committee was organized early in 1944 primarily to organize and direct the aerial gunnery assessment program of the Army Air Force and to coordinate this program with a similar one being undertaken by the Navy with the assistance of Section 7.2. Under directive AN-5 the section was asked to provide the leadership of the committee. The section asked the writer to undertake this work and he was the committee's chairman from its organization until its work terminated at the end of the war. The chairman was very ably assisted in this work by Dr. Saunders MacLane of the Applied Mathematics Group of Columbia University who was vice-chairman. After the gunnery assessment program was well under way, the committee became largely a technical forum on airborne fire control which met once each month. It was well attended by key people from both the Army and the Navy. While the committee held no authority, it received excellent cooperation from its members and succeeded in contributing substantially to the coordination and direction of the fire-control development work of both the Army Air Force and the Navy.

JOHN B. RUSSELL

Chapter 14

GENERAL SURVEY OF AERIAL GUNNERY

14.1 SITUATION AT THE START OF WORLD WAR II

When this country entered the war in December 1941, there was available practically no gunnery equipment suitable for fighting the aerial battles. Fighter planes had the new .50-caliber machine gun but no suitable gunsight to use with it. The only fighter gunsight available at that time was the old ring-and-bead sight. This ring-and-bead sight was subject to parallax errors due to the motion of the pilot's head. However, what was more serious was the fact that this sight was only a means for laying off an estimated amount of lead and was of no aid to the pilot in computing the correct amount of lead. The parallax error was eliminated before long by the introduction of a reflector-type sight. This sight produced, by optical means, an illuminated reticle which appeared to be at infinity. This gave the pilot a reticle whose direction and size were independent of the position of his head.

It was not until nearly the end of the war that a computing sight was available for the fighter airplane. This was the single-gyro type of sight developed by the British and known by them as the MK II-d. This sight had originally been developed for use with flexible guns for bombing defense, but the greater importance of fighter gunnery caused them to adapt it to the fixed-gun fighter airplane. The Army Air Force was not particularly enthusiastic about this sight because it was felt that certain inherent errors were too large. The British, however, defended the use of this sight on the basis that in spite of these errors it gave the fighter pilot a means of increasing the accuracy of his shooting several times. The Army Air Force was finally convinced of this fact through the efforts of the Navy and Section 7.2 of NDRC, and eventually adopted an American version of it in their own fighter plane. This Army Air Force version was known as the K-14 sight.

During the early phases of World War II very little development was done in this country on a fighter's gunsight. As with the British, most development work on gunsights was done on sights which were intended for use in flexible gunnery. In the last phase of the war, development work was started on a sight intended for fighter gunnery. This has resulted in the development of the A-1 gunsight which can be used not only for gunnery but for rocketry and bombing as well. In fact, the developments in the use of fighter-bombers during the war have shown the necessity of using a computing sight in the fighter which can be used for all the attack functions of the fighters.

The early B-17's had practically no defensive armament. There were no gun turrets, all guns being in a flexible mounting and hand held. There were no guns in the tail. The only sight used was a ring-and-bead sight which was subject to bad parallax error. This error was much more severe in the use of flexible guns than it was in the use of fixed guns in a fighter. In addition, the problem of estimating the lead required to hit an attacking fighter was a very difficult one. At the beginning of the war this country had no adequate rules for estimating this lead, no suitably trained gunners or any adequate means of training them. Experience gained by the British in the European Theater showed the necessity for a much greater defense of heavy bombers than was available on the B-17 or the B-24. As a result, twin gun turrets were developed for use on both these airplanes and guns were installed in the tails. Even with these additional guns and improved mounts, only a fixed sight was used. These were optical reflecting sights which eliminated the parallax error by providing an illuminated reticle projected to infinity. The control of the guns was considerably improved in the case of the turret mount by the use of a power control. In fact the added weight and size of the turret mount over that of the simple hand-held mount made it impossible for the gunner to exercise control without the assistance of some sort of power drive.

The first methods which were taught and used for estimating lead were of the so-called apparent-speed type. The gunner was expected to determine the relative motion of his target during some estimated interval of time and then set a lead computed from this observation by the use of a rule. Gunnery schools taught this method and gave the gunners practice in it largely on ground ranges. Very little time was devoted to practice in the air. A great deal of the practice on the ground was in the form of skeet shooting. This certainly gave the gunner a fair amount of experience in handling and firing his gun and in estimating lead required by a moving object fired at from a stationary platform. However, the effect of this practice was to make him lead the target in the direction in which the target appeared to be moving through the air. When this was transferred to the situation of an actual aerial combat, it often caused him to lead an attacking enemy fighter in the direction in which the fighter was moving in the air. This lead was in exactly the wrong direction because even though the fighter may have been headed in the same general direction as the bomber being defended, the actual relative motion of the fighter was usually toward the tail of the bomber. The lead should always have been in the same direction as the relative motion of the target. This fundamental error was the cause of many of the misses experienced during the early days of the war.

14.2 POSITION FIRING RULE

The British were the first to introduce a sound method of estimating the lead required to hit the attacking enemy fighter. It was known that a fighter would almost always attack a bomber in such a way that the fighter's guns would bear on the bomber during as large a portion of the attack as possible. This caused the fighter to fly a curved course approximating that known as a pursuit course. For such a course, if the fighter's true airspeed and the bomber's true airspeed were known reasonably accurately, the required lead for hitting the fighter was known and would be the same for every such attack. Furthermore, the magnitude of that lead would depend, approximately, only on the bearing angle of the fighter relative to the bomber's line of flight. The direction of this lead would be in the plane of action of the fighter, which was determined by the position of the fighter and the line of flight of the bomber. This led to the British *zone firing method* of estimating leads for bomber defense. The U. S. Navy and the Army Air Force followed the British lead and developed a similar method known as *position firing.* This method was eventually used for the defense of all U. S. Army bombers until some sort of a sighting mechanism could be provided. Subsequent tests and analyses showed that the position firing rule, while not particularly accurate, was more accurate than any of the methods previously used. It was not until almost the end of the war that anything like an adequate gunsight was available for all the various gun positions of the B-17 and B-24.

One improvement in the use of the position firing rule, which came just before the end of the war, was the introduction of the K-13 sight. This sight was eventually to be put into all gun positions not equipped with a computing sight. The K-13 sight was simply a mechanism for computing the lead required by the position firing rule, and indicating it to the gunner automatically. The biggest error in the use of the position firing rule was due to the inability of the gunnery to accurately estimate the angle off, or bearing angle, of the enemy fighter and to lay off the corresponding lead. The K-13 sight did this for the gunner and gave him an opportunity to do a better job of tracking and of estimating range. The K-13 sight was subject to all the errors inherent in the position firing method.

14.3 LEAD-COMPUTING SIGHTS

The only computing sight for flexible gunnery which saw any use during the war was one of the various forms of the Sperry lead-computing sight. The early models of this were the K-3 and K-4 sights which were used in the upper and lower turrets of the B-17, respectively. This sight computed a lead in terms of the range to the target and its relative angular velocity with respect to the bomber. It gave a lead which was satisfactory for a variety of

types of attack. One of the fundamental disadvantages of the position firing method was that the leads determined by this method were correct only for one type of relative course on the part of the attacking fighter. As soon as the fighter deviated from this course the leads given by the position firing method were in error by a very large amount. The K-3 and K-4 sights, however, were able to give leads which depend primarily on the relative motion of the target and, therefore, were usable in a wider variety of situations. They also provided leads whose accuracy depends on the actual course flown by the target but to a much less extent than for the K-13 sight.

Considerable work was done during the war on the development of an improved lead-computing sight. The British led the way with the first major improvement by introducing the single-gyro type of sight which they call the MK II-c and which was later modified for use as a fighter sight. In this country, most of the development effort put on sighting systems for flexible guns was devoted to elaborate remote-control systems. The benefit to be gained by a suitable lead-computing sight for use on planes such as the B-17 or B-24 was not fully appreciated. When the British MK II-c was first demonstrated to the Army Air Force it met with surprising lack of interest. It was largely through the efforts of Section 7.2 of NDRC that the Army Air Force finally gave the MK II-c serious consideration. This sight provided the same type of lead computation as the Sperry lead-computing sights, with one important added advantage; namely that it computed the lead in terms of the actual angular rate of the target relative to the bomber and, therefore, was not affected by any angular motion of the bomber. It did not have all the accuracy that the Army Air Force desired but would provide a much more accurate means of shooting than had previously been available. The principal source of inaccuracy of both the Sperry and the single-gyro lead-computing sights was the inability of the gunner to range on the target and track in direction simultaneously with sufficient accuracy.

Another improved lead-computing sight was one developed by Fairchild and was known as the K-8 gunsight. This sight was in production at the end of the war, and a few had been installed and put into use before hostilities ended in the European Theater. The major difference between this sight and the others was that it made use of electrical circuits for computing time of flight and the ballistic corrections. It did not use a gyro and therefore was subject to the same type of errors as the Sperry K-3 and K-4. However, work was being done on the development of a gyro for use with this sight.

14.4 TURRET CONTROLS

The early turrets, while power driven, had only very simple hand controls for controlling the pointing of the gun. The simplest gun mount, which was the hand-held mount, was position controlled. This means that the gunner grasped the rear of the gun and by his own effort pointed it in the desired direction. The force necessary to move the gun and hold it against the force of the wind stream had to be provided by the gunner. This made it very difficult to point the gun accurately. The use of power control made it unnecessary for the gunner to exert any great amount of force in controlling the position of the gun. It also made it possible to use a larger and heavier mount which would provide the gunner with a seat which carried him around with the gun. The usual type of control for such turrets was a rate control in which the angular rate of motion of the gun was more or less proportional to the displacement of the control handle. Each manufacturer had his own choice for the rate-control constant. As a result, some turrets were much easier to control than others. Also, this made it difficult to do satisfactory tracking with some turrets.

At first very little was known about what the turret control design should be in order to provide optimum tracking. It was felt by many that a slightly more complicated turret control, known as aided tracking, would provide the gunner with a greatly improved means of doing accurate tracking. Section 7.2 undertook to study this situation by means of a tracking study carried out at The Franklin Institute. This study indicated that the use of aided tracking would greatly improve the accuracy of tracking with power-driven turrets. However,

it was impossible to make use of this knowledge before the end of the war. New turret controls which were under development during the latter part of the war were to include the aided tracking feature.

An additional refinement which was to be installed in these new turrets, with very little increase in space or weight, was the stabilization feature. Even with power control of turrets it was necessary for the gunner to operate the controls in such a way that the motion of the gun not only followed the relative motion of the target with respect to the bomber, but also compensated for any angular motion of the bomber itself. This meant that in rough weather or during evasive action it was difficult for the gunner to do satisfactory tracking. As a matter of fact, the accuracy of tracking obtained under these conditions was so bad that shooting was almost entirely ineffective. By stabilizing a turret, it is possible to automatically eliminate the effects of a bomber motion. This leaves the gunner with only the problem of tracking the relative motion of the target. Tests of experimental turrets under simulated conditions show that the marked improvement in tracking accuracy expected is actually attained.

14.5 CENTRAL STATION SYSTEMS

The development of the B-29 brought up a new and difficult problem in bomber defense. The large size of the airplane called for a large number of gun positions and more elaborate arrangements for coordination between these positions. The problem of lead computation was made much more difficult because of the higher speed of this bomber. The use of locally controlled turrets appeared impractical because of the decision to design the B-29 so that it could be used at high altitude under pressurized conditions. It was primarily for this reason that a remote-control type of system was chosen for the defense of the B-29. This called for sighting stations within the pressurized sections of the plane and a number of remotely controlled gun turrets which were in the unpressurized sections of the airplane. An elaborate computer was necessary to compute the required kinematic lead, the ballistic correction, and the correction for parallax due to the large distance between the sighting station and the turret under control.

Two major developments were carried out to provide an adequate fire-control system for the B-29. One of these was carried out by the Sperry Gyroscope Company and the other by the General Electric Company. Before either of these developments was completed, the Army Air Force decided to go into production on the GE system. As a result, the Sperry development was never entirely completed. Work was carried out to the point where a developmental model was completed and in operation. This model was never tested by the Army Air Force. However, it appeared to have several features which were superior to the GE system put into production, and which should be reconsidered in any future development work.

The GE system was finally produced in adequate quantities and saw service in the air war over Japan. While it appears to have done an adequate job under the conditions which existed, it was far from a satisfactory system. The sighting station was difficult to control and this resulted in rather poor ranging and tracking accuracy. The lead computation suffered from serious time delays and was limited in many ways by the particular design chosen. The computer was large, heavy and complicated and introduced a rather serious maintenance problem. While there are many features of this system which should be retained, it is probable that at the present time a more useable and more accurate system could be built which is considerably lighter, much smaller in size, less expensive and far easier to maintain. A very large amount of time was spent on getting the bugs out of the initial GE production and in adjusting the system to give satisfactory accuracy. Considerable development work was done near the end of World War II to improve the GE system by simplifying the computer and by providing a more adequate sighting station.

14.6 TRAINING

All through the development work during the war, very little attention was paid to the human factor. In all the gunnery systems which

saw any use during the war, the ability and skill of the gunner was the predominant factor in determining the overall accuracy of the system. Little information was available to the designer on the effect of the control system design on the ability of the gunner to use the equipment. Many designs, such as that of the sighting station on the B-29 system were adopted and put into production before any information was available on the ability of the gunner to use the equipment or the difficulty in training him in its use. It is now recognized that closer cooperation between the training people, psychologists, and the equipment designers would have resulted in the production of equipment which could be used with much greater accuracy. Another difficulty which arose was due to the fact that the Units in the Army Air Force responsible for training gunners seldom saw the equipment which the gunner was supposed to use until after it had actually reached an operational theater. This meant that the theaters were often receiving gunners who had had no adequate training with the equipment they were to operate. It also meant that equipment was often sent to the theaters in which the time required for adequate training was much too great. An additional difficulty due to this lack of cooperation with the training people was the fact that they were unable to develop and build training equipment in time to start training in the use of this new equipment. Many of these problems were on the way to being solved toward the end of the war, and it is hoped that this experience will not be forgotten in the future.

14.7 ASSESSMENT

Probably the most important reason for the lack of adequate direction of the development program in aerial gunnery was due to the nonexistence of any adequate method for assessing the performance of such equipment. The first program for providing such assessment facilities was started by Section 7.2. This section undertook to develop an elaborate testing machine at the University of Texas. This machine would handle a complete local turret system, including the gunner. An artificial target was provided which could be made to fly any chosen attack course. Simple motion of the gun platform or bomber could be simulated. A means was provided for measuring the accuracy of the final gun position at every point during the attack. Conditions were reproducible so that any attack could be duplicated for various systems and with various gunners. This machine provided an accurate way of determining the overall performance accuracy of the flexible gunnery systems. While it did not simulate many of the psychological conditions which existed in the air under combat, it did provide a quick and accurate means of assessing certain systems. After the original machine was completed, two additional machines were built, one which was installed at Wright Field and the other at the Naval Air Station, Patuxent River. These machines made it possible to do most of the assessment work which is necessary for studying and evaluating experimental and final models of flexible gunnery systems. It is safe to say that if these machines had been available at the beginning of the war, the present state of flexible gunnery would have been far superior to what it is at present and many man-hours of effort and considerable money would have been saved.

A second method of assessment, which was finally developed, was one which could be used in flight testing flexible gunnery equipment. This system was developed under the general direction of the airborne fire control committee and is now in use at both the Army Air Force Proving Ground, Eglin Field and the Naval Air Station, Patuxent River. The equipment under test is flown and operated under simulated combat conditions with an actual fighter airplane making simulated attacks on the bomber. It provides operating conditions which are much more realistic than any which could be provided on the Texas tester. Most of the data was recorded photographically and later analyzed. Several methods were developed for the analysis of this photographic data, one depending largely on the use of charts, and the other on the use of special machines developed by the Northwestern Technological Institute.

A third method of assessment, which reached a high state of development during the war, was that of mathematical analysis. The Applied

Mathematics Groups at Northwestern and Columbia Universities carried out mathematical analysis of a number of production and developmental gunnery systems. Such analyses are extremely valuable in predicting errors while a system is in a design stage, and in determining the various sources of these errors. These analyses led to improved designs and calibrations of such equipment. All three methods of assessment should be used in any future development of aerial gunnery equipment.

14.8 RADAR

Radar played a very important part in the winning of this war. However, its effect on the success of aerial gunnery was almost nil. Very early in the war a large amount of effort was put into developing a completely automatic tracking and ranging radar for use in the defense of heavy bombers. A system was developed which was satisfactory from a performance standpoint. However, it was complicated, costly, and large and was therefore never used. Later, simpler systems were developed which could be used for determining range only and later others were developed for manual tracking in position also. The development of a complicated, costly, automatic system at the beginning of the war considerably delayed the development of simpler systems which might have been put into use during the war. At the close of the war, the simpler systems had been put into production. The whole gunnery radar development program suffered very seriously from the lack of adequate direction from the Army Air Force and the lack of good coordination between the fire-control and the radar groups.

14.9 PLANNING AND CONTROL

Now that the war is over, future development work on aerial gunnery systems should be done with far more coordination and direction than existed during the war. Most of the leadership and initiative in such development work during the war came from industry. There was no adequate group within the Air Force which could direct the development of such equipment and provide an integrated program. The Armament Laboratory at Wright Field, which was charged with this responsibility, had neither the number nor the quality of personnel required to provide the necessary direction. Furthermore, in the early part of the war one or two individuals in authoritative positions were unable to see the advantages which might be gained by receiving the cooperation of technically competent civilians. This made it difficult to get the most from the available development facilities and considerably delayed the work of developing new gunnery systems. In fact, it is probable that more competent direction from the Army Air Force during the development of the GE system would have resulted not only in a better system but in a considerable saving in time and money. The success of any future development of this sort depends on the ability of the Army Air Force to build up and maintain a competent development group which can plan and direct such a development program.

Chapter 15

GENERAL PRINCIPLES

15.1 TARGET COURSES

THE ACCURACY of all fire-control devices depends in a varying degree on the course traversed by the target. In the field of aerial gunnery this is particularly true. Fortunately, the general type of course that an attacking fighter may fly and at the same time do effective shooting at a bomber is rather limited.[a] This philosophy has led to the development of a simple gunsight known as a vector or own-speed sight which is designed to operate against the most common type of attack, which is a pursuit course attack. The more complicated lead-computing sights and remote-control systems are, the less dependent are they upon the actual course flown by the target, but they are still sensitive to it to a certain extent. As new weapons are developed and put into use against attacking fighters, the actual attack course used by the fighter will change. This will make it necessary to alter the design of gunsights used for the defense of bombers, either to adapt them to these new courses or make them less sensitive to the type of target course.

In order to understand the way in which a gunsight is affected by the particular course flown by the target, it is necessary to understand something about the various attacks that can be flown by an airplane and how these affect the relative course of the target as viewed from the bomber being defended. During World War II, with few exceptions, fighter airplanes carried fixed guns. These guns were pointed approximately along the line of flight of the fighter airplane. They were elevated above this line of flight by a small amount to allow for the effects of gravity. During an attack the fighter was flown in such a way that during the most important portion of this attack, which was when the fighter was in effective range of the bomber, the fighter's guns were continually bearing on the bomber. The bomber under attack was normally flying a straight and level course. This was particularly true if the attack was being made while the bomber was in formation or on its bomb run. On such a course the bomber presented the fighter with a much easier target than if the bomber were taking evasive action. If the bomber for any reason had dropped out of the usual formation and was by itself, it would generally take evasive action when under attack from an enemy fighter. This made it much more difficult for the attacking fighter to get hits and was often sufficient to allow the bomber to escape. The determination of the course of the bomber and of the attacking fighter during actual combat conditions was very difficult. In fact there were no accurate means for observing the relative motion of the two planes during combat. Observations of an attacking fighter by gunners on the bomber under attack proved to be completely inadequate to give more than a very rough idea of the course of the attacking fighter.

Since it was necessary to know something about the course of an attacking fighter to properly design a gunsight for use on the bomber and to get some idea of its accuracy, it was necessary to study the operation of such sights assuming idealized or simplified fighter attack courses. The simplest course which could be conceived was that for which the attacking fighter was flying a straight-line course relative to a bomber which, itself, was flying a straight and level course. The first analyses that were made of the performance of lead-computing sights were for this type of straight-line course. It was obviously very unrealistic but served to provide some understanding of the performance of the sight. The simplest type of target course which is reasonably realistic is the pure pursuit course. For this course it is assumed that a bomber flying a straight-line course at constant speed is under attack by a fighter flying at constant speed in such a way that at all times the fighter's line of flight is pointed directly at the bomber. These assumptions do not allow

[a]This may not continue to be true in the future if turreted fighters come into use.

for the variable speed which always occurs during a fighter attack nor for the lead which the fighter must take in order to get a hit on the bomber. However, the pure pursuit course approximates an actual fighter attack course sufficiently well that a fair understanding of the operation of a flexible gunsight can be obtained by studying its use against such a course. Very early in the war an analysis of the pure pursuit course was made.[5] General equations were derived, completely specifying the relative positions of the bomber and the fighter for this situation. Tables were prepared giving all the numerical data of interest on a variety of such pure pursuit courses, including the lead required by the relative motion of the attacking fighter for accurate shooting from the bomber.

Because it was known that an attacking fighter never actually flew a pure pursuit course, there was always the question as to how good an approximation was being made by assuming that the fighter would fly such a course. The lack of any reliable experimental data made it necessary to attempt to answer this question and get more reliable information on the courses actually flown by using analytical methods. The two important factors which are not accounted for in the assumption that the fighter flies a pure pursuit course are the effect of the lead which had to be taken by the fighter, and the effect of the aerodynamic properties of the fighter airplane. It was known that the fighter's line of flight did not coincide with the axis of its guns and, in fact, that the angle between them depended on such things as the airspeed of the fighter, the curvature of its actual flight path, the loading of the airplane, and skid. One of the early attempts to predict the course actually flown by the fighter was for the case where its guns are continually pointing at a bomber in straight-line flight at constant speed. This course is known as an *aerodynamic pursuit curve*. If, in addition, the fighter flies in such a way that it always maintains the correct lead on the bomber under attack, it is said to be flying an *aerodynamic lead pursuit curve*. In each case it is assumed that the fighter is being flown perfectly, although the aerodynamic properties of the fighter airplane are being taken into account. One of the earliest attempts to derive the equations for the aerodynamic lead pursuit curve is described in reference 6. A complete set of equations[7] specifying the aerodynamic pursuit curve has also been derived. The solution of these equations for a simple case, namely an overhead attack, was made[8] by the Applied Mathematics Group at Columbia University. They computed a number of actual courses for American fighters and finally made a fairly complete solution of the aerodynamic lead pursuit course problem.[9] They eventually computed a number of aerodynamic lead pursuit courses and gave the leads which must be taken by a gun on a bomber under attack in order to get hits on a fighter flying such a course.[10] These courses represent the most complete information on pursuit types of attacks which was available at the end of the war.

While the Applied Mathematics Groups at Brown University and Columbia University were most active in the analysis of pursuit courses, several other organizations made studies of this problem. The Mount Wilson Observatory prepared a set of charts of pure pursuit curves which were very useful.[12] The Douglas Aircraft Company made a very interesting study of pursuit-course attacks on high-speed bombers.[11] They showed that with a high-speed bomber, the regions of attack might be sufficiently reduced that the bomber need only defend itself in the nose and tail cones. This was brought about by the fact that the types of pursuit attack which were possible were greatly limited by the effects of the high acceleration produced by the curvature of the attacking fighter's course. The General Electric Company was interested in the types of attack that could be made against the B-29 and made an analysis of some of these attacks.[13] The Jam Handy Organization was interested in the analysis of attack courses for two principal reasons. They were manufacturing training films for the Navy and the Army and wished to make the attack shown as realistic as possible. Also, they had under development a vector or own-speed type of gunsight. The results of some of their analyses are given in reference 14. The Jam Handy Organization also carried out a flight experiment in cooperation with the Navy, to measure the actual courses flown by an attacking fighter. The course data obtained from this experiment are not too accurate but

did compare favorably with course data computed analytically.[15]

During the latter part of the war both the Army Air Force Proving Ground Command at Eglin Field and the Naval Air Station, Patuxent River obtained a large amount of data on courses actually flown by attacking fighters. These data were obtained under excellently controlled conditions and with rather elaborate test equipment. They are also the most accurate information on courses actually flown which was available at the end of the war. Some discussion of the method used will be described later.

As has been mentioned, one reason for desiring to know the actual course flown by an attacking fighter is to determine the various factors of design in a flexible gunsight which is to be used to shoot down the fighter and to analyze the accuracy of this shooting. Another reason is to investigate the various regions of possible attack by a fighter and thereby determine the regions in which a bomber is most likely to be under attack.[11] Reference 16 gives a good summary of the status of this problem at the end of the war.

15.2 BALLISTIC LEAD AND TIME-OF-FLIGHT

When shooting at a stationary target from a stationary gun, the gun is not pointed directly at the target but above it in order to compensate for the drop of the bullet due to gravity as it travels from the gun to the target. If the target is moving, the gun must be pointed ahead of the target in order to allow for the motion of the target during the period of time in which the bullet is traveling from the gun to the target. This period of time is usually referred to as the time-of-flight of the bullet. It depends upon the initial velocity of the bullet, the distance to the target and to some extent the slowing down of the bullet due to air friction. The deviation between the actual direction of the fire and the line of sight to the target at the instant of fire, which is required by the motion of the target during the time-of-flight, is called the *kinematic lead.* The elevation of the gun required in the simple example cited, which is necessary to compensate for the fall of the bullet due to gravity, is called the *ballistic lead.*

When firing from a gun which is on a rapidly moving platform, such as an airplane, an additional ballistic effect becomes very important. This is the effect of the relative wind caused by the rapid motion of the airplane. This relative wind causes the apparent trajectory of the bullet to be curved, and must be compensated for in order to obtain a hit on the target. This windage effect is a far more important ballistic lead or correction than that due to gravity. In order to compensate for this and to determine the correct amount of kinematic lead, the effect of this relative wind and the time-of-flight of the bullet must be known accurately under a wide variety of conditions. The extreme difficulty of carrying out experiments in the air under conditions of flight makes it almost impossible to measure these effects accurately. As a result, the best information available is obtained from analyses. The Ballistic Research Laboratory at Aberdeen has done a tremendous amount of work in analyzing the motion of a bullet fired from a moving airplane.[17-20]

This work has led to the development of methods for computing the trajectories of a wide variety of bullets fired from a moving airplane. Such information has to be incorporated in the design of any sight for use in a moving airplane. An improved method for computing these trajectories was developed at the General Electric Company.[21] Tables of ballistic lead and time-of-flight are available on all the common bullets from either the Ballistic Research Laboratory or the Armament Laboratory at Wright Field. For the purposes of the design of gunsights or the analysis of their performance, it is desirable to express the information contained in the ballistic tables by some form of an equation. This was done by the Applied Mathematics Group at Columbia[22] and also at the Laredo Army Air Field.[23] These formulas were found to be extremely useful and gave the time-of-flight and the ballistic lead with exceptional accuracy.

Although all the numerical information pertaining to the trajectories of bullets fired from airplanes has been derived analytically, the Ballistic Research Laboratory finally was able to carry out an experiment in which certain trajectories were measured which could be

compared with the corresponding computed trajectories. This experiment served to check the analytical work and to further emphasize the difficulty of obtaining such information by experiment. In fact, the experiment consisted of firing from a moving airplane at a water target. While this was sufficient to check the analytical work experimentally, it was not a measurement of an actual trajectory under conditions that would normally be used in combat.

15.3 TRUE LEAD

For any particular course being followed by an attacking fighter, it is possible to determine what true lead must be taken by the defending bomber in order to obtain a hit on the attacking fighter. The most obvious way to compute this lead, when the type of ammunition and the target course are known, is by the use of future time-of-flight. By future time-of-flight is meant the time of travel of the bullet from the instant of firing to the point where the target will be when it is struck by the bullet. If a particular point on the course of a target is taken as the future position or collision point for the target, it is possible to work backwards in time and determine from what point the bullet must have been fired in order to arrive at this future target position. In order for this process to be possible, the target course relative to the bomber must be given and the bomber must be flying in a straight-line course at a constant speed. Ballistic tables will give the time-of-flight of the bullet for the particular range between the bomber and the future position of the target, and will also give the necessary ballistic lead for the various conditions existing. The separation between the future position of the target and its position relative to the bomber at an instant of time earlier by this time-of-flight will be the desired kinematic lead. The accuracy of this method for determining true lead is limited only by the accuracy of the information available on the ballistics of the bullet and the actual target course. It can be applied to cases where an assumed target course has been computed, such as pure pursuit courses, or where the target course has been determined experimentally. This method has been used extensively at the Army Air Force Proving Ground Command at Eglin Field and at the Naval Air Station at Patuxent River for determining the true leads corresponding to the various simulated fighter attacks made in the flight testing of flexible gunnery equipment.

In analytical work where the equations of the target course are known in mathematical form, it is often possible to derive an equation for the true lead. One such formula for the case of a pursuit course is given in reference 24. Much more complete and general formulas were worked out by the Applied Mathematics Group at Columbia University.[25] This report gives the most complete analysis available on the general problem of determining true leads. The more difficult problem of determining true lead when the bomber is taking evasive action was studied by the Applied Mathematics Group at Columbia University,[26] and by the Applied Mathematics Group of Northwestern University.[167] Up until the end of the war, no attempts had been made to actually compute true leads in the case where the bomber was taking evasive action. A great deal of work had been done, however, in computing true leads for pursuit type of attack courses with the bomber flying in a straight line at constant speed. Tables giving this information are available in references 27, 28, and 10. They are also available in the results of tests at the Army Air Force Proving Ground Command at Eglin Field and the Naval Air Station at Patuxent River.

15.4 PRINCIPLES OF LEAD COMPUTATION

There are a number of basic principles which have been used in the mechanisms for computing lead in an actual gunsight. In general, ballistic lead and kinematic lead are computed separately in two different parts of the gunsight and then combined to give the total lead. In all sights, some sort of an approximation is made to the true lead in order to simplify the mechanism since sufficient data are not known from which the true lead may be determined. In the simplest case, that of the fixed gunsight, the only information on the target course which is made use of is its bearing angle in the plane of action relative to the flight path of the bomber. The lead which the gunner is expected

to use, and which is expressed by the position firing rules, is based on the assumption that the fighter is flying a pursuit course. It is also assumed that the only requirement necessary for obtaining a hit on the target is to give the bullet a velocity directed toward a fighter in the air mass at the instant of firing. Certain corrections are made for the curvature of the fighter's path and the aerodynamic characteristics of the airplane, and these will be discussed in more detail in Section 16.2.

In order to mechanize the position firing rule, and thereby produce an own-speed sight, it is necessary to build a mechanism which will produce the vector sum of a vector representing the velocity of the bomber airplane and a vector representing the velocity of the bullet relative to that airplane. The sum of these two vector velocities is a third vector whose direction gives the initial direction of the bullet relative to the air mass. This has been accomplished in two ways. The first way is by means of linkages so arranged that they form a mechanical model of three vectors in question. The second way is to derive the equation for the required deflection and approximate the solution of these equations by means of a set of cams. In such a sight the output is a combination of both the kinematic and ballistic leads, although only to a first approximation. In the more complicated gunsights, which are of the lead by time-of-flight type, the kinematic and ballistic leads are computed separately.

In general, the ballistic lead is computed by means of a set of cams whose position is determined by the position of the gun relative to the bomber airplane and certain hand-set inputs allowing for the bomber's speed, altitude, etc. Some of the cams are one dimensional, meaning that the output of the cam represents a function of a single input variable. Such a cam is a type most often seen in ordinary mechanisms. It may be a plate of irregular shape which, as it rotates, moves a follower on its edge. A variation of this plate-type cam is where the follower is moved radially along the side of the plate in a slot. A second variation is where the cam takes the form of a cylinder in which a follower is moved along the cam axially by means of a slot in the surface of the cylinder. This type of cam is usually described as a *cylindrical cam*, whereas the two previous types are referred to as *plate cams*. In all three cases the important variable is represented by the rotation of the plate or cylinder and the output is represented by the linear displacement of the follower. Where the output function depends upon two or more variables, several of these one-dimensional cams may be put in cascade, or one or more two-dimensional cams used. A two-dimensional cam is represented by an irregular surface which can be rotated and can also be moved axially. As these two motions take place a follower bearing on the surface is caused to move linearly, depending on the distance of the surface from the axis of the cam to the point of contact.

An electrical method analogous to these mechanical cam systems has also been used. This consists of electrical potentiometers in which the output voltage is determined by the position of a contact. The potentiometer winding is so designed as to give the desired functional relation between output voltage and the input rotation. It is analogous to the one-dimensional mechanical cam. Cascade combinations of these potentiometers, where the output of one potentiometer supplies the input voltage for a second one, provides a means of multiplying functions of two variables. The main advantage of the electrical type of cam or potentiometer over the mechanical cams is its small size, simplicity of construction and low cost. This means that an electrical computing device using these potentiometers can contain a great many more of these electrical cams than would be possible in an analogous computer using mechanical cams.

A third method of computing a function of one or more variables is by the use of mechanical linkages. This method was not used in any flexible gunsights but was used to some extent in other computing systems. It shows considerable promise but as yet has not been exploited to any great extent.

In general, the ballistic computer part of a gunsight was designed to give an output which approximated to an acceptable degree of accuracy the ballistic data tabulated in the ballistic table. In one case a ballistic computer was built involving a combination of linkages and gears which mechanized to the first order

of approximation the equations giving the ballistic deflections of the bullet. This was used in the GE computer for the B-29 and is discussed in more detail later. All of these various ballistic computers were used to compute the ballistic deflections due to windage and the time-of-flight of the bullet which was necessary for the computation of the kinematic lead.

The usual method of computing kinematic lead was to determine the relative angular rate of the target by the best method possible, and multiply it by the approximate time-of-flight. The accuracy of this method was the subject of considerable analysis, and has been discussed in a number of reports which will be referred to in connection with the description of various types of lead-computing sights. The principal component for computing a kinematic lead by this method was some device for measuring the apparent angular rate of the target. The first form of mechanism for carrying out this angular measurement and for multiplying by time-of-flight was a mechanical variable-speed drive.[29] It was first used for this purpose in Sperry antiaircraft directors and the early Sperry airborne lead computing sight. The principal objection to the variable-speed drive as a means for computing kinematic lead is the fact that it is driven by the position of the gun relative to the bomber airplane and, therefore, measures an angular rate relative to that airplane. This means that any angular motion of the bomber airplane introduces an error into the determination of the relative angular motion of the target. An analogous electrical method has been used by Fairchild. In this case the angular velocity of the gun relative to the bomber is determined by means of an electrical tachometer whose field is excited by a voltage which is approximately proportional to the time-of-flight setting.

In order to avoid the error in such systems (introduced by any angular motion of the bomber), various gyroscopic methods of measuring angular rate have been used. The simplest of these is a rate gyro which is similar to the gyros used in aircraft rate-of-turn instruments. This rate gyro is constrained so that it can rotate about only one axis which is perpendicular to its axis of spin. A spring is attached which tends to prevent rotation about this axis. If the spin axis of the gyro is made to rotate about a third axis which is perpendicular both to the spin axis and allowable rotation axis of the gyro, it will be displaced against the spring by an amount which is proportional to this forced rotation of the spin axis. Since such a constrained-rate gyro can give an indication of rotation about only one axis, two such gyros are necessary in order to measure the two components of any two-dimensional rotation.

A second method of using a gyro to determine angular rate is by means of a free gyro. This gyro is mounted in gimbals so that mechanically it is free to move in any direction. Torques are applied to it in one of several ways in order to cause it to precess so that its spin axis will continually point at the object in space whose relative angular rate is to be determined. These precessional torques may be applied by springs acting on the gimbal supporting the gyro, by an electric torque motor acting on these gimbals, or by means of electric coils actually mounted on the gyro frame. One very ingenious method for using a free gyro was developed by the British and was used in the only gyro sight to see service during this war. In this case the gyro axis supported a copper dome which rotated with the spin rotation of the gyro and was subjected to a magnetic field produced by means of a magnet mounted on the housing supporting the gyro gimbal. The eddy current set up in the gyro dome due to its rotation produced the desired precessional torques.

A discussion of the principle involved in the use of both the rate gyro and the free gyro in a computing gunsight will be found in a report.[30] The equations of operation of such a gyro gunsight for a simple case are discussed in references 31 and 32. More detailed analysis of the operation of gyro gunsights will be found in the discussion of the various types of sights, which appears in a later section.

One problem which arises in the analysis and understanding of practically all gunsights is that of axis conversion.[33] For various practical reasons it is often necessary to compute the necessary lead in terms of two components. This is true not only for the computation of ballistic lead but also for the computation of kinematic lead. It is also necessary because in most cases the indication of computed lead is

given by the rotation of an optical line of sight about a pair of axes. In the early days of the war a number of mistakes were made in the analysis and development of gunsights because this problem was not fully appreciated. What is even more confusing is the fact that in many gunsights the axes with respect to which the two components of the ballistic lead are computed are different than the two axes with respect to which the two components of the kinematic lead are computed. This axis conversion problem is much more difficult in remote-control systems than it is for those in locally controlled turrets.

Chapter 16

LOCAL CONTROL SYSTEMS

16.1 FIXED SIGHT SYSTEMS

At the beginning of World War II, the most common sight used in flexible gunnery was the *fixed sight.* The earliest form of this was the iron *ring-and-bead sight.* This was later replaced by an *optical reflector type sight* which produced an illuminated reticle projected to infinity. This eliminated the parallax error which was so annoying with the ring-and-bead type of sight. The standard model of the Army reflector-type fixed sight was known as the N-8 and a similar sight used by the Navy was known as the Mark 9. The major problem in developing these optical sights was to eliminate the difficulties due to airplane vibration and to provide a sufficiently large field of view.

The early rules for laying off lead by means of these sights were the apparent speed rules. To apply these rules it was necessary to track the target long enough to get the reticle centered on the target and then hold the gun and sight stationary for a predetermined length of time. This time was measured by saying some key word such as "elephant" which was supposed to take a length of time equal to the time of flight of the bullet at the opening range. The apparent relative motion of the target during this time interval was observed and then the gun and sight were moved so as to point ahead of the target by an amount equal to this apparent motion.

There were two very serious difficulties with this system. The first is that it was impossible for a gunner to carry out these operations exactly. The second is that even if these rules were exactly followed, the leads computed by their application were seriously in error. This was realized by the Army Air Forces and the Navy in the early days of the war and considerable effort was made to determine what these errors were and to develop a more suitable set of rules. An elaborate analytical study of these rules and the proposed position firing rule was made by the Applied Mathematics Group at Columbia.[34] It was shown that even if the rules for the apparent-speed method were followed exactly, the resulting error in the leads would be as much as 25 to 50 mils in certain cases. The position firing rule which was finally adopted and used during most of the war on the B-17 and B-24 airplanes was found to be reasonably satisfactory. The errors run as high as 15 mils and have an average of perhaps 8 to 10 mils. This may be considered as satisfactory in view of all the other difficulties which were encountered in pointing the gun and estimating the opening range.

Since the only variable which the gunner takes into account in applying this rule is the position angle of the target, it will give satisfactory results only for the assumed set of conditions. This means that if the target attack course is materially different from that assumed in determining the rule, considerable error will be introduced into the lead. Also, the required lead will vary with the type of ammunition used, the altitude of the airplane and its indicated airspeed. All during the war this rule was under more or less continuous study and it was revised to meet changing conditions. Examples of this are the modification of the position firing rule for use as an emergency sighting system for the B-29[35] and the revised rule for use on the A-26.[36]

Even though the position firing rule is a relatively simple one, putting it into operation presented many difficulties. The gunner had to estimate the range to the attacking fighter in order to open fire at the appropriate maximum range. Experience showed that in general this estimation of range was in error by a factor of about two and that he therefore opened fire at about twice the desired opening range. Another difficulty was the inability of the gunner to properly estimate the bearing angle of the target and the plane of action of the target. In the waist position the gunner was aided considerably in the estimation of the target bearing angle by the relative position

of the gun with respect to the periphery of the waist hatch and the relative position of the wing and tail surfaces. In the tail position no part of the airplane was visible to aid in the estimation of the target bearing angle. Fortunately this was not too serious since the size of the required leads was relatively small.

In a turret the estimation of the target bearing angle is also difficult. This difficulty of estimating a target bearing angle led many people to propose mechanisms for aiding the gunner in this estimation. In general these took the form of what is known as own-speed sights which mechanized the position firing rule and presented the gunner with information on the required lead. Toward the end of the war enemy fighters were making attacks which were somewhat different than the pursuit type of attack and therefore the position firing rule being used was considerably in error. This also applies to the own-speed type of sights which will be discussed shortly. As a result, interest in apparent-speed rules was revived. A revised evaluation of these methods was made and is contained in reference 37.

16.2 OWN-SPEED SIGHTS

During World War II a great number of own-speed sights were proposed. The idea goes back to World War I during which the so-called wind vane sight was proposed. The own-speed sights proposed during this war were of two general types. The first consisted of a linkage type mechanism generally containing gear drives which was a geometrical model of the various vector velocities which determined the required lead. The second type made use of cams for mechanizing the equations giving the required lead in terms of two components. Reference 38 contains a good summary of the various types of own-speed sights proposed. The first own-speed sights seriously considered were the K-10 and K-11 which were developed by Sperry. These were designed for tail and nose positions, respectively. They used three dimensional cams rather than linkages and were, therefore, somewhat limited in the range of coverage. This limitation prevented them from being used in any but the nose and tail positions. These particular sights were also excessively heavy and large and rather expensive. A description of the K-10 and K-11 sights and their underlying theory will be found in three Sperry reports.[39,40,41]

Because of the simplicity of the own-speed type of sight and the lack of any lead-computing sight that was considered satisfactory, considerable effort was made to develop a suitable own-speed sight and get it into use as soon as possible. Section 7.2 started such a development at The Franklin Institute[45] in Philadelphia quite early in the war. This development served to stimulate interest on the part of other development organizations as well as on the part of the Army Air Force. Developments using the own-speed principle were also carried on by the Jam Handy Organization. The original purpose of this development was to produce an own-speed type of assessor but it also led to the development of an own-speed sight. A description of the Jam Handy own-speed sight has been prepared by them.[42] The Jam Handy Organization (under a contract with the AAF) also developed a similar sight for the tail position of the B-25. A report on this development has been made.[43] One example of the wide variety of the sights proposed using the own-speed principle is given by the proposed modification of the standard B-17 tail sight.[44]

Since the accuracy of the own-speed type of sight, as is the case with the position firing rules, depends so greatly on the actual course of the attack fighter, considerable effort was spent on ways of adapting the own-speed sight to meet these different conditions. If the attacking fighter made the assumed type of attack, it was only necessary for the flexible gunner to keep the reticle of his sight squarely on the target. The main indication that the fighter was flying a course different from the pursuit type of course assumed was the apparent attitude of the attacking fighter. The most important situation in which this condition arose was when an attempt was being made to provide support fire for protection of another bomber in the formation. In order to provide satisfactory support fire, using the own-speed sight, it was necessary for a gunner not only to apply rules which met this situation but to estimate the general aspect angles of attacking fighters.

This situation was studied in some detail toward the end of the war, and the various proposed rules and their evaluation are discussed in four reports — 46, 47, 48, and 49.

With all the various proposals for an own-speed type of sight, it soon became necessary to make some sort of an evaluation of them and to decide on one or two best suited for the needs of the Air Force. The sight chosen was the Sperry K-13 which made use of a linkage type of mechanism. This sight was selected not necessarily because it was the most accurate sight available but because its accuracy was adequate and it could be put into use in the operational theaters most rapidly. The details of this sight are given in the Sperry instruction manual.[50] An analysis of the errors of this sight under the conditions for which it was designed is contained in reference 51.

Theory will show that the own-speed type of sight will accurately predict the required lead when the bomber is flying a straight course at constant speed, the attacking fighter is flying a pure pursuit course and the bullet has no slow down. In this case the input to the sight which accounts for the airspeed of the bomber is taken as the full value of the bomber's airspeed. Due to the fact that the fighter is not flying a pure pursuit curve because of its aerodynamic properties and its own lead on the bomber, the use of the full value of the bomber's airspeed will give erroneous results. This effect can be compensated for by using an appropriate percentage of the bomber's airspeed as the airspeed input to the sight. Various attempts[52,53,54] were made to evaluate this situation and determine the optimum way to use the own-speed sight.

In addition to the factor already mentioned, which might cause substantial error in the lead computation by the own-speed sight, there are several other factors of importance. One of these is the fact that the actual course of the attacking fighter does not lie in a plane of action having a constant elevation relative to the bomber. In particular, for high side attacks, the attacking fighter has an actual instantaneous velocity which tends to carry it below the bomber's line of flight. This has to be continuously corrected by the fighter pilot and causes the instantaneous plane of action to continuously decrease in elevation. This results in an error in the lead produced by the own-speed sight which causes it to shoot too high. This error is partially compensated for by the gravity drop of the bullet which is not computed by the own-speed sight. Another important effect is that due to the attack angle of the bomber. It is assumed in using the own-speed sight that the actual vector velocity of the bomber is known. The direction of this velocity in space is normally not along the thrust axis of the bomber. In fact the angle between the actual velocity of the bomber and its thrust axis, which is known as the attack angle, is not constant. It varies with the airspeed and loading of the bomber. An average value of this attack angle is used in harmonizing the various own-speed sights with the guns. An attempt to take this factor into consideration was made in The Franklin Institute sight and in some of the Jam Handy developments.

16.3 LEAD-COMPUTING SIGHTS

The usual lead-computing sight is one in which the kinematic lead is computed by multiplying an appropriate angular rate by a time of flight. It is so arranged that when a target is tracked with it, a measurement of angular rate is made continuously and this is multiplied by a time of flight setting which is derived from a measurement of range to the target. This measurement of range is ordinarily made stadiometrically. To do this the gunner operates the range control of sight so as to keep a reticle of varying size constantly bracketing the target. Toward the end of the war there were several projects whose objective was to introduce the use of radar determined range into the sight. This will be discussed in more detail in a later section.

One basic difference between the lead-computing sight and the own-speed sight is the manner in which the reticle moves with respect to the gun. In the own-speed sight the reticle is coupled to the gun in such a way that it always moves with the gun and gives the gunner very direct control over its motion. In this sight the control of the reticle by the gunner is very direct and he would ordinarily not be cognizant of any

motion of the reticle due to a change in lead. In the case of the lead-computing sight the reticle is floating. This means that a motion of the gun does not necessarily cause a direct corresponding motion of the reticle. This is because the lead indicated by the reticle of the lead-computing sight depends not only upon the gun position but also upon its rate of change of position. This causes the lead-computing sight to be somewhat more difficult to track with than the own-speed sight. This difficulty of tracking is one important factor which must be taken into account in the sight design.

The floating character of the reticle in the lead-computing sight also introduces a problem in regard to picking up and getting on the target. In general, during the preliminary slewing, the high angular rates produced will generate an excessive lead which must be removed before the gun is in correct position for firing. The transient response of the sight, which results in the removal of this excessive lead, may last for a time interval approximately equal to the time of flight setting of the sight. It also depends upon the design constants of the sight and is inseparably connected with the tracking characteristics of it. This transient effect may also introduce an error into the lead when firing against a target having angular acceleration. This is because the transient effect is similar to a time lag and the lead which the sight computes is actually based on old data. An analysis of the lead-computing sight and its various effects is given in reference 55, which also discusses the effects of computing the lead in terms of two components instead of directly in the plane of action of the target.

The only lead-computing sights for flexible gunnery to see any extensive use in this war were the Sperry K-3 and K-4 sights. These sights were installed in the upper and lower turrets of the B-17. They use a mechanical type of rate-measuring device and multiplier. Rather severe approximations were made in designing the time of flight and the ballistic lead computer with the result that these sights are not too accurate. However, they gave considerably better performance than could be obtained with either position firing or the use of an own-speed sight. The sights were severely criticized by many people in the field and it was even suggested that they be removed and position firing rules and a fixed sight be used in their place. Fortunately, this was not done and it was subsequently shown that the K-3 and K-4 sights were far more accurate than a good many people realized. A description of these sights and an analysis of some of their errors appear in two Sperry reports.[56,57] Toward the end of the war an improved version of these sights was developed by Sperry and is known as the K-12.[58] The K-12 sight provides a somewhat better computation of time-of-flight and ballistic lead. The errors in the computed kinematic lead that are inherent in any lead-computing sight of this sort, can be minimized only by the proper choice of design constants and the proper calibration of the time-of-flight setting. The errors of the K-3 and K-12 sights have been the subject of rather detailed analysis.[59]

One of the fundamental sources of error in these sights is the fact that the angular rate of the target is measured with respect to the bomber. This means that an incorrect angular rate will be determined if the bomber has any angular motion of its own. This was the source of one of the most severe criticisms of the Sperry lead-computing sights. This would be particularly serious in a situation where it was necessary to use the sights while the bomber was taking evasive action. Fortunately this seldom occurred. The only important case was when an isolated bomber was being attacked and evasive action was being used as a means of protection. In the European Theater, this occurred when a single bomber had to drop out of formation for any number of reasons. In this case the bomber was generally under attack by several enemy fighters and it would have been difficult to defend it even with most ideal sighting systems.

The first gyro type of lead-computing sight to see extensive use was not for plane-to-plane fire but was for antiaircraft fire. This contained two rate-measuring gyros which were used to compute the two components of kinematic lead. The sight was used extensively with 20-mm and 40-mm antiaircraft guns on shipboard. The sight was built by Sperry and was known as the Mark 14 gunsight. A rather detailed investigation of the theory and errors of

this sight has been made.[60] Even though this sight was designed and used for antiaircraft fire, its operation and its errors are similar to those that would be found in using such a sight for plane-to-plane fire. One error which is inherent in any lead-computing sight which determines lead in terms of two components is that due to the rotation of the sight about the line of sight. This particular error is described in reference 61.

The most successful gyro type of lead-computing sight for aerial gunnery was the single-gyro sight developed by the British. This sight was originally developed for flexible gunnery for bomber protection. However, before it could be put into any extensive use for bomber defense, the need for its use as a fighter gunsight became very great. As a result the only use which this type of sight saw during the war was in fixed gun fighters. The Navy version of the gyro sight for flexible gunnery is known as the Mark 18 gunsight.[63] The Army version is known as the K-15. A brief summary of the single-gyro type of sight as used in this country for both fixed and flexible gunnery is given in reference 62.

When the single-gyro type of sight was first brought to this country by the British, the AAF showed very little interest in it. It was only through the efforts of Section 7.2 of NDRC that they finally gave it any serious thought and eventually adopted it for use in fixed gun fighters. On the other hand, the Navy was very interested in this sight from the beginning and was largely responsible for getting it into production, even though eventually the Army took over most of this production. A great deal of the work that was done by the Navy to redesign the sight for American use and get it into production was done by the Lucas-Harold Corporation.[64,65] Their reports give an analysis of the sight and many of the design details.

Because of the importance of the single-gyro type of sight, a great amount of effort was put into analyzing it and attempting to improve its design. Two detailed studies of the operation of the gyro and its supporting Hook's joint were made.[66,68] A study of the general equations of the sight will be found in references 67 and 70, and a detailed analysis of the optical systems of the Mark 18 and K-15 in reference 69. An analysis of the errors of the Mark 18 sight when used against pure pursuit courses has also been made.[71,72]

The time-lag error which is inherent in any lead-computing sight can be minimized by a proper choice of the time-of-flight setting. If the time-of-flight setting which is used is the same as the actual time-of-flight to the present position of the target, the errors in the kinematic lead will be smallest when the sight is used against a straight-line target course. However, in plane-to-plane fire the sight will normally be used against a pursuit-type of attack course. In this case, if the time-of-flight setting is taken as the time-of-flight to the present position of the target, the generated kinematic lead will be in error by about 10 per cent. To compensate for this the time-of-flight setting is normally reduced by about 10 per cent. This tends to minimize the errors in kinematic lead for the pursuit-type of attack course. However, in doing this a sight is given an error of about the same amount for straight-line courses. This effect is typical of all types of fire-control equipment.

All sight systems are sensitive in varying degrees to the type of attack course. As was pointed out earlier, the own-speed type of sight is very sensitive to the type of attack course. The lead-computing type of sight is sensitive to the type of attack course to a much smaller degree. In general it is possible to calibrate the time-of-flight setting of a lead-computing sight so that the errors in kinematic lead are minimized for some particular type of attack course.[74,75,76] The effect of the design constants of the sight on its accuracy and ease of tracking was the subject of an extensive study by The Franklin Institute.[77,78,79]

The various lead-computing sights were the subject of several assessment programs. Some of the analytical work on the errors of the lead-computing sight have been referred to previously. The first experimental determination of the errors in the Mark 18 single-gyro type of sight was made at the University of Texas[73] on the testing machine developed under Section 7.2. The testing program started at the University of Texas on the Mark 18 was never carried very far because of the importance of getting performance data on the B-29 equipment. How-

ever, enough information was obtained to get some idea of the overall performance of the Mark 18 sight. This test was run in such a way that it is difficult to make a good comparison between performance of the Mark 18 sight and other types of sights. Also, it did not take into account many of the psychological conditions which would exist in actual flying. A test under flight conditions in which the Mark 18 was compared with the K-3 and K-13 and position firing rule was made by Eglin Field. This was the first test made by the camera assessment method which was developed with the assistance of Section 7.2. A complete description of this test and the results obtained appear in an Eglin Field report.[80] A rather early analysis of the errors in the lead-computing sight when used against a pure pursuit course is given in reference 81. A summary of the various assessments made of the various types of gunsights available during the latter part of the war has been prepared.[82] This summary presents graphically the available knowledge on the magnitude of the errors on these various systems near the end of the war. A somewhat more detailed summary of the results of these various assessment programs was also made.[83]

Almost at the end of the war, an electrical type of lead-computing sight became available in the European Theater. This was the K-8 sight developed by Fairchild. It measured angular rates by means of electrical tachometers mounted on the gun axes. In principle it was similar to the Sperry sights. However, the time-of-flight and ballistic-lead computations were very accurate and in fact were more accurate than the accuracy of the input data warranted. This sight was the basis for a newer development which resulted in the S-3 and S-4 sights.[84] These sights measured angular rates by means of two rate gyros and had stabilized lines of sight. The stabilization is of the rate type which means that the angular rate of motion of the line of sight is made to be independent of the motion of the supporting airplane and depends only on the position of the control handles. In this system the kinematic lead-computation and the stabilization feature are linked together so that the lead computation and turret control are no longer independent as they are in previous systems. This has made the over-all analysis of the errors in lead computation much more difficult.

While some analysis has been done on the performance of the S-3 system, it is not nearly as complete as that for the previous systems.[85,86] An analysis of the S-4 sight, which is similar except it is for a different caliber gun, has also been made.[87] A somewhat different stabilized system has been developed by Sperry.[88] The sight is known as the S-8. It uses a single free gyro and provides position stabilization of the line of sight. This development is not yet complete but appears to be a notable advance in flexible gunnery fire-control systems.

There is no question but what the most promising advances in the development of the gunsights for flexible gunnery will be in the direction of stabilized systems using gyros in which the turret control and lead-computer are designed as part of the same system.

Chapter 17

REMOTE CONTROL SYSTEMS

17.1 NEED FOR REMOTE-CONTROL SYSTEMS

With the development of the very heavy type of bomber, it became desirable to consider remote-control type of flexible gunnery systems. It is conceivable that such a system could be used to arm a large bomber in much the same way as the Navy uses remote-control systems for its main battery and antiaircraft armament on board ship. Locally controlled turrets offer a certain amount of simplicity over a remotely controlled system but require an operator for each gun position, and as normally used cannot put concentrated fire power under the control of a single gunner. On the other hand a remote-control system offers a great deal of flexibility and appears to have certain advantages. To date there has been no adequate comparative evaluation of locally controlled turret systems and the remote-controlled turret systems. The factors which favor one or the other system are so varied that the choice has been largely a function of the design of the particular aircraft concerned and its intended use.

The main impetus during World War II for the development of a remote-control system was for the defense of very heavy bombers such as the B-29. Perhaps the predominant factor leading to the choice of this system comes from the requirements of pressurization. Since it is undesirable to pressurize the entire fuselage of a very heavy bomber, it is possible, using a remote-control system, to locate the sighting and control stations within the pressurized area and to place the turrets in unpressurized portions of the airplane. It is conceivable that an adequate locally controlled turret system could be developed in which certain of the turrets are independently pressurized. Such a scheme has been considered but no great effort has been made to develop it.

Because of the separation of the turret and the sighting station in a remote-control system, a number of problems arise which are not present in the locally controlled turret. It is necessary to transmit accurate position data between various points on the airplane and to compensate for the effect of parallax produced by the separation of the sighting station and the turret. The separation also increases the complexity of the computer for determining the necessary lead. This separation also introduces rather serious problems in regard to the maintenance of alignment and harmonization. On very large aircraft, this is seriously affected by strains in the airplane which are caused by many factors. On the other hand, on new high-speed bombers it is possible, using a remote-control system, to locate gun turrets where it would be physically impossible to locate a locally controlled turret. One outstanding example of this is the proposed installation of turrets on the wing tips. Another advantage of the remote-control system is that it is possible to reduce considerably the size of that portion of the turret which is exterior to the normal surface of the airplane. This is important in reducing the drag due to the turrets and is most important in the case of new high-speed airplanes.

17.2 EARLY DEVELOPMENTS

The development of remote-control systems started before the United States entered the war, with the initiation of the development of a completely automatic radar system for gunlaying. When the first contract for such a system was let, the necessity for a suitable computer was not appreciated. The system consisted only of one or more radar sighting stations which pointed one or more remotely controlled gun turrets. It was pointed out by Section 7.2 (then Section D-2) that for all-round firing it was absolutely necessary to provide a suitable computer as part of this system in order to get any firing accuracy at all. This original development for a complete radar gunlaying system, known as AGL, was carried on by the General Electric Company[90,91]

with the cooperation of the Radiation Laboratory at MIT. The system was known as AGL-1. The AGL-1 system was obsolete before it was ever finished and was never tested. However, it formed the basis for the development of the manually operated remote-control system which was finally used operationally in the B-29's over Japan. In fact, while this AGL-1 system was never produced or used, it made some of the most important contributions to the use of radar in airborne fire-control systems and to the general development of computers for such systems.

Before long a number of developments on remote-control systems were under way. Most of these were for the purpose of providing completely automatic radar tracking and ranging and multiple operation of remotely controlled turrets. An early report on these various systems is reference 89, which describes all the important remote-control systems which were under development during World War II, with the exception of a Westinghouse system which was not developed until the latter part of the war. During the early part of the war there was great interest in developing these completely automatic systems. The concentration of such a great amount of effort on completely automatic fire-control radars for aircraft use resulted in a delay in the simpler radar systems which were eventually found to be of much greater practical use. In fact it was not long before interest in the completely automatic systems decreased to almost nil and the effort was concentrated on the simpler fire-control radars, which will be discussed later.

17.3 GENERAL ELECTRIC SYSTEM

The main contribution of this early development was the ground work that it laid for the development of computers for remote-control systems. The only remote-control system which saw use during the war was the General Electric system installed in the B-29 airplane.[92,93,94] This system is manually operated and except for a few installations at the end of the war made no use of radar. Because of the importance of this system, considerable effort was exerted in determining its performance by test and analysis and in increasing its accuracy. References 95, 96, and 97 give a good description of the theory of the operation of this system. This study was brought about by the fact that the system, as originally produced, did not have sufficient accuracy and was unsatisfactory in many ways.

Modifications for improving the GE system were almost continuously under development. The remotely controlled turrets of the GE system were of excellent design and gave exceedingly good service. The data transmission system which was used was very accurate and contributed negligible error to the overall operation. In fact it can be said that the turret and data transmission system design was entirely satisfactory. The computer which was finally produced was large and complicated and could not be considered as entirely satisfactory. It was not the only computer designed under development by GE but was chosen because it promised to be the computer which could most easily be put into production and meet the rather rigid time schedules. It is very probable that if the choice of a computer could have been delayed another six months a considerably improved computer would have resulted.

The most unsatisfactory component of the General Electric B-29 system was the sighting station. This was a pedestal-type sight which was operated directly by the gunner without benefit of power controls. It was awkward to handle both for ranging and tracking. It required extensive training and practice in its use and even then the tracking and ranging errors were excessive. In the preproduction model kinematic lead was computed by means of a free gyro which was contained in the sight head. This gyro was precessed by electrical means so as to remain parallel to the optical line of sight. The servo system used for producing this precession was operated by a set of contacts mounted on the gyro and the sight case. The biggest difficulty resulted from the use of these contacts in the servo systems. Another serious difficulty encountered was an instability which made it impossible to use a single gyro. This could be overcome only by providing two separate gyros at the sighting stations. The sight actually put into production was the two-gyro sight. Another serious difficulty which never was overcome satisfactorily was the sen-

sitivity of the system to the character of the tracking errors.

This latter difficulty was realized very soon after the first systems were put into operation and steps were taken to develop a more suitable sighting station. The first such improved sighting station resulted in what was known as the free-gyro lead computer. This consisted of an entirely new sighting station containing a single free gyro. Only minor modifications were required in the computer to adapt the system to its use. This free-gyro computing system (described in detail in reference 98) was given fairly extensive tests and gave an accuracy of performance which was substantially greater than that of the standard system. By the time the war ended, plans were already under way for producing this free-gyro system in quantity and eventually replacing the standard system with it.

A second improvement was started much earlier than the free-gyro system. This is the development of a gyro-stabilized sight[99] having power control. This gyro-stabilized sight provided much more accurate lead computation and also provided tracking and ranging controls that were a very great improvement over either the standard system or the free-gyro system. This sight promised to be considerably better than that of the standard B-29 system or the free-gyro sight. However, in spite of this the development of the gyro-stabilized sight was considerably slower than that of the free-gyro sight. This is largely due to the lack of any strong central planning group within the Air Force which could competently evaluate the various proposals for the development of airborne fire-control equipment and which could decide on which line of endeavor to follow. The gyro-stabilized sight promised to eliminate all the serious difficulties that were encountered with the standard system and yet the effort put on its development was exceedingly small. Perhaps if adequate assessment devices had been available earlier in the war, this would have helped to direct GE's efforts into more fruitful channels.

Even after the standard B-29 fire-control system was in production and installed in operating airplanes, its operation was still not entirely satisfactory. As a result a number of studies were made to eliminate the major difficulties. The portions of the computer which determine the ballistic lead and the corrections necessary on account of parallax did so with an accuracy which was considerably greater than was necessary. In fact one major criticism of the standard B-29 computer was the poor design balance between the kinematic lead computer and the ballistic lead and parallax computers. The two most important difficulties were the awkwardness of the pedestal sight as a tracking device, which made it very hard to track and range satisfactorily, and the rather large time lag inherent in the computer, which seriously affected the computation of the kinematic lead. The time lag introduced by the computer in the computation of the kinematic lead was most serious in the case of nose attacks. Such attacks proved to be rather popular with the Japanese and this made it even more important to improve the performance of the standard computer. Studies and recommendations on improving the standard B-29 computer were made by the Applied Mathematics Group at Columbia. Their results are given in a series of reports.[101-104]

At the time that the Army Air Force decision was made to freeze the development work and get the present B-29 computer into production, Section 7.2 contracted with GE to continue the computer development work. The Section felt that there was a good possibility of producing a new computer which was considerably superior to the standard computer. This was particularly true since at the time the production design was decided on, GE had not completed any developmental computer nor had an opportunity to test and study such a computer. In fact the development of the gyro-stabilized sight was a direct outcome of this continuing development work under NDRC. The computer which GE was developing for Section 7.2 was not quite completed at the time the contract had to be terminated because of the end of the war. However, enough work has been done to indicate that this new computer was considerably superior to the standard B-29 computer. This new computer will undoubtedly be completed by GE under the direction of the Army Air Forces. Perhaps the most important contribution which was made through its develop-

ment was the design of a second new computer which was considerably smaller and less complicated than any previous one.

17.4 EVALUATION OF GE SYSTEM

The standard B-29 fire-control system was first tested by the Army Air Forces Proving Ground Command in Eglin Field. This is the first test in which official recognition was given to the fact that this system was not entirely satisfactory in its operation. This particular test and the conclusions drawn from it are given in a report of the Army Air Forces Board.[105] While this test served to stimulate considerable effort for the purpose of improving the operation of the standard B-29 system, it cannot be considered as a particularly reliable evaluation of the fire-control system. Plans were eventually made for re-evaluating the B-29 system by means of a satisfactory method, but for various reasons these plans were never carried out. In fact no completely satisfactory evaluation of the B-29 fire-control system has ever been made.

Under directive AC-92 from the Army Air Forces, the Applied Mathematics Panel of NDRC undertook to make a rather extensive and elaborate tactical evaluation of the B-29. The principal part of this work was carried out by the University of New Mexico in Albuquerque. The re-evaluation of the B-29 fire-control system by Eglin Field was a part of this program. However, the New Mexico group found it necessary to run some tests on the accuracy of the system. The results of this test are given in one of their reports.[106] The whole AC-92 program was intended to be a quick and not too thorough study of use of the B-29, and the test run by the New Mexico group on the accuracy of the fire-control system was carried out with this in view. As a result this test can be taken only as an indication of the general accuracy of the B-29 system and cannot be considered as being particularly reliable.

Toward the end of the war the University of Texas Testing Machine, which will be discussed later, became available and was used quite extensively in testing and studying the B-29 system. The problem of defending the B-29 against nose attacks was so critical that the problem of improving the performance of the computer for such attacks was given first priority. All the important proposals for modifying the computer were studied[107] and additional tests were run.[108] Eventually the University of Texas Testing Machine was used to make a comparison of the standard B-29 system with the free-gyro sighting system and the stabilized sighting system. The results of these studies as well as those previously mentioned are discussed in reference 109.

Toward the end of the war the General Electric Company developed an excellent assessment facility at Brownsville, Texas, and succeeded in running some adequate tests on the standard B-29 system and some of its modifications. No report is available on the results of this test but the proposed program is described.[168]

17.5 SPERRY SYSTEMS

Although the General Electric Company developments were the ones which were finally accepted by the Army Air Forces and put into production, there were other developments of remote-control systems which might possibly have been superior to the GE system. The most important of these is a series of developments by the Sperry Gyroscope Company. Very early in the war, Sperry undertook to develop a completely automatic radar gunlaying system using remote-control turrets. The computer which was used in the first phase of this radar fire-control system was the Sperry P-4 computer. This computer was an adaptation of the Sperry K-3 and K-4 sights to the central station type of computer. The sight station was a double ended periscope mounted directly on the computer. The periscope was power controlled from a pair of handle bars and made use of aided tracking. The remotely controlled turrets developed by Sperry were very accurately positioned by means of a selsyn data transmission system and a hydraulic turret servo.

There were several serious objections to the P-4 computer. The field of view of the periscope was only 30 degrees and this made it virtually impossible to carry out a search operation with the periscope alone. This eventually led to the

development of what was known as a presighting station which consisted primarily of a pedestal-type sight that could be pointed directly by an operator. This sight was pointed at a target as soon as it was picked up and the information automatically relayed to the computer, which resulted in automatically putting the line of sight of the periscope almost on the target. Preliminary tests indicated that this system was very successful. Perhaps the most fundamental objection to the P-4 computer was the fact that, like the K-3 and K-4 sights, it measured angular rates with respect to the bomber. This introduced serious errors into the computation of kinematic lead when the bomber takes evasive action. A lesser difficulty, but one which is still fundamental, is due to the fact that the P-4 computer did not correct for parallax between the sighting station and the turret and contained rather substantial errors as a result of not properly carrying out the various required axis conversion.

The P-4 computer was in production very early in the war and could have been used long before the General Electric system for the B-29 was actually in production. At the time this Sperry system was developed there was no really adequate means of assessing it so that the importance of the various known theoretical errors was never fully evaluated. It is very probable that like the K-3 and K-4 sights, which were eventually tested, the Sperry central fire-control system would have been found to be at least as adequate for the defense of the B-29 as the standard GE system. The P-4 computers which were produced were never used. A description of this complete system will be found in the preliminary instructions for it which were prepared by Sperry.[111]

The first phase of the complete radar gunlaying system developed by Sperry was essentially the central fire-control system just discussed with the addition of a completely automatic X-band radar. In this first-phase system the position data obtained by the radar was used to provide a target on a cathode-ray tube which was tracked by the gunner who operated the control handles of the P-4 computer. It was realized at the time that this arrangement would probably not be particularly satisfactory and that the accuracy of the P-4 computer left much to be desired. However, this first-phase system was the first complete remote-control system to be completed and flown and could have been used to provide some very useful information on the general technical value of such a system. Such a study was never carried out primarily because the AAF decided to concentrate on the development and production of the GE system and the Sperry system never was given very serious further consideration. This first-phase AGL system is described in a Sperry report.[110]

At the same time that the first phase of the Sperry central fire-control system development was being finished, work was started on the design and development of a completely new computer which was to be a part of a final-phase system, and on the modification of a P-4 computer for use in an intermediate-phase system. The modified P-4 computer would be associated with a single free gyro which would be a part of the kinematic lead computer, and would also serve to stabilize the line of sight. This single-gyro system would eventually be made a component of the final-phase computer. The intermediate phase development was completed about a year before the end of the war and was given rather extensive tests by Sperry. Within the limitations of the P-4 computer, which still contained some of its fundamental errors, this intermediate-phase system proved to be very accurate and easy to handle. This particular system was never tested by the AAF. In fact, when the development had been completed, all further work on such systems by Sperry was stopped. A complete description of this intermediate-phase system and the tests which Sperry ran on it are contained in one of their reports.[112] The design of the computer for the final-phase system was completed by Sperry but no parts were made. At the same time a double periscope having a 70-degree field was developed and built which eliminated most of the objections to the original double periscope. A brief description of this final-phase computer design is also contained in the report just mentioned.[112]

17.6 OTHER SYSTEMS

The difficulties which were encountered with the standard GE computer for the B-29, and

which delayed its production almost a year, led NDRC to start another central station computer development by means of a contract with the Fairchild Camera and Instrument Company. This particular development was to be based on the Fairchild K-8 lead-computing sight which already was beginning to go into production. The Fairchild computer was almost entirely electrical in nature and used two rate-type gyros for the computation of kinematic lead. The computer, as finally designed, was mounted in the same case as the standard B-29 computer and was interchangeable with it except for the gyros in the sighting station. Because of the different type of kinematic lead computation, the two GE gyros had to be replaced by the two Fairchild gyros for use with the Fairchild computer. One model of this computer was completed but never tested. A theoretical consideration of the Fairchild computer design would indicate that it would compare very favorably in performance with the standard GE computer. A good description of this computer development is given in the Fairchild final report to NDRC.[113] Very late in the war one other central station computer development was started. This was carried out by Westinghouse and brought to the point where one computer and sighting station were available for test. This Westinghouse system used a gyro type of kinematic lead computer and provided stabilization of the line of sight.

The British were interested in remote-control systems but did not undertake to develop anything so elaborate as any of the American systems. They developed an excellent remotely controlled 20-mm turret and data transmission system but made no attempt to develop a central station type of computer. The computer which they used was a Mark II-c type of lead-computing sight which was mounted so that it could be operated by power control from a pair of handle bars. The computed gun position determined by this sight was then relayed to the remote-control turret through a simple mechanical type of parallax computer. A discussion of this system and its performance will be found in two British reports from the Royal Aircraft Establishment, Farnsborough[114-115]. It is interesting to note that largely as a result of this work by the British the Americans became interested in trying the same sort of an arrangement in the B-29. However, this was never carried out.

Chapter 18

TRACKING AND RANGING

18.1 GENERAL DISCUSSION

THE MOST IMPORTANT operation which is carried out by any fire-control system is that of accurately locating the position of the target. In all airborne fire-control systems, this is done in terms of a spherical coordinate system. This means that the data on the target position is given in terms of the distance to the target, which is called range, and the elevation and azimuth bearing angles of the target. The process of operating the direction-finding equipment with which the angular position of the target is determined is known as *tracking*. Operation of the range measuring equipment is known as *ranging*. While it would be more appropriate to refer to these operations as angle tracking and range tracking, the previous terms are very well established and have come to have the meanings given.

The simplest form of tracking in flexible gunnery occurs in the use of the flexible gun such as is found at the waist positions of a bomber like the B-17. In this case there is no range-measuring device other than a set of stadia rings in the fixed sight. Tracking is accomplished by manually moving the gun about without the aid of any power control. Such control is known as direct control. The accuracy of tracking with a flexible gun mount such as this is extremely poor. One of the important factors that contributes to this inaccuracy is the large mechanical load which the gunner must overcome due to the weight of the gun and the various aerodynamic forces on it. The first step in improving the tracking accuracy of the gunner was to mount the gun in a power-driven turret. This permitted the use of more than one gun and allowed the operator to ride around in azimuth with the gun. The power required to move the gun and the turret is supplied by a motor which is controlled by the position of some sort of a control handle. In the common turrets, a pair of control handles was used which adjusted the speed of the motor driving the turret. The usual arrangement was to have the motor drive produce an angular rate in the gun position which was approximately proportional to the displacement of the control handles. This type of control is known as *rate control*.

For practical reasons the turret was normally mounted so that it rotated in azimuth about the vertical axis. The guns which were mounted in the turret were arranged so that they could be rotated in elevation about a horizontal axis carried on the turret. The control handles rotated about a similar pair of axes and the displacement of the handle bar about these axes controlled the rate-of-motion about the corresponding axes of the gun turret. Such a turret arrangement is referred to as a cylindrical turret. This type of turret was the most common type to be used during this war. In some cases, a turret was so arranged that it rotated both in azimuth and in elevation, and carried the gunner and the guns with it. This type of turret is known as a *spherical turret*. In this case, also, the displacement of the control handles determine the rate-of-motion of the turret and gun about the two axes of rotation. A description of all the standard turrets used during World War II will be found in the AAF Technical Order series, 11-45, on aircraft turrets.

Experience showed that even with power-driven turrets using rate control, the tracking accuracy obtained was not satisfactory. The ability of a gunner to track accurately with such a system depends on many factors such as the amount and quality of his training, the type of target course being followed, the design constants of the control system and the number and type of other functions the gunner has to carry out simultaneously with the tracking. Among these other functions are such things as ranging, identification of the target, operation of the trigger and various control switches.

The method usually used for measuring the target range was the *stadiometric method*. This is accomplished by adjusting the size of the reticle until it bracketed the target, of assumed

size. From the assumed target size and the known size of the reticle, the range of the target could then be determined. This was accomplished in the mechanism controlling the reticle size so that the computed target range was continuously available and inserted into whatever computer was making use of such information. The range obtained by this stadiometric method was usually very inaccurate. In fact, in any of the lead-computing systems which made use of range data, the greatest inaccuracies in the computed lead were those due to the ranging errors.

There were two general ways in which the target's position data could be improved in accuracy. The first of these was to improve the tracking and ranging controls and the training of the gunner so that the system as a whole could be used with greater accuracy. The second of these was to replace the manually operated range and tracking system by one which was automatic in either tracking or ranging, or both. This could be accomplished by the use of radar and some systems were developed for doing it. This will be discussed later.

18.2 TRACKING AND RANGING STUDIES

To improve the accuracy of the manually operated system, Section 7.2 undertook a number of studies on tracking and ranging. The most important of these was carried out by The Franklin Institute in Philadelphia. The Franklin Institute developed mechanisms for simulating the dynamics of various types of tracking controls and of the standard lead-computing sights which were to be used with them. A general summary of the work carried out on the tracking study at The Franklin Institute will be found in their final report on this study.[116]

The first part of the study was carried out using a mechanical device for simulating the turret tracking control and the lead-computing sight. One objective of this study was to evaluate the use of aided tracking control on a turret. This type of control consists of a combination of the usual rate control and of direct control. A second objective of the study was to determine the best choice of the design constants for the lead-computing sight. The first results of this work are given in a report made in August of 1943.[117] The most outstanding result was the conclusive evidence that aided tracking control was substantially superior to rate control. Further work substantiated this and was reported to the Services soon thereafter.[118] Additional work[119] was done using the mechanical simulator with a considerably greater variety of control constants and lead-computing sight constants.

In any fire-control system the gunner must decide when to fire and when not to fire. In general, he should withhold his fire until the target is within suitable range and then should fire only when he feels that his guns are pointed with sufficient accuracy to get effective hits. When using a lead-computing sight, his only available method is to fire when he considers that his sight is accurately pointed at the target. A study was carried out to evaluate the gunner's ability to judge when to fire and when not to fire. The results of this study are given in two extensive reports.[120,121] The general conclusion reached is that the gunner's judgment of when to pull the trigger is quite unreliable. This is due to the fact that the gunner's judgment of his tracking error is very poor and also to the fact that with a lead-computing sight the intervals during which the tracking error is small is not necessarily those during which the gun error is small.

As has been indicated, the early work on tracking which was done at The Franklin Institute made use of a mechanical simulating mechanism. With this device it was possible to study only target courses having constant or variable angular rates at constant range. Such target courses are quite different from those actually found in practice. This was brought about due to a physical limitation contained in the design of the mechanical simulator. Since the use of such unrealistic courses in the study might have some influence on the final result, a new simulating mechanism was built on which realistic courses could be used. This mechanism was largely electrical in nature and somewhat more accurate in performance than the mechanical simulator. It also provided for a much greater variety of tracking-control constants and lead-computing sight constants. Studies on the electric simulator verified the results of the previous work and gave additional information

on the best choice of tracking-control constants and lead-computing sight design. Three reports on these results have been prepared.[77,78,79]

On all the work previously mentioned, the complete turret control and the lead-computing sight were simulated with the gunner remaining stationary. In an actual turret system the gunner is subjected to certain forces as a result of the motion of the turret in which he rides. Since this effect had not been considered in the previous studies, it was decided to continue the work using a system involving an actual turret. This was done and the results of that study have been reported.[122] An analysis of the work using the turret system was made to obtain additional data on the optimum tracking-control and sight constants.[123]

One additional factor, which was considered in these studies was the effect of training and practice. All through the work a statistical evaluation of the results was made to be sure that the conclusions regarding the tracking controls and the lead-computing sight constants were not unduly affected by variations in the subjects used in the study. A special study[124] was made to determine the effect of practice. It is interesting and somewhat discouraging to note that in spite of the fact that very early in the war the tracking studies at The Franklin Institute showed the superiority of aided tracking over velocity tracking it was not until very near the end of the war that any turrets were produced with aided tracking control.

With the development of the University of Texas Testing Machine another source of information on the accuracy of tracking became available. The main objective of this machine was to evaluate the overall performance of a flexible gunnery system. However, in the process of carrying out this evaluation, data on tracking and ranging accuracy also became available. The first data of this sort which was obtained at the University of Texas will be found in their report on the Mark 18 gunsight.[73] While the work at The Franklin Institute was entirely on tracking, the studies at the University of Texas were on both tracking and ranging. Before any extensive studies could be made on lead-computing sight systems, the B-29 central fire-control system became of greater importance. As has been previously mentioned, the accuracy of tracking with the GE pedestal sight was quite unsatisfactory. As a result, a series of tests on this system were undertaken by the University of Texas. The first results on tracking with the GE sight appear in an early report.[125] Additional data were accumulated on the studies of the overall accuracy of the GE system, which have been previously mentioned.[107,108,109]

The University of New Mexico also accumulated a large amount of data on tracking and ranging accuracy. One of their reports,[126] deals with the use of the K-3 sight mounted in a standard Sperry turret. Another[127] is concerned with tracking accuracy obtained with the GE B-29 system. Of all the studies thus far mentioned, only those at The Franklin Institute were sufficiently extensive to give any useful information on the choice of constants for the systems considered. A considerable amount of additional tracking information is available in the various tests conducted at the Army Air Forces Proving Ground Command, Eglin Field, Florida and at the Armament Test Section of the Naval Air Station, Patuxent River, Maryland. In both of these places the general objective of the the tests was the evaluation of the overall performance of the various systems and as a result there was very little analysis of tracking data. The one important exception is an analysis of the Eglin Field data.[128]

18.3 STABILIZATION

It has already been pointed out that the use of aided tracking control gives considerably better performance in tracking than the standard type of rate control. An additional improvement in the performance of manually operated tracking systems can be had by the use of stabilization. It is known that with the ordinary tracking-control system the accuracy of tracking is substantially reduced by the influence of the motion of the gunner's platform. This means that tracking accuracy obtained while a bomber is carrying out maneuvers or evasive action is much poorer than when the bomber is flying a straight and level course. This is principally due to the fact that under conditions of evasive action the gunner operating the track-

ing controls has not only to produce turret or sight motions which correspond to the motion of the target, but must also produce turret and sight motions which compensate for the motions of the bomber. The problem of compensating for the bomber's motion makes the tracking problem sufficiently more difficult so that the resultant accuracy decreases very greatly. By the use of a single or double gyro in the turret-control system, it is possible to arrange the controls so that the motions of the bomber are automatically compensated. This leaves the gunner with only the problem of providing the motions which are necessary to track the target.

The first stabilized turret control was developed very early in the war by the Ford Instrument Company. This turret was intended to be installed in the nose position of a Navy flying boat. Mainly because of its excessive size and weight this turret was never produced. It was subsequently improved by the Ford Instrument Company but only a few of these improved versions were ever built. The preliminary tests on the stabilized turret showed the great advantages to be gained by the use of stabilization. Following this Navy development there was no very serious effort to develop stabilized tracking controls until fairly well along in the war. This is surprising in view of the rather definite improvements to be gained.

Sperry developed a stabilized spherical turret for use in the nose and tail of the B-32. A number of these turrets were actually produced but very few of them were put into use. Sperry also undertook to develop a stabilized spherical tail turret which was to be used with the S-8 sighting system. In this turret the gyro which provides stabilization is also the gyro which is used for the computation of kinematic lead. This arrangement is one which is to be expected for future stabilized sighting systems. The Sperry S-8 sight system also provides for automatic S-band radar. Fairchild also developed a stabilized-control system in conjunction with the development of their S-3 sight. Both the Sperry and the Fairchild systems were still in development at the end of the war. As has already been mentioned, the General Electric Company had under development a completely stabilized sight for use with their B-29 fire-control system.[99] The first stabilized sighting system produced by Sperry was that used in the intermediate phase of their AGL development. This system was given rather extensive tests by Sperry although it was never flown or tested by the AAF. The Sperry system also made use of aided tracking control. Considerable tracking data is available in the Sperry report[112] as well as a number of comparative results showing the relative advantages of stabilization.

18.4 RADAR FOR GUNLAYING

During the war a tremendous amount of effort was put on development of various types of airborne radar sighting systems. The first developments were by Sperry[110,112] and General Electric.[90] In both of these systems the objective was to provide completely automatic position finding by means of radar. This was accomplished satisfactorily from the point of view of performance in both of these developments. However, for a number of reasons, the principal ones being size and weight, these systems were never used.

Fairly early in the war it became apparent that these completely automatic radar equipments were for use only on very heavy bombers and, therefore, were for use considerably in the future. As a result, the available effort was shifted from these systems to the development of very lightweight radars which could be used on existing aircraft. Since one of the greatest sources of error in the use of lead-computing sight systems was contributed by the very inadequate ranging performance obtained, considerable effort was made to develop a suitable radar ranging set. The first production model of this range set was the AN/APG-5. This set is described in a Navy operating manual prepared by the Radiation Laboratory.[129] Only a relatively small number of these sets were produced and these were entirely for experimental purposes.

Perhaps the biggest problem in providing radar range for use with flexible gunnery systems was that of finding suitable space for the installation of the radar set. This problem was not as severe in the case of fixed gun fighters and at the end of the war there were a number of experimental fighter systems under test which made use of the AN/APG-5 set for

automatic ranging. While a number of tests were made on fire-control systems using this radar set, no really reliable test information has been provided. Tests on radar ranging sets of the type AN/APG-5 indicate that the accuracy of range data obtained is entirely satisfactory. However, some difficulties often arise in the use of this set if there is more than one target in the field. This difficulty can be largely overcome by suitable training. Another factor in the use of radar ranging devices is that the use of accurate range may under some circumstances decrease the accuracy of the fire. This occurs when the systematic errors which are present in any computing system, are of about the same size or larger than the tracking errors. In such a case the use of accurate range may result in systematic misses, whereas with the use of manual ranging the dispersion of the ranging errors is sufficiently great to provide enough overall dispersion in the gun position to give at least some reasonable probability of getting hits. This means that accurate ranging data can be effectively used only when the computing system is properly calibrated. This effect has been amply demonstrated both analytically and experimentally.

The only radar set for aerial gunnery to be produced in any quantity was AN/APG-15B. This is described in an instruction manual prepared by the Radiation Laboratory.[130] This set provides not only automatic range information but an angular position indication which may be used for manually tracking the target. This set was eventually installed in the tail position of a number of B-29s which were sent to the Pacific Theater.

A rather good summary of the various fire-control radars available or under development at the end of the war has been prepared by the Armament Section of the Assistant Chief of Air Staff-4.[131]

Chapter 19

SIMULATION AND GUNNERY ASSESSMENT

19.1 ASSESSMENT METHODS AND SIMULATION

At the beginning of World War II, there was no adequate method and no suitable equipment available for adequately evaluating the performance of flexible gunnery systems. The usual test for determining the performance of such systems was for combat experienced gunners to use it by shooting at towed-flag targets. The quality of performance of the system was then judged by the percentage of hits obtained on the flag target. This method of test was later shown to be very inadequate. The usual experiment was not done under suitably controlled conditions, the gunners used were of unknown ability and training and the number of hits obtained on the target was usually so small as to give no reliable indication of the accuracy of shooting. In addition, the quality of performance was based largely on the gunner's opinion of the system under test.

One of the most important jobs which Section 7.2 did during the war was to assist in the establishment of suitable methods of assessing flexible gunnery systems and in the development of the necessary equipment. The Section was also largely responsible for educating the Army and the Navy in the general philosophy of quantitative experimental work.

There are several general methods that can be used for evaluating aerial gunnery systems. One such method is to study the equipment analytically and to compute its performance numerically. The main objection to this method is that it is impossible to take into account all the factors which influence the performance of such a system. This method is most useful in determining the performance of a system under assumed ideal conditions. It also has the advantage that it provides a means of breaking down the overall error into its component parts, and of determining the sources of the various parts of the error. This method was applied rather extensively to an analytical evaluation of the more common lead-computing sight systems. The work was carried out by the Applied Mathematics Group at Columbia University under the direction of the Applied Mathematics Panel. This work was known as Study 104 on The Analytical Assessment of Certain Lead Computing Sights. A large number of reports resulted and most of these have already been referred to under the sections of this report pertaining to the particular sighting system.

A second method of assessment is that of simulation. In this method the system under study is represented by a model or simulator which is then subjected to the various assessment experiments. The principal use of this method was in the various tracking studies conducted under the direction of Section 7.2. The tracking studies carried out at The Franklin Institute used such simulators and have already been discussed. The three simulators which were used in that study are described in three reports by The Franklin Institute.[132,133,134]

In any study of a gunnery system, the operator, or gunner, plays a very important part. It is very important that the errors in the overall performance of the system contributed by the gunner be known. In fact one of the most difficult problems in such assessment work is to design the experiment in such a way that the effect of the gunner's performance can be evaluated. In carrying out experiments with various types of simulators, it would be very desirable to have some sort of an artificial gunner. Some attempt was made to determine the general dynamic properties of a typical gunner which then could be built into an artificial gunner. While such work did not progress very far, enough was done to indicate the future possibilities. The earliest work appears to have been done in England.[135] The analysis of the dynamics of a human being was based on the use of sinusoidal courses of varying frequencies. The method of analysis used is that so commonly applied by electrical engineers. A similar but somewhat briefer study was made

by Columbia University using a slightly different technique of the electrical engineer. In this case the courses used for analysis were of the transient type. This work is described in the final report of the Columbia University project.[136]

19.2 UNIVERSITY OF TEXAS TESTING MACHINE

In addition to the two methods of assessment just described there are several entirely experimental methods which can be used to evaluate a complete gunnery system, including the gunner. One obvious such method is some form of flight test. This leads to certain practical difficulties which are present in any flight test. Even if the experiment is properly designed so that all the necessary factors are controlled, there are always the difficulties due to the inoperability of airplanes, the effects of weather, and the rather large time and cost involved. These difficulties can be largely overcome by the use of a device on the ground which will allow the use of a full-size gunnery system and provide the gunner with reasonably realistic target courses. The desirability of such a testing machine was pointed out early in the war by Section 7.2 (then Section D.2).[137] At the same time the general specifications for such a machine were also prepared.[138] Both the Army and Navy agreed to the desirability of such a ground testing machine for evaluating performance of flexible gunnery equipment, and the development of such a machine was carried out by Section 7.2 at their request. The work was done at the University of Texas and the machine developed by them has come to be known as the Texas Testing Machine. A very large amount of effort was put into its development and a great many proposals were studied for its design.

It was originally intended to build a machine which projected target spots onto a large screen which could then be followed by the gunner operating the equipment under test. On the basis of experience with such devices which had been obtained with the Waller gunnery trainer, this arrangement appeared to be feasible. However, preliminary study indicated that with such an arrangement the parallax introduced by the separation between the various axes of the testing machine and of the gunnery system were almost insurmountable. The arrangement finally accepted provides an optical target which can be moved in space to simulate the desired target motion. In its final form, the target can be made to travel any desired course whether ideal, such as a pure pursuit course, or real, such as had been actually determined by experiments. The final form of the optical target system also provides a moving picture of an airplane in its correct size and attitude. The image of the optical target and its position is controlled by a set of cams and therefore provides a target course which can be accurately and readily reproduced. While the work of preparing such a set of cams, which is required for each target course, is rather extensive, once these cams are prepared only a small amount of work is necessary to replace them with cams for another course. For each particular course the correct gun position is known and this is compared with the actual gun position obtained in the system under test. The gun error is continuously recorded so that the actual elevation and azimuth components of the gun error are known for each point on the course. In the case of certain gunnery equipment, such as the GE central fire-control system, it is possible to drive the sight mechanically so as to produce perfect tracking.

The original machine which was developed at the University of Texas has been in operation for some time and some of the results obtained from it have been referred to in previous sections of this report. Two additional machines were built. One of these is installed at the Armament Laboratory at Wright Field and the other at the Naval Air Station, Patuxent River, Maryland. Results obtained have shown the great usefulness of this machine, and without question these machines have already saved the cost of the original development. The present Texas Testing Machine is described in two reports from the University of Texas.[139,140] While a large amount of work was done on various phases of the development of this machine, there are very few reports available on this work. Even before the machine was finished, alternative methods of carrying out some of the operations performed were being

studied and designs prepared. The only other available material on this work is a series of informal progress reports.[141-146] In the future, it should be possible to use the three available Texas Testing Machines for most of the assessment work required on experimental and preproduction models of flexible gunnery systems. This will provide far more reliable data than was previously available and at a considerable saving of time and money.

19.3 PHOTOGRAPHIC METHODS FOR FLIGHT TESTS

The first adequate flight tests of flexible gunnery equipment were performed by the British.[147-151] This test was done by the Gunnery Research Unit at Exeter and was an assessment of the Mark II gyro gunsight. The first facility in the United States for adequately assessing flexible gunnery systems by flight test was developed by the AAF Proving Ground Command at Eglin Field under general guidance of Section 7.2. Camera equipment was developed and installed in the airplane carrying the equipment under test which would record the actual relative position of a fighter plane making a simulated attack. A pair of gyros were used to determine the roll, pitch and yaw of the bomber airplane. Other cameras were used for photographing the position of the target relative to the line of the gun and the position of the target and sight reticle as viewed through the sight. After the various films were processed the necessary data was read from them and the overall error in gun pointing was computed. Great care was taken to design the experiment and to control the various conditions so that the computed results were statistically reliable.

A large amount of time was required to develop the equipment needed by this test and to train the necessary personnel. Probably the most difficult problem in establishing this method was to educate the people concerned in the general philosophy of quantitative testing. It was found exceedingly difficult to get the people running the test to follow the actual operation schedule laid out, and to make sure that the equipment under test was in proper operating condition. Much time was lost due to missions being run under improper circumstances. The method of flexible gunnery assessment developed at Eglin Field has been completely described in one of their reports.[80]

Once a test had been planned, a relatively large amount of data could be collected in a short time. At first this was not the case because of breakdowns of the test equipment and the general difficulties of weather and airplane failures which are common to all flight tests. Even with the delays encountered in taking data, the time necessary to analyze the results and compute the gun errors was many times that necessary to take the original data. Any improvement in the method, particularly that of analyzing the data, promised large savings in time. For this reason an analysis of this assessment method was undertaken by the Applied Mathematics Group at Northwestern University.[152] This analysis was in effect an assessment of the assessment method being used at Eglin Field. It was found that certain approximations which had been used to simplify computation led to errors which were greater than desired. Improved methods of computation were recommended and several sets of results were computed using them.[153] As this assessment work continued other improvements were developed.[154] Another improvement which was developed at Eglin Field was the use of gnomonic charts[155,156] for carrying out a large part of the computation.

Before the flexible gunnery assessment method had been completely developed at Eglin Field, the Navy became interested and requested NDRC to develop a similar facility at the Naval Air Station, Patuxent River, Maryland. As part of this work, the Applied Mathematics Group at Northwestern University undertook a study of existing assessment methods, including the one under development at Eglin Field. The papers referred to above report part of that study. An excellent analysis of the general photographic method of gunnery assesement is given in reference 157. Other reports were prepared on the problem of determining the roll, pitch and yaw of the bomber airplane by means of gyros and the use of this information in correcting the original data for this motion.[158-162]

In carrying out the request of the Navy,

Section 7.2 of NDRC contracted with the Northwestern Technological Institute to do the work necessary to provide the Navy with the flexible gunnery testing facility it desired. Northwestern, after reviewing the Eglin Field method of analysis, decided that the analysis work could be done with less effort by means of special mechanisms. A set of such mechanisms was developed and built and put into use for the Navy. Duplicates were subsequently built for Eglin Field. The most complete description of the theory and operation of this photographic method of flexible gunnery assessment is given in a rather extensive set of reports prepared by Northwestern. These reports[163] should be required reading for anyone undertaking to understand this method and its use. Northwestern also undertook to prepare a computer's manual for processing the aerial gunnery assessment film obtained by this method. This manual is not yet complete but the first two parts have been finished. The first part is included in the report previously mentioned[163] and the second part is given in a separate report.[164] One useful computing device which was suggested for use in carrying out some of the work of analyzing the assessment films was suggested by the Applied Mathematics Group at Columbia University. This is known as the stereographic spherimeter, and is described in two reports prepared by them.[165,166]

In the flexible gunnery assessment work done both at Eglin Field and Patuxent River, it was always assumed that the bomber carrying the equipment under test was flying a straight-line course at constant speed. Under any other conditions errors would be introduced into the result. Since it was desirable to carry on some tests with the bomber taking evasive action, some work was done to modify the method so as to adapt it to this situation. This is described in reference 167.

The original evaluation test of the B-29 central fire-control system, which was carried out by Eglin Field, was done before the photographic method of assessment had been developed. After the development of this assessment method, the AAF planned to re-evaluate the B-29's using it. However, the General Electric Company decided that it would be to their advantage to have a testing facility of their own which could be used in the improvement of the present B-29 system and the development of new systems. They developed such a facility at Brownsville, Texas and succeeded in getting a fair amount of data by the end of the war. This method used a somewhat different technique than that in use at either Eglin Field or Patuxent River. It was one which was particularly adaptable to remote control turrets using electric servos. No adequate report of the details of the method is available but the program for the first set of extensive tests is described.[168]

19.4 OTHER METHODS

Two other methods of flexible gunnery assessment were proposed. The first of these[169] made use of distant reference points. This method was applied by the University of New Mexico in a few tests which they performed on the B-29. It was never fully analyzed and therefore its errors are not known. However, it is considerably simpler than any of the methods just discussed and should not be overlooked in future work. The other method[170] is essentially a vector method. It is also considerably simpler than the Eglin Field or Patuxent River methods. A preliminary study of this method[171] indicates that its accuracy is comparable.

One interesting training device which was developed during the war is the frangible bullet. This is a .30-caliber bullet which breaks up on striking very light armor. It can be fired at pursuit airplanes while they are making simulated attacks on the bomber. By modifying standard computing systems for the ballistics of this bullet, it is possible to give a gunner training under very realistic conditions. In the system finally developed, special armored P-63's were used to make the simulated attacks and these contained a hit indicator which counted the hits and flashed a light when a hit was being made. This system provided one psychological condition which is missing in all the assessment methods, namely the ability to fire live ammunition at the target in flight. For this reason, it was suggested that frangible bullets be used as an assessment means. However, this has two serious objections which were not fully understood by those suggesting its use. The first

is that it is not possible to accurately transpose the results obtained with frangible bullets into results which would have been obtained with standard live ammunition. The second is that a record of the number of hits on the target, which was a rather small percentage of the total rounds fired, is very inadequate evidence of the performance of the system. This is discussed in reference 172.

Although this report is concerned with flexible gunnery systems, some mention should be made of methods for assessing fighter gunnery. Considerable work on this was done by the Applied Mathematics Group at Northwestern, and their results are given in six reports.[173-178]

19.5 DISPERSION AND THE FIGURE OF MERIT

The usual results obtained by any test of a flexible gun system are given as the error in the gun position. These error data in their simple form are not particularly useful. This is because the effect of such an error on the ability of the system to bring down an attacking fighter is not at all evident. In fact the given angular error in gun position is much more serious at longer ranges than at shorter ranges. In addition, its effect is controlled to a large extent by the dispersion pattern of the bullets. A number of tests were run to determine the actual dispersion pattern of various flexible gunnery systems. By far the most outstanding of these tests were run by the Flexible Gunnery School at Laredo, Texas and the results of such tests on the B-17, B-24 and B-29 are given in two of their reports.[180,181] A detailed description of the testing method has also been prepared by them.[182]

The final measure of effectiveness of accuracy of a gunnery system should be expressed in terms of expected damage at the target. Considerable effort was expended in finding a suitable figure of merit of this sort and in developing methods for computing it. The general philosophy of such a figure of merit is discussed in reference 179. The figure of merit which was finally adopted for use in all the flexible gunnery assessment programs was the probability of a hit when a single bullet is fired. The details of this figure of merit and methods of computing it from gun error data will be found in a group of four reports.[183,184,185,186] A very complete and detailed analysis of this problem was made in England.[187]

A much more realistic method of evaluating the effectiveness of aerial gunnery is based on the idea of a duel between a fighter and a bomber. In this case an attempt is made to determine which airplane would be shot down first or which side would be successful in a duel between groups of fighters and bombers. The problem of answering such questions by evaluating the probability of either plane shooting down the other, while much more realistic, is considerably more difficult. A fair amount of work on this has already been accomplished, principally in England,[188,189,190,191] but considerably more needs to be done.

Chapter 20

DISCUSSION ON FUTURE WORK

20.1 INTRODUCTION

FUTURE DEVELOPMENT WORK in airborne fire-control systems should be of two types. The first of these is the continual improvement of systems now in existence or under development. The main objective of such work will be to obtain the best performance from the systems that can be put into use in the immediate future. If another national emergency arises soon, this is the only type of equipment that will be available for use. It is unlikely that such work will lead to any startling improvements in performance over that of present systems. The second type of development work is relatively long term in nature and would involve a search for systems which may be quite different than those in existence at present. The greatest possibility of making large improvements in performance is provided by such long-term work. However, such work must be based on estimates of the character of future aerial warfare and would have very little effect on equipment which would have to be used in the next few years. One major problem which must be solved is that of deciding what proportion of the total available effort is to be put on these two types of work. It goes without saying that neither one can be neglected. In this discussion only that work pertaining to the immediate future will be considered.

20.2 TRACKING CONTROLS

Evidence is already available that very worth-while improvements in the accuracy of gunnery systems can be obtained by improved methods of locating target position. One way in which this can be accomplished in the case of manual tracking is by the use of aided tracking controls. It is very probable that this should be made a part of all new tracking-control systems. While it was difficult to modify turrets produced during the war so that they would have aided tracking control, very little effort is required to provide it in new turret designs. Another very important aid to the gunner is line of sight stabilization. This has already been given considerable consideration and has been shown to contribute a substantial increase in accuracy in the performance of the system. Here also the probable type of future fire-control systems makes it possible to include stabilization in future designs with the expenditure of very little additional effort.

With present fire-control systems, by far the largest source of error has been the poor quality of the ranging. This means that one of the best ways of improving the accuracy of future systems is to improve the accuracy of ranging. The ranging controls used in manually operated systems during World War II were relatively crude. In addition, the necessity of doing ranging simultaneously with tracking made it very difficult to do either satisfactorily. Perhaps the best way of obtaining better range data is by the use of radar. This cannot be accomplished satisfactorily until present radar range sets are improved to the point where they can be easily installed and are less susceptible to the effects of multiple targets.

The use of completely automatic radar for tracking and ranging offers considerable promise. However, the cost, weight, and complexity of such equipment may make it unprofitable to rely on it completely. There is no doubt that for fighting at night or in bad weather, radar offers the only possible method. In any event, the use of both radar and manual operation should be the subject of further study. One of the major problems still unsolved is that of determining how these two tracking and ranging methods can best be exploited.

20.3 COMPUTERS

A wide variety of gunnery computers of both the local and remote-control types was developed during World War II. It was found relatively easy to design these computers so that they would determine the required ballistic

lead accurately. In fact, in most cases the accuracy of the ballistic lead provided was considerably greater than was warranted in view of inaccuracies present in other parts of the system. The main requirement which should be met by future ballistic computers is that they should be so designed as to make it possible to adapt them to various types of ammunition. This has already been accomplished to a large extent in some designs. The other requirement is that the ballistic computers should be simplified as much as possible so that the performance provided by them is in a better balance with that of the other parts of the system.

The largest errors present in existing computers occur in the computation of kinematic lead. Present computers are sensitive to the type of target course largely because the only course data used as inputs are present angle and range. The effects of range rate and angular acceleration are taken into account only through a calibration procedure which adapts the computer to a particular course. The next major step to be made in the improvement of kinematic-lead computers is to provide them with range rate and angular rate data and design them to use this additional information. This has been impossible in the past largely because the accuracy of such data as were available was inacceptable. The use of improved tracking and ranging methods will make it possible to provide range rate and angular rate data which can be used by improved kinematic-lead computers. In addition, all future computers should be based on the use of gyros. This calls for some improvement in the gyros themselves. Considerable work has already been done in this direction by Sperry, General Electric, Fairchild, and Westinghouse, and this work should be evaluated and made the basis for the future development of aerial gunnery equipment.

20.4 PSYCHOLOGICAL DESIGN

The psychological problems arising in aerial gunnery systems have been given far too little consideration in the past. The evidence that such problems existed first arose in the training field and a large amount of effort was put into solving them. One general problem which has not been fully appreciated has been the necessity of designing the controls of equipment so that they can be operated by a man in the best way. One of the biggest opportunities for improvement in performance lies in this field. It is also very important that people concerned with training be given an opportunity to evaluate new equipment, while still in the development stages, from the viewpoint of training. This will not only assist the training people in developing an adequate training program but will often prevent equipment from getting into the field which is difficult to operate. As an aid to these problems, it is desirable to learn more about the performance of a human being as a component in a tracking and ranging control system.

20.5 ASSESSMENT

The tremendous advantage of using a testing machine such as the Texas Tester over the usual flight test has already been demonstrated. This does not mean that the necessity for flight test has been eliminated but it does mean that most of the testing of experimental equipment and preproduction models can be accomplished on such a machine. This also makes it possible to carry out tests at a very early stage in development. It also makes it possible to select the most promising equipment from a number of competing developments, and thereby concentrate the effort and save considerable time and money.

This same sort of result can also be obtained by early mathematical analysis of various proposed systems. Such an analysis may often provide very valuable evaluation on such systems while they are in a very early stage of development. It also offers a very useful tool in planning the development program. During World War II most of the mathematical analysis of this sort was on equipment either in production or in a very late stage of development. In the future, such analysis should be one of the first things done when a new system is proposed.

There is ample room for improvement in the various existing methods of gunnery assessment. The present Texas Testing Machine is only the first model of such a machine. The needs for it were so pressing that every effort

was made to complete three of these as soon as possible rather than to provide a machine of improved performance at a much later date. The usefulness of this machine is so great that it will be well worth while to put some effort into producing a model which is more flexible than the present one and which will handle a much wider variety of equipment and target courses. The present photographic method of flight testing has reached a high state of perfection and it is doubtful if any large effort should be put into its improvement. However, the distance reference point method and the vector or own-speed method offer great promise. It is conceivable that either or both of these might provide a method with accuracy equal to that of the present method, but requiring far less work in analyzing the results. The development of both of these methods should be carried forward to the point where they can be compared with the present methods. In addition, some facility should be developed which can be used for carrying out tactical studies on the use of aerial gunnery equipment. This facility should be capable of providing all the control conditions and quantitative data that are required by any good experimental work.

20.6 WEAPONS

The standard weapon used in airplanes for bomber defense during World War II was the .50-caliber machine gun. Its performance is excellent and also considerably better than any of the small caliber weapons which were previously developed and used. However, this weapon was never used at its full effectiveness because of the inaccuracies of the fire-control system of which it was a part. No one questions the desirability of developing a weapon of larger caliber and greater range and higher-firing rate. The probable extent of improvements in this direction, however, does not offer as much overall improvement in the performance of gunnery systems as improvements in computer performance. Developments in new weapons should certainly continue, but they should not be made at the expense of developments which will lead to increased accuracy in other parts of the gunnery system. The most profitable direction for weapon development is in the direction of large caliber projectiles. One such projectile is the spin-stabilized rocket. While this probably is a relatively long-term development, it is possible that a suitable projectile of this sort would become available in the next few years.

20.7 PLANNING AND CONTROL

Perhaps the most serious lack in the fire control development work which was done during World War II was the lack of an adequate planning and control group within the Army Air Force. Most of the initiative for these developments came from industry. The Army Air Force exerted very little direction over this work and appeared to have no coordinated plan. As a result, a lot of competing developments were carried on simultaneously and the choice of the final system was not made on any logical basis. To avoid this difficulty in the future, it is necessary for the Army Air Forces to have a central planning group which has the authority to control the overall development program. It is also necessary to provide much greater use of highly trained and experienced civilians. In the past the direction of important development work has been given to officers or civilians who had neither the background nor the ability to do it properly. This situation has been further aggravated by the fact that the same organization has been responsible for procurement and production as well as research and development. There is ample experience in industry to show that these two types of functions should be separate.

APPENDIX

INDEX OF DIFFERENTIAL ANALYZER SOLUTIONS FOR GUIDED BOMB TRAJECTORIES (AZON AND RAZON)

REMARK

THE present index does not attempt to explain guided bombing or even to give an account of the use of the differential analyzer in the study to date of the guiding problem. It is merely a ready-reference record from which the various trajectory solutions may be conveniently selected. In the foreword which follows, the equations solved by the analyzer are given together with some of the more pertinent physical parameters, initial and final conditions, and so on. There is also included a brief explanation of the tabular form of the index itself.

FOREWORD

If it is assumed that the lift force acting on a bomb as a result of elevator control remains parallel to the original vertical plane of projection when rudder control is applied, then the differential equations for the bomb trajectory are

$$\ddot{x} = -\dot{x}\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\left[d(\delta_E,\delta_R,z) + \frac{s(\delta_R,z)\dot{y}}{\sqrt{\dot{x}^2+\dot{z}^2}}\right] - \frac{l(\delta_E,z)\,\dot{z}}{\sqrt{\dot{x}^2+\dot{z}^2}}(\dot{x}^2+\dot{y}^2+\dot{z}^2)$$

$$\ddot{y} = \sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\cdot\left[-d(\delta_E,\delta_R,z)\,\dot{y} + s(\delta_R,z)\sqrt{\dot{x}^2+\dot{z}^2}\right] \tag{1}$$

$$\ddot{z} = -\dot{z}\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\left[d(\delta_E,\delta_R,z) + \frac{s(\delta_R,z)\,\dot{y}}{\sqrt{\dot{x}^2+\dot{z}^2}}\right] + \frac{l(\delta_E,z)\,\dot{x}}{\sqrt{\dot{x}^2+\dot{z}^2}}(\dot{x}^2+\dot{y}^2+\dot{z}^2) - y$$

where x, y, z are Cartesian coordinates in a system whose origin is stationary in the air mass and at ground level directly beneath the point of release of the bomb, and whose x axis is in the direction of the velocity of the airplane with respect to the air at release. The functions $d(\delta_E,\delta_R,z)$, $s(\delta_R,z)$, $l(\delta_E,z)$ are defined by

$$d(\delta_E,\delta_R,z) = \frac{A}{M}\frac{\rho}{2}C_D(\delta_E,\delta_R),$$

$$s(\delta_R,z) = \frac{A}{M}\frac{\rho}{2}C_S(\delta_R), \tag{2}$$

$$l(\delta_E,z) = \frac{A}{M}\frac{\rho}{2}C_L(\delta_E),$$

where δ_E and δ_R are the elevator and rudder deflections measured in degrees from the position of zero control, A is the area of cross section of the bomb, ρ is the air density at altitude z, and C_D, C_S, C_L are ballistic coefficients of the bomb, given as empirical functions of δ_E and δ_R. M is the mass of the bomb.

All trajectories listed under RDA No. 119 are for range-only control. In this case equations (1) were used, where δ_E and δ_R are zero. In the case of rudder control the following approximate equations were used:

$$\ddot{x} = -\left[\dot{x}\left(d(\delta_E,\delta_R,z) + \frac{s(\delta_R,z)\,\dot{y}}{\sqrt{\dot{x}^2+\dot{z}^2}}\right) + l(\delta_R,z)\dot{z}\right]\cdot\sqrt{\dot{x}^2+\dot{z}^2},$$

$$\ddot{y} = \left[-d(\delta_E,\delta_R,z)\dot{y} + s(\delta_R,z)\sqrt{\dot{x}^2+\dot{z}^2}\right]\sqrt{\dot{x}^2+\dot{z}^2},$$

$$\ddot{z} = -\left[\dot{z}\left(d(\delta_E,\delta_R,z) + \frac{s(\delta_R,z)\,\dot{y}}{\sqrt{\dot{x}^2+\dot{z}^2}}\right) - l(\delta_E,z)\dot{x}\right]\cdot\sqrt{\dot{x}^2+\dot{z}^2} - y. \tag{3}$$

These were obtained by replacing $\dot{x}^2+\dot{y}^2+\dot{z}^2$ in the drag and side-force terms by $\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\sqrt{\dot{x}^2+\dot{z}^2}$, and in the lift-force term by $\dot{x}^2+\dot{z}^2$. All runs labeled RDA 120 are for equations (3). These equations reduce to equations (1) when s and $\dot{y}$ are zero. In RDA No. 121 equations (1) were used, with range-only control. In these runs the functions C_D and C_L were plotted against δ_E on input tables and fed in continuously during the course of the solution whenever control was applied. For all other runs the full value of C_D, C_S or C_L for any specified elevator or rudder position was set in at the indicated times.

Except where otherwise noted the function used for density was

$$\frac{\rho}{\rho_0} = 0.959\, e^{-2.96\times 10^{-5}z}$$

where $\rho_0 = 0.002378$ slug/ft³ is air density at sea level.

In RDA No. 119, Edition 1, and RDA No. 120, Edition 2, the value used for A was 1.865 ft², and for M 1,000 lb. In all other editions except where

noted the values used for both Azon and Razon were in the case of the 1,000-lb bomb $A = 1.89$ ft^2, $M = 1{,}020$ lb, and for the 2,000-lb bomb $A = 2.89$ ft^2, $M = 2{,}160$ lb.

The sources of the various values of C_D, C_S, C_L used are given for each edition. For Editions 8 and 9 the coefficients were obtained by varying the Wyckoff coefficients by ± 20 per cent. These runs are to be used to obtain, by interpolation or extrapolation, data for bombs with different ballistic coefficients. Also included in Edition 8 are some runs, Nos. 123-129, using coefficients corresponding to half or three-quarters control for an early Razon bomb with long horizontal fins or "sideburns."

The initial velocity of the bomb, v_0, is in the horizontal direction and given in miles per hour, except where otherwise specified. The time of flight, t_f, is given in seconds. Rudder and elevator deflection δ_R and δ_E are in degrees. The range and sidewise deflection at impact, $x(t_f)$, $y(t_f)$, are given in feet. The column headed t_1 gives the time of application of the control indicated on that same line. Where there is a blank, no control was applied and the trajectory is free fall. When for any run the first value of t_1 listed is not zero, then no control has been applied up to that instant. This control is understood to continue until a new value of t_1 with different control is listed, or until impact if no change in control is indicated.

RDA No. 119 Edition No. 1. Initial Altitude 15,000 feet.
1,000-pound Razon (Octagonal Shroud) Range-Only Control

(Akron Coefficients, 10-27-43)

Run No.	v_0	δ_E	C_D	C_L	t_1	t_f	$x(t_f)$
·	100	0°	0.406	0		32.294	4218
116	100	20°	0.692	0.981	0	36.284	8505
131	100	—20°	0.692	—0.981	0	33.301	195
113	100	20°	0.692	0.981	8	35.852	8282
130	100	—20°	0.692	—0.981	8	33.408	503
112	100	20°	0.692	0.981	15.5	34.852	7280
129	100	—20°	0.692	—0.981	15.5	33.186	1447
111	100	20°	0.692	0.981	23	33.249	5597
128	100	—20°	0.692	—0.981	23	32.678	2945
110	100	20°	0.692	0.981	27	32.619	4750
127	100	—20°	0.692	—0.981	27	32.428	3712

RDA No. 119 Edition No. 1 (*Continued*)

Run No.	v_0	δ_E	C_D	C_L	t_1	t_f	$x(t_f)$
	175	0°	0.406	0		32.524	7285
115	175	20°	0.692	0.981	0	38.342	12306
122	175	—20°	0.692	—0.981	0	32.430	2754
109	175	20°	0.692	0.981	8	37.426	11986
126	175	—20°	0.692	—0.981	8	32.832	3285
108	175	20°	0.692	0.981	15.5	35.747	10762
125	175	—20°	0.692	—0.981	15.5	32.882	4381
107	175	20°	0.692	0.981	23	33.680	8822
124	175	—20°	0.692	—0.981	23	32.644	5952
106	175	20°	0.692	0.981	27	32.849	7877
123	175	—20°	0.692	—0.981	27	32.499	6745
101	250	0°	0.406	0		32.578	10245
71	250	20°	0.692	0.981	0	41.100	16355
144	250	—20°	0.692	—0.981	0	31.481	4965
105	250	20°	0.692	0.981	8	39.492	15897
118	250	—20°	0.692	—0.981	8	32.300	5804
104	250	20°	0.692	0.981	15.5	37.013	14335
120	250	—20°	0.692	—0.981	15.5	32.636	7120
103	250	20°	0.692	0.981	23	35.590	12442
119	250	—20°	0.692	—0.981	23	32.651	8808
102	250	20°	0.692	0.981	27	33.139	10917
117	250	—20°	0.692	—0.981	27	32.606	9655
73	250	20°	0.692	0.981	8		
		0°	0.406	0	27		
		—20°	0.692	—0.981	29	36.865	12953
136	250	—20°	0.692	—0.981	0		
		0°	0.406	0	18		
		20°	0.692	0.981	19.3	32.482	9121
137	225	—20°	0.692	—0.981	0		
		0°	0.406	0	18		
		20°	0.692	0.981	19.3	32.696	8465
138	200	—20°	0.692	—0.981	0		
		0°	0.406	0	18		
		20°	0.692	0.981	19.3	32.897	7763
139	175	—20°	0.692	—0.981	0		
		0°	0.406	0	18		
		20°	0.692	0.981	19.3	33.066	7017
140	225	—20°	0.692	—0.981	0		
		0°	0.406	0	16		
		20°	0.692	0.981	17.3	33.371	9495
141	200	—20°	0.692	—0.981	0		
		0°	0.406	0	16		
		20°	0.692	0.981	17.3	33.502	8743
142	200	—20°	0.692	—0.981	0	32.123	3531
143	225	—20°	0.692	—0.981	0	31.807	4267
148	250	20°	0.692	0.981	8		
		0°	0.406	0	20		
		—20°	0.692	—0.981	22	35.040	10157
149	250	20°	0.692	0.981	8		
		0°	0.406	0	21		
		—20°	0.692	—0.981	23	35.281	10568
150	250	20°	0.692	0.981	8		
		0°	0.406	0	20.3		
		—20°	0.692	—0.981	22.3	35.107	10288

1,000-pound Azon (Square-tail)
(Akron Coefficients, 10-27-43)

Run No.	v_0	δ_E	C_D	C_L	t_1	t_f	$x(t_f)$
132	100	0°	0.265	0		31.678	4306
133	175	0°	0.265	0		31.766	7471
134	250	0°	0.265	0		31.881	10565

RDA No. 119 Edition No. 2. Initial Altitude 15,000 feet.
1,000-pound Azon (Preset elevator control)
(R. D. Wyckoff Coefficients 12-21-43)

Run No.	v_0	δ_R	δ_E	C_D	C_L	t_1	t_f	$x(t_f)$
23	250	0°	0°	0.262	0		31.855	10577
24	250	0°	1°	0.263	0.047	0	32.06	10857
25	250	0°	—1°	0.263	—0.047	0	31.659	10293
26	250	0°	—2°	0.264	—0.093	0	31.481	10026
27	250	0°	2°	0.264	0.093	0	32.289	11140
28	250	0°	3°	0.270	0.140	0	32.547	11422
29	250	0°	—3°	0.270	—0.140	0	31.332	9747
30	250	0°	—4°	0.276	—0.178	0	31.222	9522
31	250	0°	4°	0.276	0.178	0	32.771	11650
32	250	0°	5°	0.286	0.216	0	33.023	11875
33	250	0°	—5°	0.286	—0.216	0	31.139	9294

RDA No. 119 Edition No. 2 (*Continued*)
2,000-pound Razon
(R. D. Wyckoff Coefficients 12-21-43)

Run No.	v_0	δ_E	C_D	C_L	t_1	t_f	$x(t_f)$
41	250	0°	0.261	0		31.489	10751
46	250	20°	0.487	0.739	0	35.379	13969
42	250	20°	0.487	0.739	8	34.512	13684
43	250	20°	0.487	0.739	15.5	33.384	12832
44	250	20°	0.487	0.739	23	32.171	11607
47	250	20°	0.487	0.739	27	31.687	11031
48	250	—20°	0.487	—0.739	8	31.045	8078
49	250	—20°	0.487	—0.739	15.5	31.336	8912
50	250	—20°	0.487	—0.739	23	31.463	9974
51	250	—20°	0.487	—0.739	27	31.483	10477
52	175	0°	0.261	0		31.405	7577
63	175	20°	0.487	0.739	0	34.070	10192
55	175	20°	0.487	0.739	8	33.594	10010
54	175	20°	0.487	0.739	15.5	32.837	9361
62	175	20°	0.487	0.739	23	31.936	8332
57	175	20°	0.487	0.739	27	31.565	7820
58	175	—20°	0.487	—0.739	8	31.383	5270
59	175	—20°	0.487	—0.739	15.5	31.487	5930
60	175	—20°	0.487	—0.739	23	31.463	6872
61	175	—20°	0.487	—0.739	27	31.424	7334

RDA No. 120 Edition No. 2. Initial Altitude 15,000 feet.
1,000-pound Razon (Octagonal Shroud)
(Akron Coefficients, 10-27-43)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
83	100	0°	0°	0.406	0	0		32.286	4218	0
4	100	20°	0°	0.692	0.981	0	8	34.386	4052	3758
2	100	20°	0°	0.692	0.981	0	15.5	33.814	4139	2797
5	100	20°	0°	0.692	0.981	0	23	32.930	4201	1297
6	100	20°	0°	0.692	0.981	0	27	32.518	4215	513
	175	0°	0°	0.406	0	0		32.524	7285	0
10	175	20°	0°	0.692	0.981	0	8	34.697	6970	4174
9	175	20°	0°	0.692	0.981	0	15.5	34.003	7134	3016
8	175	20°	0°	0.692	0.981	0	23	33.094	7254	1379
7	175	20°	0°	0.692	0.981	0	27	32.664	7290	552

RDA No. 120 Edition No. 2 (*Continued*)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
15	250	0°	0°	0.406	0	0		32.591	10251	0
14	250	20°	0°	0.692	0.981	0	8	35.105	9749	4768
13	250	20°	0°	0.692	0.981	0	15.5	34.333	10029	3325
12	250	20°	0°	0.692	0.981	0	23	33.308	10202	1491
11	250	20°	0°	0.692	0.981	0	27	32.869	10237	535
87	250	20°	20°	0.978	0.981	0.981	8	41.663	14246	5952
90	250	20°	20°	0.978	0.981	0.981	15.5	38.862	13537	4181
91	250	20°	20°	0.978	0.981	0.981	23	35.062	11875	1774
92	250	20°	20°	0.978	0.981	0.981	27	33.439	10908	669
104	250	20°	—20°	0.978	0.981	—0.981	8	34.695	5645	4122
102	250	20°	—20°	0.978	0.981	—0.981	15.5	34.294	7057	2941
101	250	20°	—20°	0.978	0.981	—0.981	23	33.341	8803	1362
100	250	20°	—20°	0.978	0.981	—0.981	27	32.861	9651	564

RDA No. 120 Edition No. 2 (*Continued*)
1,000-pound Azon (Square-tail)
(Akron Coefficients, 10-27-43)

Run No.	v_0	δ_R	C_D	C_S	t_1	t_f	$x(t_f)$	$y(t_f)$
36	100	0°	0.265	0		31.685	4307	0
43	100	20°	0.487	0.513	8	32.844	4204	1960
41	100	20°	0.487	0.513	15.5	32.543	4259	1469
42	100	20°	0.487	0.513	23	32.176	4308	671
37	100	20°	0.487	0.513	27	31.812	4304	249
29	175	0°	0.265	0		31.770	7472	0
31	175	20°	0.487	0.604	8	33.118	7260	2604
32	175	20°	0.487	0.604	15.5	32.737	7380	1868
35	175	20°	0.487	0.604	23	32.173	7453	818
30	175	20°	0.487	0.604	27	31.910	7470	300
54	175	20°	0.487	0.513	15.5	32.694	7389	1584
20	250	0°	0.265	0		31.882	10563	0
27	250	20°	0.487	0.604	8	33.364	10229	3010
23	250	20°	0.487	0.604	15.5	32.923	10424	2076
26	250	20°	0.487	0.604	23	32.312	10536	887
28	250	20°	0.487	0.604	27	32.033	10560	323
55	250	20°	0.487	0.513	8	33.249	10251	2539
49	250	20°	0.487	0.513	15.5	32.850	10431	1754
46	250	20°	0.487	0.513	23	32.293	10541	754
45	250	20°	0.487	0.513	27	32.032	10563	277

RDA No. 120 Edition No. 3. Initial Altitude 15,000 feet.
1,000-pound Azon
(R. D. Wyckoff Coefficients, 12-21-43)

Run No.	v_0	δ_R	C_D	C_S	t_1	t_f	$x(t_f)$	$y(t_f)$
1	250	0°	0.262	0		31.853	10573	0
9	250	20°	0.580	0.637	0	34.050	9755	3841
5	250	20°	0.580	0.637	5	33.932	9994	3396
7	250	20°	0.580	0.637	10	33.684	10194	2884
34	250	20°	0.580	0.637	15	33.288	10363	2229
35	250	20°	0.580	0.637	20	32.765	10486	1427
36	250	20°	0.580	0.637	25	32.232	10554	613
15	250	20°	0.580	0.637	2.5			
		0°	0.262	0	7.5	31.966	10417	522
14	250	20°	0.580	0.637	7.5			
		0°	0.262	0	12.5	32.083	10452	623
22	250	20°	0.580	0.637	12.5			
		0°	0.262	0	17.5	32.199	10444	751

RDA No. 120 Edition No. 3 (*Continued*)

Run No.	v_0	δ_R	C_D	C_S	t_1	t_f	$x(t_f)$	$y(t_f)$
17	250	20°	0.580	0.637	17.5			
		0°	0.262	0	22.5	32.283	10487	819
18	250	20°	0.580	0.637	22.5			
		0°	0.262	0	27.5	32.262	10535	712
20	250	20°	0.580	0.637	27.5	32.017	10568	278
23	250	20°	0.580	0.637	5			
		−20°	0.580	−0.637	10			
		20°	0.580	0.637	15			
		−20°	0.580	−0.637	20			
		20°	0.580	0.637	25			
		−20°	0.580	−0.637	30	33.399	10083	312

RDA No. 120 Edition No. 4. Initial Altitude 15,000 feet. 2,000-pound Razon (R. D. Wyckoff Coefficients, 12-21-43)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
138	175	0°	0°	0.261	0	0	0	31.404	7576	0
100	175	0°	2°	0.265	0	0.155	0	31.796	8152	0
101	175	0°	−2°	0.265	0	−0.155	0	31.140	7028	0
102	175	0°	−5°	0.289	0	−0.303	0	30.987	6505	0
103	175	0°	5°	0.289	0	0.303	0	32.150	8671	0
104	175	0°	7°	0.312	0	0.384	0	32.443	8963	0
105	175	0°	−7°	0.312	0	−0.384	0	30.992	6226	0
106	175	0°	−10°	0.325	0	−0.485	0	30.951	5854	0
107	175	0°	10°	0.325	0	0.485	0	32.856	9318	0
108	175	2°	0°	0.265	0.155	0	0	31.429	7570	584
109	175	5°	0°	0.289	0.303	0	0	31.556	7542	1130
112	175	7°	0°	0.312	0.384	0	0	31.688	7519	1437
111	175	10°	0°	0.352	0.485	0	0	31.855	7466	1801
92	175	2°	2°	0.269	0.155	0.155	0	31.760	8129	595
93	175	−2°	−2°	0.269	−0.155	−0.155	0	31.168	7024	−579
94	175	−5°	−5°	0.307	−0.303	−0.303	0	31.105	6484	−1094
95	175	5°	5°	0.307	0.303	0.303	0	32.292	8642	1189
96	175	7°	7°	0.363	0.384	0.384	0	32.700	8878	1516
97	175	−7°	−7°	0.363	−0.384	−0.384	0	31.230	6171	−1370
98	175	−10°	−10°	0.443	−0.485	−0.485	0	31.398	5772	−1701
99	175	10°	10°	0.443	0.485	0.485	0	33.266	9106	1916
137	175	20°	0°	0.487	0.739	0	8	32.410	7422	2330
136	175	20°	0°	0.487	0.739	0	15.5	32.120	7510	1662
135	175	20°	0°	0.487	0.739	0	23	31.698	7545	709
134	175	20°	0°	0.487	0.739	0	27	31.496	7574	240

(See RDA No. 119, Ed. No. 2 for $\delta_R = 0°$, $\delta_E = \pm 20°$)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
81	175	−20°	−20°	0.713	−0.739	−0.739	8	32.357	5181	−2157
82	175	20°	20°	0.713	0.739	0.739	8	34.627	9725	2568
83	175	20°	20°	0.713	0.739	0.739	15.5	33.622	9236	1820
84	175	−20°	−20°	0.713	−0.739	−0.739	15.5	32.183	5893	−1557
85	175	−20°	−20°	0.713	−0.739	−0.739	23	31.744	6863	−701
86	175	20°	20°	0.713	0.739	0.739	23	32.243	8309	777
79	175	20°	20°	0.713	0.739	0.739	27	31.661	7826	247
80	175	−20°	−20°	0.713	−0.739	−0.739	27	31.512	7332	−231
49	250	0°	0°	0.261	0	0		31.488	10745	0
39	250	0°	2°	0.265	0	0.155	0	32.017	11444	0
40	250	0°	−2°	0.265	0	−0.155	0	31.042	10072	0
41	250	0°	5°	0.289	0	0.303	0	32.680	12119	0
42	250	0°	−5°	0.289	0	−0.303	0	30.737	9426	0
43	250	0°	7°	0.312	0	0.384	0	33.068	12468	0
45	250	0°	−7°	0.312	0	−0.384	0	30.624	9072	0
46	250	0°	10°	0.352	0	0.485	0	33.652	12908	0
48	250	0°	−10°	0.352	0	−0.485	0	30.565	8637	0
36	250	2°	0°	0.265	0.155	0	0	31.521	10734	735
37	250	5°	0°	0.289	0.303	0	0	31.666	10682	1431
34	250	7°	0°	0.312	0.384	0	0	31.793	10631	1806
35	250	10°	0°	0.352	0.485	0	0	32.009	10550	2270

RDA No. 120 Edition No. 4 (*Continued*)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
50	250	2°	2°	0.269	0.155	0.155	0	32.056	11431	757
51	250	2°	−2°	0.269	0.155	−0.155	0	31.075	10058	708
53	250	5°	5°	0.307	0.303	0.303	0	32.801	12050	1524
54	250	5°	−5°	0.307	0.303	−0.303	0	30.875	9388	1346
55	250	7°	7°	0.363	0.384	0.384	0	33.402	12329	1955
56	250	7°	−7°	0.363	0.384	−0.384	0	30.924	8985	1669
57	250	10°	10°	0.443	0.485	0.485	0	34.220	12643	2449
59	250	10°	−10°	0.443	0.485	−0.485	0	31.072	8494	2057
12	250	20°	0°	0.487	0.739	0	0	32.751	10274	3394
133	250	20°	0°	0.487	0.739	0	8	32.584	10490	2715
132	250	20°	0°	0.487	0.739	0	15.5	32.257	10643	1852
131	250	20°	0°	0.487	0.739	0	23	31.789	10727	770
130	250	20°	0°	0.487	0.739	0	27	31.588	10743	265

(See RDA No. 119, Ed. No. 2 for $\delta_R = 0°$, $\delta_E = \pm 20°$)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
52	250	20°	20°	0.713	0.739	0.739	0	36.571	13178	3836
68	250	−20°	−20°	0.713	−0.739	−0.739	8	32.118	7934	−2407
69	250	20°	20°	0.713	0.739	0.739	8	35.644	13196	3116
66	250	20°	20°	0.713	0.739	0.739	15.5	34.219	12626	2100
67	250	−20°	−20°	0.713	−0.739	−0.739	15.5	32.064	8850	−1676
64	250	−20°	−20°	0.713	−0.739	−0.739	23	31.754	9959	−720
65	250	20°	20°	0.713	0.739	0.739	23	32.504	11574	842
62	250	20°	20°	0.713	0.739	0.739	27	31.798	11030	273
63	250	−20°	−20°	0.713	−0.739	−0.739	27	31.589	10479	−253
10	250	20°	0°	0.487	0.739	0	5			
		0°	0°	0.261	0	0	18			
		−20°	0°	0.487	−0.739	0	20	32.275	10474	254
11	250	0°	20°	0.487	0	0.739	0	35.379	13969	0
12	250	20°	0°	0.487	0.739	0	0	32.751	10274	3394
14-15	250	20°	0°	0.487	0.739	0	5			
		0°	0°	0.261	0	0	18			
		−20°	5°	0.515	−0.739	0.303	20			
		−20°	−10°	0.578	−0.739	−0.485	30	32.555	10914	221
16	250	20°	0°	0.487	0.739	0	5			
		0°	0°	0.261	0	0	15			
		−10°	0°	0.352	−0.485	0	17	31.928	10525	9
17	250	20°	0°	0.487	0.739	0	5			
		0°	0°	0.261	0	0	15			
		−10°	0°	0.352	−0.485	0	17	31.952	10533	−12
19	250	20°	0°	0.487	0.739	0	5			
		0°	0°	0.261	0	0	15			
		−10°	0°	0.352	−0.485	0	17	31.947	10530	26
21-23	250	20°	0°	0.487	0.739	0	0			
		7°	0°	0.312	0.384	0	8.5			
		−7°	0°	0.312	−0.384	0	15	31.757	10491	149
24	250	20°	0°	0.487	0.739	0	0			
		7°	0°	0.312	0.384	0	8.5			
		−7°	0°	0.312	−0.384	0	14	31.760	10496	15
25	250	20°	20°	0.713	0.739	0.739	0			
		7°	7°	0.363	0.384	0.384	8.5			
		−7°	7°	0.363	−0.384	0.384	14	33.705	12235	−131
26	250	20°	20°	0.713	0.739	0.739	0			
		7°	0°	0.312	0.384	0	8.5			
		−7°	0°	0.312	−0.384	0	15	32.554	10768	72
27	250	20°	20°	0.713	0.739	0.739	0			
		7°	0°	0.312	0.384	0	8.5			
		−7°	0°	0.312	−0.384	0	14	32.316	10702	−29
28	250	20°	20°	0.713	0.739	0.739	0			
		7°	0°	0.312	0.384	0	8.5			
		−7°	0°	0.312	−0.384	0	14.5	32.548	10766	13
29	250	−20°	20°	0.713	−0.739	−0.739	0	36.565	13177	−3774
30	250	20°	20°	0.713	0.739	0.739	0	36.576	13181	3842
31	250	20°	20°	0.713	0.739	0.739	0			
		7°	7°	0.363	0.384	0.384	8.5			
		−7°	−7°	0.363	−0.384	−0.384	14	32.470	9914	−15

RDA No. 120 Edition No. 4 (*Continued*)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
33	250	20°	0°	0.487	0.739	0	0			
		7°	0°	0.312	0.384	0	8.5	31.887	10470	2152
70	250	7°	7°	0.363	0.384	0.384	8			
		7°	— 7°	0.363	0.384	—0.384	20	32.192	10734	1438
73	250	7°	7°	0.363	0.384	0.384	0			
		7°	— 7°	0.363	0.384	—0.384	20	32.634	10828	1828
75-76	250	7°	7°	0.363	0.384	0.384	8			
		7°	— 7°	0.363	0.384	—0.384	23	32.434	11186	1468
77	250	7°	7°	0.363	0.384	0.384	8			
		7°	— 7°	0.363	0.384	—0.384	17	31.956	10304	1402
87	250	7°	7°	0.363	0.384	0.384	8			
		7°	0°	0.312	0.384	0	15			
		7°	— 7°	0.363	0.384	—0.384	23	32.020	10626	1424
90	253.8	7°	7°	0.363	0.384	0.384	8			
	(10° Glide)	7°	— 7°	0.363	0.384	—0.384	20	30.286	10310	1356
140	250	7°	0°	0.312	0.384	0	8			
		— 7°	0°	0.312	—0.384	0	19	31.678	10696	19
143	250	20°	0°	0.487	0.739	0	8			
		—20°	0°	0.487	—0.739	0	19.5	32.303	10522	122
144	250	20°	0°	0.487	0.739	0	8			
		—20°	0°	0.487	—0.739	0	19	32.325	10530	— 19
146	250	0°	0°	0.261	0	0		31.487	10745	0
147	250	0°	7°	0.312	0	0.384	8			
		0°	— 7°	0.312	0	—0.384	19	31.848	10651	0
148	250	0°	— 7°	0.312	0	—0.384	8			
		0°	7°	0.312	0	0.384	19	31.502	10724	0
141	175	7°	0°	0.312	0.384	0	8			
		— 7°	0°	0.312	—0.384	0	19	31.579	7544	— 60
142	175	7°	0°	0.312	0.384	0	8			
		— 7°	0°	0.312	—0.384	0	19.5	31.578	7544	9
149	175	0°	— 7°	0.312	0	—0.384	8			
		0°	7°	0.312	0	0.384	19	31.487	7634	0
150	175	0°	7°	0.312	0	0.384	8			
		0°	— 7°	0.312	0	—0.384	19	31.674	7449	0
151	175	0°	0°	0.261	0	0		31.405	7576	0
88	175	7°	7°	0.363	0.384	0.384	8			
		7°	— 7°	0.363	0.384	—0.384	20	31.975	7539	1223

RDA No. 120 Edition No. 5. Initial Altitude 15,000 feet. 1,000-pound Razon

(R. D. Wyckoff Coefficients, 12-21-43)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
3	175	0°	0°	0.262	0	0	0	31.739	7475	0
102	175	0°	— 2°	0.264	0	—0.093	0	31.520	7030	0
103	175	0°	2°	0.264	0	0.093	0	32.010	7934	0
98	175	0°	— 5	0.286	0	—0.216	0	31.350	6429	0
99	175	0°	5°	0.286	0	0.216	0	32.494	8530	0
94	175	0°	— 8°	0.334	0	—0.320	0	31.362	5888	0
95	175	0°	8°	0.334	0	0.320	0	33.120	9033	0
90	175	0°	—10°	0.371	0	—0.382	0	31.435	5595	0
91	175	0°	10°	0.371	0	0.382	0	33.491	9280	0
114	175	2°	0°	0.264	0.093	0	0	31.762	7473	474
111	175	5°	0°	0.286	0.216	0	0	31.911	7437	1100
110	175	8°	0°	0.334	0.324	0	0	32.201	7363	1639
108	175	10°	0°	0.371	0.382	0	0	32.422	7307	1925
85	175	— 2°	— 2°	0.266	—0.093	—0.093	0	31.538	7024	— 479
86	175	2°	2°	0.266	0.093	0.093	0	32.029	7929	496
81	175	— 5°	— 5°	0.310	—0.216	—0.216	0	31.516	6396	—1067
82	175	5°	5°	0.310	0.216	0.216	0	32.665	8483	1160
77	175	— 8°	— 8°	0.406	—0.324	—0.324	0	31.820	5802	—1562

RDA No. 120 Edition No. 5 (*Continued*)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
78	175	8°	8°	0.406	0.324	0.324	0	33.558	8875	1743
72	175	—10°	—10°	0.480	—0.382	—0.382	0	32.111	5471	—1812
73	175	10°	10°	0.480	0.382	0.382	0	34.174	9046	2051
58	175	0°	20°	0.580	0	0.637	8	35.007	10196	0
59	175	0°	—20°	0.580	0	—0.637	8	32.316	4731	0
60	175	0°	—20°	0.580	0	—0.637	15.5	32.291	5522	0
61	175	0°	20°	0.580	0	0.637	15.5	33.971	9531	0
62	175	0°	20°	0.580	0	0.637	23	32.620	8386	0
63	175	0°	—20°	0.580	0	—0.637	23	32.013	6623	0
64	175	0°	—20°	0.580	0	—0.637	27	31.841	7166	0
65	175	0°	20°	0.580	0	0.637	27	32.031	7803	0
66	175	20°	0°	0.580	0.637	0	8	33.514	7188	2653
67	175	20°	0°	0.580	0.637	0	15.5	33.038	7347	1939
68	175	20°	0°	0.580	0.637	0	23	32.292	7451	854
69	175	20°	0°	0.580	0.637	0	27	31.932	7513	309
43	175	20°	20°	0.898	0.637	0.637	8	36.822	9646	2908
44	175	—20°	—20°	0.898	—0.637	—0.637	8	33.743	4575	—2416
45	175	—20°	—20°	0.898	—0.637	—0.637	15.5	33.586	5464	—1794
46	175	20°	20°	0.898	0.637	0.637	15.5	35.377	9260	2109
48	175	20°	20°	0.898	0.637	0.637	23	33.228	8332	913
49	175	—20°	—20°	0.898	—0.637	—0.637	23	32.583	6611	— 811
50	175	—20°	—20°	0.898	—0.637	—0.637	27	32.032	7162	— 301
51	175	20°	20°	0.898	0.637	0.637	27	32.236	7795	323
28	250	0°	0°	0.262	0	0		31.853	10576	0
100	250	0°	2°	0.264	0	0.093	0	32.290	11140	0
101	250	— 0°	— 2°	0.264	0	—0.093	0	31.481	10025	0
96	250	0°	5°	0.286	0	0.216	0	33.019	11871	0
97	250	— 0°	— 5°	0.286	0	—0.216	0	31.136	9292	0
92	250	0°	8°	0.334	0	0.320	0	33.869	12469	0
93	250	— 0°	— 8°	0.334	0	0.320	0	31.024	8629	0
87	250	0°	10°	0.480	0	0.382	0	34.405	12759	0
88	250	0°	—10°	0.480	0	—0.382	0	31.041	8269	0
113	250	2°	0°	0.264	0.093	0	0	31.880	10569	596
136	250	5°	0°	0.286	0.216	0	0	32.049	10504	1398
109	250	8°	0°	0.334	0.324	0	0	32.378	10376	2047
104	250	10°	0°	0.371	0.382	0	0	32.644	10286	2404
83	250	2°	2°	0.266	0.093	0.093	0	32.315	11132	625
84	250	— 2°	— 2°	0.266	—0.093	—0.093	0	31.524	10025	— 595
79	250	5°	5°	0.310	0.216	0.216	0	33.217	11788	1478
80	250	— 5°	— 5°	0.310	—0.216	—0.216	0	31.327	9235	—1312
74	250	8°	8°	0.406	0.324	0.324	0	34.385	12197	2227
75	250	— 8°	— 8°	0.406	—0.324	—0.324	0	31.541	8481	—1889
70	250	10°	10°	0.480	0.382	0.382	0	35.145	12337	2611
71	250	—10°	—10°	0.480	—0.382	—0.382	0	31.798	8057	—2176
19	250	0°	20°	0.580	0	0.637	8	36.204	13765	0
21	250	0°	—20°	0.580	0	—0.637	8	31.991	7415	0
22	250	0°	—20°	0.580	0	—0.637	15.5	32.134	8405	0
23	250	0°	20°	0.580	0	0.637	15.5	34.695	12937	0
24	250	0°	20°	0.580	0	0.637	23	32.940	11604	0
25	250	0°	—20°	0.580	0	—0.637	23	32.024	9642	0
26	250	0°	—20°	0.580	0	—0.637	27	31.927	10231	0
27	250	0°	20°	0.580	0	0.637	27	32.210	10945	0
38	250	20°	0°	0.580	0.637	0	8	33.800	10117	3104
39	250	20°	0°	0.580	0.637	0	15.5	33.236	10378	2148
40	250	20°	0°	0.580	0.637	0	23	32.452	10539	927
41	250	20°	0°	0.580	0.637	0	27	32.056	10568	339
29	250	20°	20°	0.898	0.637	0.637	0	39.075	12543	4150
30	250	20°	20°	0.898	0.637	0.637	8	38.113	12875	3457
31	250	—20°	—20°	0.898	—0.637	—0.637	8	33.874	7171	—2693
32	250	—20°	—20°	0.898	—0.637	—0.637	15.5	33.484	8302	—1912
33	250	20°	20°	0.898	0.637	0.637	15.5	36.194	12559	2419
34	250	20°	20°	0.898	0.637	0.637	23	33.591	11526	1019
35	250	—20°	—20°	0.898	—0.637	—0.637	23	32.629	9622	— 859
36	250	—20°	—20°	0.898	—0.637	—0.637	27	32.124	10225	— 324
37	250	20°	20°	0.898	0.637	0.637	27	32.434	10933	360
4	250	8°	8°	0.406	0.324	0.324	8			
		8°	— 8°	0.406	0.324	—0.324	20	32.918	10451	1632

RDA No. 120 Edition No. 5 (*Continued*)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
6	250	8°	8°	0.406	0.324	0.324	8			
		8°	—8°	0.406	0.324	—0.324	20.5	32.990	10523	1633
7-8	250	8°	8°	0.406	0.324	0.324	8			
		8°	—8°	0.406	0.324	—0.324	23.5	33.254	11010	1673
9	175	8°	8°	0.406	0.324	0.324	8			
		8°	—8°	0.406	0.324	—0.324	20.5	32.678	7425	1399
55	250	20°	0°	0.580	0.637	0	8			
		—20°	0°	0.580	—0.637	0	19	33.419	10164	—105
56	250	20°	0°	0.580	0.637	0	8			
		—20°	0°	0.580	—0.637	0	20	33.417	10165	203
119	250	20°	0°	0.580	0.637	0	0			
		—20°	0°	0.580	—0.637	0	12			
		0°	0°	0.262	0	0	20	32.671	9929	95
120	250	20°	0°	0.580	0.637	0	0			
		—20°	0°	0.580	—0.637	0	11			
		0°	0°	0.262	0	0	20	32.677	9931	—142
121	250	20°	0°	0.580	0.637	0	0			
		—20°	0°	0.580	—0.637	0	11.6			
		0°	0°	0.262	0	0	20	32.672	9931	9
122	250	0°	—20°	0.580	0	—0.637	0	31.454	6715	0
123	250	0°	—20°	0.580	0	—0.637	0			
		0°	0°	0.262	0	0	20	31.008	7895	0
125	250	0°	—20°	0.580	0	—0.637	0			
		0°	0°	0.262	0	0	21	31.039	7722	0
126	250	0°	—20°	0.580	0	—0.637	0			
		0°	20°	0.580	0	0.637	21	32.020	8780	0
130	250	0°	—8°	0.334	0	—0.324	8			
		0°	8°	0.334	0	0.324	20	31.921	10357	0
131-132	250	0°	—8°	0.334	0	—0.324	8			
		0°	8°	0.334	0	0.324	19.5	31.965	10445	0
133	250	0°	—8°	0.334	0	—0.324	8			
		0°	8°	0.334	0	0.324	19	32.019	10538	0
134	175	0°	—8°	0.334	0	—0.324	8			
		0°	8°	0.334	0	0.324	19	31.972	7540	0
135	175	0°	—8°	0.334	0	—0.324	8			
		0°	8°	0.334	0	0.324	19.5	31.936	7458	0

RDA No. 120 Edition No. 6. Initial Altitude 15,000 feet.
1,000-pound Razon
Effect of increase in

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
1	250	0°	0°	0.524	0	0		33.164	10003	0
2	250	8°	8°	0.668	0.324	0.324	8			
		8°	—8°	0.668	0.324	—0.324	20.5	34.248	9880	1534
3	250	8°	8°	0.668	0.324	0.324	8			
		8°	—8°	0.668	0.324	—0.324	21	34.289	9952	1538
4	250	0°	—8°	0.596	0	—0.324	8	32.696	8479	0

RDA No. 119 Edition No. 7. Initial Altitude 28,000 feet.
1,000-pound Razon
(R. D. Wyckoff Coefficients, 12-21-43)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
1	250	0°	0°	0.262	0	0		44.205	14332	0
3	250	8°	8°	0.406	0.324	0.324	8			
		8°	—8°	0.406	0.324	—0.324	30	46.616	14429	4128
4	250	0°	8°	0.334	0	0.324	8			
		0°	—8°	0.334	0	—0.324	30	45.106	14653	0

RDA No. 119 Edition No. 7 (*Continued*)

Run No.	v_0	δ_R	δ_E	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
5	250	0°	8°	0.334	0	0.324	8			
		0°	—8°	0.334	0	—0.324	29	45.011	14354	0
6	250	0°	—8°	0.334	0	—0.324	8	43.374	10236	0
7	250	0°	8°	0.334	0	0.324	0			
		0°	—8°	0.334	0	—0.324	29	45.333	14413	0
13	250	0°	20°	0.580	0	0.637	0			
		0°	—8°	0.334	0	—0.324	20	46.262	12502	0
14	250	0°	—20°	0.580	0	—0.637	0			
		0°	—8°	0.334	0	—0.324	20	43.629	8523	0
15	250	0°	20°	0.580	0	0.637	0			
		0°	—8°	0.334	0	—0.324	25	47.201	13966	0
16	250	0°	20°	0.580	0	0.637	0			
		0°	—8°	0.334	0	—0.324	26.5	47.509	14462	0
17	250	0°	20°	0.580	0	0.637	0			
		0°	—8°	0.334	0	—0.324	26	47.438	14303	0
8	175	0°	8°	0.334	0	0.324	8			
		0°	—8°	0.334	0	—0.324	29	45.126	10194	0
9	175	0°	0°	0.262	0	0	0	44.073	10119	0
10	175	8°	8°	0.406	0.324	0.324	8			
		8°	—8°	0.406	0.324	—0.324	30	46.173	10252	0
11	175	0°	—8°	0.334	0	—0.324	8	44.107	6438	0

RDA No. 120 Edition No. 8. Initial Altitude 15,000 feet.
1,000-pound Razon
(Synthetic Coefficients)

Run No.	v_0	F.F.C_D	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
1	250	0.2	0.2	0	0		31.545	10719	0
6	250	0.2	0.5	0	0.55	8	35.271	13430	0
7	250	0.2	0.5	0	—0.55	8	31.717	7892	0
10	250	0.2	0.5	0	0.55	23	32.469	11585	0
11	250	0.2	0.5	0	—0.55	23	31.714	9911	0
13	250	0.2	0.5	0	0.75	8	36.656	14896	0
14	250	0.2	0.5	0	—0.75	8	31.446	7048	0
19	250	0.2	0.5	0	—0.75	23	31.645	9645	0
20	250	0.2	0.5	0	0.75	23	32.715	11966	0
21	250	0.2	0.7	0	0.55	8	36.154	12995	0
22	250	0.2	0.7	0	—0.55	8	32.610	7738	0
28	250	0.2	0.7	0	0.55	23	32.799	11549	0
29	250	0.2	0.7	0	—0.55	23	32.005	9895	0
30	250	0.2	0.7	0	—0.75	23	31.936	9633	0
31	250	0.2	0.7	0	0.75	23	33.052	11920	0
36	250	0.2	0.7	0	0.75	8	37.481	14317	0
37	250	0.2	0.7	0	—0.75	8	32.360	6922	0
97	250	0.2	0.5	0.55	0	23	32.059	10682	792
100	250	0.2	0.5	0.55	0	8	33.289	10295	2722
102	250	0.2	0.5	0.75	0	23	33.616	10245	3754
105	250	0.2	0.5	0.75	0	23	32.117	10679	1083
106	250	0.2	0.7	0.55	0	8	34.196	10058	2616
109	250	0.2	0.7	0.55	0	23	32.368	10659	2319
110	250	0.2	0.7	0.75	0	8	34.521	10011	3609
113	250	0.2	0.7	0.75	0	23	32.427	10656	1074
42	250	0.3	0.3	0	0		32.043	10485	0
40	250	0.3	0.5	0	0.55	23	32.834	11379	0
43	250	0.3	0.5	0	—0.55	23	32.072	9675	0
47	250	0.3	0.5	0	0.55	8	35.281	13311	0
48	250	0.3	0.5	0	—0.55	8	31.759	7797	0
50	250	0.3	0.5	0	—0.75	23	32.010	9404	0
53	250	0.3	0.5	0	0.75	23	33.079	11761	0
54	250	0.3	0.5	0	—0.75	8	31.511	6964	0
57	250	0.3	0.5	0	0.75	8	36.627	14747	0
58	250	0.3	0.7	0	—0.55	23	32.381	9660	0

RDA No. 120 Edition No. 8 (*Continued*)

Run No.	v_0	$F.F.C_D$	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
61	250	0.3	0.7	0	0.55	23	33.174	11341	0
62	250	0.3	0.7	0	0.55	8	36.163	12882	0
65	250	0.3	0.7	0	—0.55	8	32.667	7653	0
66	250	0.3	0.7	0	—0.75	23	32.315	9360	0
69	250	0.3	0.7	0	0.75	23	33.432	11718	0
71	250	0.3	0.7	0	0.75	8	37.505	14191	0
74	250	0.3	0.7	0	—0.75	8	32.422	6838	0
79	250	0.3	0.5	0.55	0	8	33.326	10195	2709
82	250	0.3	0.5	0.55	0	23	32.422	10463	808
85	250	0.3	0.5	0.75	0	23	32.489	10460	1109
86	250	0.3	0.5	0.75	0	8	33.649	10146	3730
89	250	0.3	0.7	0.55	0	8	34.232	9961	2603
90	250	0.3	0.7	0.55	0	23	32.744	10439	799
93	250	0.3	0.7	0.75	0	23	32.806	10435	1096
94	250	0.3	0.7	0.75	0	8	34.550	9915	3584
2	175	0.2	0.2	0	0		31.456	7560	0
5	175	0.2	0.5	0	0.55	23	32.235	8334	0
8	175	0.2	0.5	0	—0.55	8	32.009	5133	0
9	175	0.2	0.5	0	0.55	8	34.300	9888	0
12	175	0.2	0.5	0	—0.55	23	31.705	6829	0
15	175	0.2	0.5	0	—0.75	8	31.896	4361	0
16	175	0.2	0.5	0	0.75	8	35.200	11022	0
17	175	0.2	0.5	0	0.75	23	32.387	8652	0
18	175	0.2	0.5	0	—0.75	23	31.674	6577	0
24	175	0.2	0.7	0	0.55	8	35.162	9620	0
25	175	0.2	0.7	0	—0.55	8	32.866	5044	0
26	175	0.2	0.7	0	—0.55	23	31.992	6820	0
27	175	0.2	0.7	0	0.55	23	32.520	8305	0
32	175	0.2	0.7	0	0.75	23	32.699	8621	0
33	175	0.2	0.7	0	—0.75	23	31.959	6569	0
34	175	0.2	0.7	0	—0.75	8	32.758	4291	0
35	175	0.2	0.7	0	0.75	8	36.036	10672	0
98	175	0.2	0.5	0.55	0	23	31.944	7538	728
99	175	0.2	0.5	0.55	0	8	33.053	7297	2350
103	175	0.2	0.5	0.75	0	8	33.331	7268	3239
104	175	0.2	0.5	0.75	0	23	31.996	7536	998
107	175	0.2	0.7	0.55	0	8	33.913	7148	2275
108	175	0.2	0.7	0.55	0	23	32.237	7523	721
111	175	0.2	0.7	0.75	0	8	34.189	7118	3133
112	175	0.2	0.7	0.75	0	23	32.292	7521	988
39	175	0.3	0.3	0	0		31.915	7426	0
44	175	0.3	0.5	0	—0.55	23	32.037	6682	0
45	175	0.3	0.5	0	0.55	23	32.546	8215	0
46	175	0.3	0.5	0	0.55	8	34.316	9821	0
49	175	0.3	0.5	0	—0.55	8	32.049	5082	0
51	175	0.3	0.5	0	—0.75	23	32.005	6425	0
52	175	0.3	0.5	0	0.75	23	32.719	8540	0
55	175	0.3	0.5	0	—0.75	8	31.945	4309	0
56	175	0.3	0.5	0	0.75	8	35.207	10949	0
59	175	0.3	0.7	0	—0.55	23	32.333	6673	0
60	175	0.3	0.7	0	0.55	23	32.863	8190	0
63	175	0.3	0.7	0	0.55	8	35.181	9558	0
64	175	0.3	0.7	0	—0.55	8	32.911	4991	0
67	175	0.3	0.7	0	0.75	23	32.300	6418	0
68	175	0.3	0.7	0	0.75	23	33.045	8511	0
72	175	0.3	0.7	0	0.75	8	36.050	10606	0
73	175	0.3	0.7	0	—0.75	8	32.809	4234	0
75	175	0.3	0.7	0	—0.75	23	32.299	6419	0
80	175	0.3	0.5	0.55	0	8	33.087	7239	2348
81	175	0.3	0.5	0.55	0	23	32.274	7408	745
84	175	0.3	0.5	0.75	0	23	32.329	7407	1019
87	175	0.3	0.5	0.75	0	8	33.362	7212	3231
88	175	0.3	0.7	0.55	0	8	33.944	7089	2276
91	175	0.3	0.7	0.55	0	23	32.580	7393	737
92	175	0.3	0.7	0.75	0	23	32.637	7390	1013
95	175	0.3	0.7	0.75	0	8	34.220	7063	3128

RDA No. 120 Edition No. 8 (*Continued*)
(Coefficients for Line of Sight Control)

Run No.	v_0	$F.F.C_D$	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
114	250	0.2	0.2	0	0		31.543	10718	0
115	250	0.2	0.8	0.75	0.75	8			
			0.8	0.75	—0.75	19	35.581	9501	3603
116	250	0.2	0.8	0.75	0.75	8			
			0.8	0.75	—0.75	22	36.166	10371	3733
117	250	0.2	0.8	0.75	0.75	8			
			0.8	0.75	—0.75	23.5	36.459	10806	3806
119	250	0.2	0.6	0.75	0.75	8			
			0.6	0.75	—0.75	22	35.296	10725	3918
123	250	0.3	0.51	0.79	0.79	8			
			0.51	0.79	—0.79	20.25	34.689	10183	4094
125	250	0.3	0.51	0.79	0.79	8			
			0.51	0.79	—0.79	23	35.290	11117	4274
126	250	0.3	0.51	0.79	0.79	8			
			0.51	0.79	—0.79	21.2	34.894	10504	4151
127	250	0.3	0.3	0	0		32.042	10489	0
128	250	0.3	0.61	1.19	1.19	8			
			0.61	1.19	—1.19	21.2	37.177	10108	6526
129	250	0.3	0.61	1.19	1.19	8			
			0.61	1.19	—1.19	22.2	37.505	10544	6653
130	250	0.3	0.4	0	0.79	8			
			0.4	0	—0.79	19.5	33.230	10241	0
131	250	0.3	0.4	0	0.79	8			
			0.4	0	—0.79	20.5	33.437	10607	0
132	250	0.3	0.4	0	0.79	8			
			0.4	0	—0.79	20.2	33.372	10497	0
133	250	0.3	0.4	0	—0.79	8			
			0.3	0	0	20	30.184	7540	0
135	250	0.3	0.4	0	—0.79	0			
			0.3	0	0	20	30.184	7537	0
136	250	0.3	0.456	0	1.19	0			
			0.456	0	—1.19	22	36.260	11151	0
137	250	0.3	0.456	0	1.19	0			
			0.456	0	—1.19	21	35.978	10670	0
138	250	0.3	0.456	0	1.19	0			
			0.456	0	—1.19	20.4	35.786	10384	0
139	250	0.3	0.456	0	—1.19	0			
			0.3	0	0	25	29.801	4832	0

RDA No. 120 Edition No. 9. Initial Altitude 15,000 feet.
2,000-pound Razon
(Synthetic Coefficients)

Run No.	v_0	$F.F.C_D$	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
1	250	0.2	0.2	0	0		31.266	10847	0
6	250	0.2	0.5	0	0.55	8	33.808	12805	0
10	250	0.2	0.5	0	—0.55	8	31.290	8708	0
11	250	0.2	0.5	0	—0.55	23	31.358	10261	0
14	250	0.2	0.5	0	0.55	23	31.885	11457	0
15	250	0.2	0.5	0	0.75	8	34.602	13774	0
18	250	0.2	0.5	0	—0.75	8	31.052	8071	0
19	250	0.2	0.5	0	—0.75	23	31.308	10068	0
22	250	0.2	0.5	0	0.75	23	32.029	11712	0
23	250	0.2	0.7	0	0.55	8	34.471	12531	0
26	250	0.2	0.7	0	—0.55	8	31.954	8586	0
27	250	0.2	0.7	0	—0.55	23	31.572	10250	0
30	250	0.2	0.7	0	0.55	23	32.107	11434	0
31	250	0.2	0.7	0	0.75	23	32.254	11685	0
34	350	0.2	0.7	0	—0.75	23	31.513	10055	0

RDA No. 120 Edition No. 9 (*Continued*)

Run No.	v_0	$F.F.C_D$	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
35	250	0.2	0.7	0	—0.75	8	31.702	7968	0
38	250	0.2	0.7	0	0.75	8	35.252	13452	0
40	250	0.2	0.5	0.55	0	8	32.451	10547	2010
43	250	0.2	0.5	0.55	0	23	31.610	10824	565
44	250	0.2	0.5	0.75	0	8	32.620	10521	2762
47	250	0.2	0.5	0.75	0	23	31.640	10823	772
48	250	0.2	0.5	0.55	0	8	33.112	10371	1956
51	250	0.2	0.5	0.55	0	23	31.825	10808	565
52	250	0.2	0.5	0.75	0	8	32.606	10192	2565
55	250	0.2	0.5	0.75	0	23	31.853	10807	767
56	250	0.3	0.3	0	0		31.627	10680	0
59	250	0.3	0.5	0	0.55	23	32.150	11304	0
65	250	0.3	0.5	0	0.55	8	33.820	12715	0
66	250	0.3	0.5	0	—0.55	8	31.335	8652	0
67	250	0.3	0.5	0	—0.55	23	31.631	10092	0
69	250	0.3	0.5	0	—0.75	8	31.095	8012	0
71	250	0.3	0.5	0	—0.75	23	31.574	9894	0
72	250	0.3	0.5	0	0.75	23	32.296	11562	0
75	250	0.3	0.5	0	0.75	8	34.602	13674	0
76	250	0.3	0.7	0	0.55	8	34.479	12445	0
79	250	0.3	0.7	0	0.55	23	32.379	11281	0
80	250	0.3	0.7	0	—0.55	23	31.841	10080	0
83	250	0.3	0.7	0	—0.55	8	31.989	8522	0
84	250	0.3	0.7	0	—0.75	8	31.746	7905	0
87	250	0.3	0.7	0	—0.75	23	31.783	9885	0
88	250	0.3	0.7	0	0.75	23	32.530	11535	0
91	250	0.3	0.7	0	0.75	8	35.251	13349	0
92	250	0.3	0.5	0.55	0	8	32.481	10474	2005
96	250	0.3	0.5	0.55	0	23	31.875	10664	572
97	250	0.3	0.5	0.75	0	8	32.648	10448	2742
100	250	0.3	0.5	0.75	0	23	31.904	10662	784
101	250	0.3	0.7	0.55	0	23	32.095	10648	569
104	250	0.3	0.7	0.55	0	8	33.142	10301	1948
105	250	0.3	0.7	0.75	0	8	33.308	10275	2677
108	250	0.3	0.7	0.75	0	23	32.123	10646	778
2	175	0.2	0.2	0	0		31.202	7637	0
7	175	0.2	0.5	0	0.55	8	33.131	9312	0
8	175	0.2	0.5	0	—0.55	8	31.532	5832	0
12	175	0.2	0.5	0	—0.55	23	31.362	7107	0
13	175	0.2	0.5	0	0.55	23	31.707	8176	0
16	175	0.2	0.5	0	0.75	8	33.653	10071	0
17	175	0.2	0.5	0	—0.75	8	31.400	5256	0
20	175	0.2	0.5	0	—0.75	23	31.329	6927	0
21	175	0.2	0.5	0	0.75	23	31.806	8391	0
24	175	0.2	0.7	0	0.55	8	33.766	9141	0
25	175	0.2	0.7	0	—0.55	8	32.147	5753	0
28	175	0.2	0.7	0	—0.55	23	31.560	7100	0
29	175	0.2	0.7	0	0.55	23	31.918	8161	0
32	175	0.2	0.7	0	0.75	23	32.019	8376	0
33	175	0.2	0.7	0	—0.75	23	31.526	6919	0
36	175	0.2	0.7	0	—0.75	8	32.012	5193	0
37	175	0.2	0.7	0	0.75	8	34.279	9871	0
41	175	0.2	0.5	0.55	0	8	32.286	7451	1723
42	175	0.2	0.5	0.55	0	23	31.527	7620	521
45	175	0.2	0.5	0.75	0	8	32.430	7435	2365
46	175	0.2	0.5	0.75	0	23	31.552	7620	709
49	175	0.2	0.7	0.55	0	8	32.908	7339	1681
50	175	0.2	0.7	0.55	0	23	31.730	7610	516
53	175	0.2	0.7	0.75	0	8	33.047	7323	2309
54	175	0.2	0.7	0.75	0	23	31.757	7609	704
57	175	0.3	0.3	0	0		31.533	7538	0
61	175	0.3	0.5	0	—0.55	23	31.603	7005	0
62	175	0.3	0.5	0	—0.55	8	31.561	5792	0
63	175	0.3	0.5	0	0.55	23	31.949	8092	0
64	175	0.3	0.5	0	0.55	8	33.153	9261	0
69	175	0.3	0.5	0	—0.75	8	31.430	5222	0
70	175	0.3	0.5	0	—0.75	23	31.569	6820	0
73	175	0.3	0.5	0	0.75	23	32.051	8313	0
74	175	0.3	0.5	0	0.75	8	33.658	10010	0
77	175	0.3	0.7	0	0.55	8	33.783	9096	0

RDA No. 120 Edition No. 9 (*Continued*)

Run No.	v_0	$F.F.C_D$	C_D	C_S	C_L	t_1	t_f	$x(t_f)$	$y(t_f)$
78	175	0.3	0.7	0	0.55	23	32.164	8077	0
81	175	0.3	0.7	0	—0.55	23	31.803	6997	0
82	175	0.3	0.7	0	—0.55	8	32.175	5717	0
85	175	0.3	0.7	0	—0.75	8	32.039	5159	0
86	175	0.3	0.7	0	—0.75	23	31.770	6814	0
89	175	0.3	0.7	0	0.75	23	32.270	8296	0
90	175	0.3	0.7	0	0.75	8	34.293	9825	0
93	175	0.3	0.5	0.55	0	8	32.312	7409	1716
94	175	0.3	0.5	0.55	0	23	31.769	7527	526
98	175	0.3	0.5	0.75	0	8	32.456	7395	2360
99	175	0.3	0.5	0.75	0	23	31.795	7527	722
102	175	0.3	0.7	0.55	0	23	31.977	7517	524
103	175	0.3	0.7	0.55	0	8	32.933	7298	1686
106	175	0.3	0.7	0.75	0	8	33.072	7282	2306
107	175	0.3	0.7	0.75	0	23	32.004	7516	717

RDA No. 120 Edition No. 10. Initial Velocity 250 mph.
1,000-pound Azon
(R. D. Wyckoff Coefficients, 12-21-43)
Altitude Effects

Run No.	Altitude	δ_R	C_D	C_S	t_1	t_f	$x(t_f)$	$y(t_f)$
2	5000	0°	0.262	0		18.025	6232	0
7	5000	20°	0.580	0.637	0	18.593	5934	1003
13	5000	20°	0.580	0.637	15.5	17.997	6210	47
3	10000	0°	0.262	0		25.782	8723	0
8	10000	20°	0.580	0.637	0	27.104	8127	2294
14	10000	20°	0.580	0.637	15.5	26.356	8640	825
1	15000	0°	0.262	0		31.853	10576	0
9	15000	20°	0.580	0.637	0	34.046	9753	3831
15	15000	20°	0.580	0.637	15.5	33.248	10381	2151
19	15000	20°	0.580	0.637	27	32.057	10568	339
4	20000	0°	0.262	0		37.040	12153	0
10	20000	20°	0.580	0.637	0	40.165	11115	5548
16	20000	20°	0.580	0.637	15.5	39.382	11791	3797
20	20000	20°	0.580	0.637	27	37.901	12102	1435
5	25000	0°	0.262	0		41.651	13554	0
11	25000	20°	0.580	0.637	0	45.731	12325	7375
17	25000	20°	0.580	0.637	15.5	44.970	13007	5625
21	25000	20°	0.580	0.637	27	43.387	13410	2993
6	30000	0°	0.262	0		45.833	14831	0
12	30000	20°	0.580	0.637	0	50.870	13444	9286
18	30000	20°	0.580	0.637	15.5	50.252	14124	7633
22	30000	20°	0.580	0.637	27	48.571	14564	4841

RDA No. 120 Edition No. 10 (*Continued*)
Initial Altitude 15,000 feet.
Velocity Effects

Run No.	v_0	δ_R	C_D	C_S	t_1	t_f	$x(t_f)$	$y(t_f)$
23	175	0°	0.262	0		31.743	7477	0
28	175	20°	0.580	0.637	0	33.703	6999	3127
29	175	20°	0.580	0.637	15.5	33.042	7349	1943
24	250	0°	0.262	0		31.854	10574	0
27	250	20°	0.580	0.637	0	34.048	9753	3831
30	250	20°	0.580	0.637	15.5	33.239	10377	2151
54	325	0°	0.262	0		31.979	13601	0
56	325	20°	0.580	0.637	0	34.445	12343	4728
55	325	20°	0.580	0.637	15.5	33.477	13328	2422

RDA No. 120 Edition No. 10 (*Continued*)

Run No.	v_0	δ_R	C_D	C_S	t_1	t_f	$x(t_f)$	$y(t_f)$
			Special Effects					
33	250	20°	0.580	0.637	0	34.044	9754	3832
34	250	20°	0.580	0.637	0			
		—20°	0.580	—0.637	10	33.731	9835	—1816
35	250	20°	0.580	0.637	0			
		—20°	0.580	—0.637	15	33.601	9828	— 549
36	250	20°	0.580	0.637	0			
		—20°	0.580	—0.637	20	33.555	9804	900
37	250	20°	0.580	0.637	0			
		—20°	0.580	—0.637	25	33.667	9776	2342
38	250	20°	0.580	0.637	0			
		0°	0.262	0	10	32.077	10149	1033
39	250	20°	0.580	0.637	0			
		0°	0.262	0	15	32.394	9990	1663
40	250	20°	0.580	0.637	0			
		0°	0.262	0	20	32.844	9867	2374
41	250	20°	0.580	0.637	0			
		0°	0.262	0	25	33.377	9791	3083
42	250	20°	0.580	0.637	0			
		0°	0.262	0	10			
		—20°	0.580	—0.637	20	33.080	10057	2474
			Trainer Data					
43	200 kts	0°	0.262	0		31.836	9773	0
44	200 kts	20°	0.580	0.637	15	33.232	9578	2166
45	200 kts.	20°	0.580	0.637	0	33.951	9046	3624
			Check Cases					
			$\frac{\rho}{\rho_0} = e^{-3.10 \times 10^{-5}z}$; $A = 1.87$ ft²; $M = 31.1$ slugs					
50	250	0°	0.26	0		31.894	10561	0
64	250	20°	0.58	0.86	15			
		0°	0.26	0	30	33.512	10313	2949
65	250	20°	0.58	0.86	15	33.711	10310	3174
			$\frac{\rho}{\rho_0} = e^{-2.96 \times 10^{-5}z}$					
57	250	0°	0.26	0		31.909	10549	0
58	250	20°	0.58	0.86	15	33.748	10293	3199

RDA No. 121 Edition No. 1. Initial Altitude 15,000 feet.
2,000-pound Razon
(R. D. Wyckoff Coefficients, 12-21-43)

Run No.	v_0	δ_E	C_D	C_L	t_1	t_f	$x(t_f)$
1	250	0°	0.261	0		31.485	10746
2	250	20°	0.487	0.739	0	35.375	13968
3	250	20°	0.487	0.739	5		
		—20°	0.487	—0.739	21	33.142	10990
4	250	20°	0.487	0.739	5		
		—20°	0.487	—0.739	16		
		20°	0.487	0.739	28	33.447	10052
5	250	20°	0.487	0.739	5		
		—20°	0.487	—0.739	19		
		0°	0.261	0	26		
		—20°	0.487	—0.739	30	32.819	11018
6	250	20°	0.487	0.739	5		
		—20°	0.487	—0.739	19		
		20°	0.487	0.739	29.5	32.887	10781

RDA No. 121 Edition No. 1 (*Continued*)

Run No.	v_0	δ_E	C_D	C_L	t_1	t_f	$x(t_f)$
9	250	20°	0.487	0.739	5		
		—20°	0.487	—0.739	21	33.148	11021
10	250	20°	0.487	0.739	5		
		—20°	0.487	—0.739	20		
		20°	0.487	0.739	31.5	32.985	10817
11	250	20°	0.487	0.730	5		
		—20°	0.487	—0.739	19		
		0°	0.261	0	27	32.821	10921
12	250	20°	0.487	0.739	5		
		—20°	0.487	—0.739	19		
		0°	0.261	0	29	32.824	10708
13	250	20°	0.487	0.739	0		
		—20°	0.487	—0.739	19		
		20°	0.487	0.739	30	33.361	10780
14	250	20°	0.487	0.739	0		
		—20°	0.487	—0.739	21	33.620	11027

Differential Analyzer Trajectories for Razon

The following few runs were made at the request of Gulf for the purpose of comparison with actual tests at Tonopah. Some of the parameters used differ from those of previous differential analyzer solutions.

$$M = 1{,}200 \text{ lb}$$
$$A = 1.89 \text{ ft}^2$$
$$\rho/\rho_0 = 0.765e^{-2.96 \times 10^{-5}z}$$
$$\rho_0 = 0.002378 \text{ slug/ft}^3$$
$$v_0 = 225 \text{ mi/hr}$$

The initial altitude is 15,000 feet. The same notation is used as in the other indices. C_s is zero for all runs.

RDA No. 119 Edition No. 4 and 4A.

Run No.	C_D	C_L	t_1	t_f	$x(t_f)$
2	0.225	0		31.286	9766
3	0.34	—0.36	0		
	0.225	0	15.6		
	0.340	—0.36	17.9	30.879	8448
4	0.31	—0.36	0		
	0.225	0	15.6		
	0.34	0.36	17.9	31.370	9737
5	0.34	0.36	0	32.699	11064
6	0.34	—0.36	0	30.863	8304
7	0.335	0		31.649	9622
8	0.627	—1.0	0		
	0.335	0	15.6		
	0.627	—1.0	17.9	30815	6294
9	0.627	—1.0	0		
	0.335	0	15.6		
	0.627	1.0	17.9	32.006	9657
10	0.627	1.0	0	36.246	13528
11	0.627	—1.0	0	30,834	5946

SUPPLEMENTARY INDEX OF DIFFERENTIAL ANALYZER SOLUTIONS FOR RAZON TRAJECTORIES

This index is of data made at the request of R. D. Wyckoff for the purpose of studying variation in time-of-flight as a result of control. Only range control is applied. The same differential equations are used as in previous RDA No. 119 solutions, with full elevator, either up or down, applied at the indicated times t_1. No allowance is made for the time the elevator takes to move from neutral position to that of full control, so that the aerodynamic coefficients are changed discontinuously. Control applied at a time t_1 continues until a change is made at a new value of t_1, or until impact. Time-of-flight and range-at-impact are denoted by t_f and $x(t_f)$.

The coefficients used are for the VB-3 Mk II, a 1,000-lb Razon model. The parameters used are as follows:

$$\text{Mass} = 1{,}100\ \text{lb}$$
$$\text{Area} = 1.89\ \text{sq ft}$$

The density function used is the same as for previous trajectories

$$\rho / \rho_0 = 0.959e^{-2.96 \times 10^{-5}z}$$

where

$$\rho_0 = 0.002378\ \text{slug/ft}^3$$

is the density of air at sea level.

RDA No. 119 Edition No. 8.
Initial Altitude 15,000 feet.
Initial Velocity (horizontal) 250 mph.

Run No.	C_D	C_L	t_1 (sec)	t_f (sec)	$x(t_f)$
1	0.230	0	—	31.608	10692
8	0.345	0.430	0	34.270	13049
2	0.345	0.430	5	33.889	12944
3	0.345	0.430	10	33.447	12683
4	0.345	0.430	15	32.926	12247
5	0.345	0.430	20	32.376	11682
6	0.345	0.430	25	31.895	11102
7	0.345	0.430	30	31.626	10722
9	0.345	—0.430	0	30.727	8272
10	0.345	—0.430	5	31.005	8499
11	0.345	—0.430	10	31.225	8823
12	0.345	—0.430	15	31.394	9257
13	0.345	—0.430	20	31.510	9781
14	0.345	—0.430	25	31.577	10309
15	0.345	—0.430	30	31.606	10662
16	0.345	0.430	10		
	0.230	0	15	32.042	11078
17	0.345	0.430	10		
	0.230	0	20	32.511	11579
18	0.345	0.430	10		
	0.230	0	25	32.978	12122

RDA No. 119 Edition No. 8 (*Continued*)

Run No.	C_D	C_L	t_1 (sec)	t_f (sec)	$x(t_f)$
19	0.345	0.430	10		
	0.230	0	30	33.347	12564
20	0.345	—0.430	10		
	0.230	0	15	31.385	10209
21	0.345	—0.430	10		
	0.230	0	20	31.245	9657
22	0.345	—0.430	10		
	0.230	0	25	31.199	9141
23	0.345	—0.430	10		
	0.230	0	30	31.283	8843
24	0.345	0.430	15		
	0.230	0	20	32.064	11202
26	0.345	0.430	20		
	0.230	0	25	32.025	11231
27	0.345	0.430	25		
	0.230	0	30	31.868	11062
28	0.345	—0.430	15		
	0.230	0	20	31.441	10126
29	0.345	—0.430	20		
	0.230	0	25	31.510	10141
30	0.345	—0.430	25		
	0.230	0	30	31.575	10335
32	0.345	0.430	10		
	0.345	—0.430	15		
	0.230	0	30	31.773	9634
33	0.345	—0.430	10		
	0.345	0.430	15		
	0.230	0	30	32.550	11634
34	0.345	0.430	10		
	0.345	—0.430	20		
	0.230	0	30	32.331	10630
35	0.345	—0.430	10		
	0.345	0.430	20		
	0.230	0	30	31.847	10515
36	0.345	0.430	10		
	0.345	—0.430	25		
	0.230	0	30	32.881	11692
37	0.345	—0.430	10		
	0.345	0.430	25		
	0.230	0	30	31.383	9457
38	0.345	0.430	10		
	0.345	—0.430	15		
	0.345	0.430	20		
	0.345	—0.430	25		
	0.230	0	30	32.194	10650
39	0.345	—0.430	10		
	0.345	0.430	15		
	0.345	—0.430	20		
	0.345	0.430	25		
	0.230	0	30	31.941	10515
40	0.345	0.430	10		
	0.230	0	15		
	0.345	0.430	20		
	0.230	0	25	32.484	11657
41	0.345	—0.430	10		
	0.230	0	15		
	0.345	—0.430	20		
	0.230	0	25	31.312	9676

PRELIMINARY INDEX OF DIFFERENTIAL ANALYZER SOLUTIONS FOR ROC TRAJECTORIES

This index records the data from differential analyzer trajectories for the Roc guided missile (00-1000-V) with range-control only. The same differential equations were used as for the Azon and Razon bombs.

The value used for the control area A was 9 ft^2; for M, 1,700 pounds. The density function used was

$$\rho = 0.959\rho_0 e^{-2.96 \times 10^{-5} z}$$

where $\rho_0 = 0.002378$ slug/ft^3 is air density at sea level. The time of application of control is t_1. Where no control was applied this item is left blank. The indicated control continues until a new control at a new value of t_1 is applied, or until impact if no change in control is listed. Time of flight and range at impact are given by t_f and $x(t_f)$. The maximum value for the bomb velocity is given in miles per hour, and the trail angle in mils.

The value of the drag coefficient C_D depends on whether or not brakes are used. For full brakes

$$C_D = 0.413 + 0.310\, C_L^2,$$

where C_L is zero for no control and ± 0.65 for maximum sail or dive.

For no brakes

$$C_D = 0.122 + 0.310\, C_L^2.$$

In some runs fractional amounts of brakes were applied. If this fraction is K, then

$$C_D = 0.122 + 0.291\, K + 0.310\, C_L^2.$$

All runs here recorded are made for an initial altitude of 15,000 feet, and initial velocity of 250 miles per hour. Runs will be obtained later for different altitudes and velocities.

RDA No. 119 Edition No. 5. Initial Altitude 15,000 feet.
Initial Velocity 250 mph.
00-1000-V Roc

Run No.	Brakes	Control	C_L	C_D	t_1 (sec)	t_f (sec)	$x(t_f)$ (feet)	Max Roc Velocity (mph)	Trail (mils)
1	Full	None	0	0.413	—	36.332	8744	428	302
2	Full	None	0	0.413	—				
	Full	Full Sail	0.650	0.543	8	51.600	18152	348	52
3	Full	None	0	0.413	—				
	Full	Full Dive	−0.650	0.543	8	37.774	1519	389	701
4	Full	None	0	0.413	—				
	Full	75% Sail	0.488	0.487	8				
	Full	75% Dive	−0.488	0.487	22	39.224	8440	402	384
5	Full	None	0	0.413	—				
	Full	75% Sail	0.488	0.487	8				
	Full	75% Dive	0.488	0.487	23	39.465	8860	402	391
6B	None	None	0	0.122	—	32.306	10385	597	100
7	None	None	0	0.122	—				
	None	Full Sail	0.650	0.252	8	71.095	33523	383	−470
8	None	None	0	0.122	—				
	None	Full Dive	−0.650	0.252	8	33.160	1918	502	610
9	None	None	0	0.122	—				
	None	75% Sail	0.488	0.196	8				
	None	75% Dive	−0.488	0.196	20.5	35.124	10072	544	188
10	None	None	0	0.122	—				
	None	75% Sail	0.488	0.196	8				
	None	75% Dive	−0.488	0.196	21	35.285	10377	542	172
11	Full	None	0	0.413	—				
	Full	75% Dive	−0.488	0.487	8	35.582	3998	411	553
12	None	None	0	0.122	—				
	None	75% Dive	−0.488	0.196	8	31.626	4183	547	459

Run No.	Brakes	Control	C_L	C_D	t_1 (sec)	t_f (sec)	$x(t_f)$ (feet)	Max Roc Velocity (mph)	Trail (mils)
13	None	None	0	0.122	—				
	None	75% Sail	0.488	0.196	8				
	None	75% Dive	−0.488	0.196	18	34.311	8703	544	258
14	None	None	0	0.122	—				
	None	75% Sail	0.488	0.196	8				
	None	75% Dive	−0.488	0.196	16.75	33.942	8044	545	303
15	Full	None	0	0.413	—				
	Full	75% Sail	0.488	0.487	8				
	Full	75% Dive	−0.488	0.487	18.5	38.390	7009	355	448
16	Full	None	0	0.413	—				
	Full	75% Sail	0.488	0.487	8				
	Full	75% Dive	−0.488	0.487	17.75	38.299	6713	406	463
18	None	None	0	0.122	—				
	None	75% Sail	0.488	0.196	8				
	None	75% Dive	−0.488	0.196	16.75				
	None	None	0	0.122	25	33.673	9725	582	176
19	Full	Full Sail	0.650	0.543	0				
	Full	Full Dive	−0.650	0.543	30	45.542	11090	392	364
20	Full	Full Sail	0.650	0.543	0				
	Full	Full Dive	−0.650	0.543	24	43.719	8212	383	489
21	Full	Full Sail	0.650	0.543	0				
	Full	Full Dive	−0.650	0.543	25	44.000	8692	382	469
22	None	Full Sail	0.650	0.252	0				
	None	75% Dive	−0.488	0.196	20	38.349	9723	538	287
23	None	Full Sail	0.650	0.252	0				
	None	75% Dive	−0.488	0.196	21	38.771	10254	537	263

RDA No. 119 Edition No. 6. Initial Altitude 15,000 feet.
Initial Velocity 250 mph.

00-1000-V Roc

Run No.	Brakes	Control	C_L	C_D	t_1 (sec)	t_f (sec)	$x(t_f)$ (feet)	Max Roc Velocity (mph)	Trail (mils)
2	75%	None	0	0.341	—	35.345	9109	461	256
3	75%	None	0	0.341	—				
	75%	75% Dive	—0.488	0.415	8	35.039	3599	432	563
4	25%	None	0	0.196	—	33.308	9929	542	154
13	25%	None	0	0.196	—				
	25%	75% Sail	0.488	0.270	8				
	Full	75% Dive	—0.488	0.487	19	37.604	8532	411	343
14	25%	None	0	0.196	—				
	25%	75% Sail	0.488	0.270	8				
	Full	75% Dive	—0.488	0.487	22	38.216	10084	411	261
15	25%	None	0	0.196	—				
	25%	75% Sail	0.488	0.270	8				
	Full	75% Dive	—0.488	0.487	16	37.067	7067	410	418
16	25%	75% Sail	0.488	0.270	—				
	Full	75% Dive	—0.488	0.487	19	39.594	8397	409	395
17	25%	None	0	0.196	—				
	25%	75% Sail	0.488	0.270	8				
	Full	83.5% Dive	—0.543	0.504	19	37.741	8148	407	370
18	25%	None	0	0.196	—				
	25%	50% Sail	0.325	0.229	8				
	Full	75% Dive	—0.488	0.487	19	36.564	8024	416	374
19	25%	None	0	0.196	—				
	25%	75% Sail	0.488	0.270	8				
	Full	50% Dive	—0.325	0.446	20	37.676	10135	422	245

SUPPLEMENTARY INDEX OF DIFFERENTIAL ANALYZER SOLUTIONS FOR ROC TRAJECTORIES

This index, like the preceding one, is of data on differential analyzer trajectories for the Roc guided missile (Douglas symbol: 00-1000-V) with range-only control, using the same constants.

Control area: $A = 9$ sq ft

Mass: $M = 1{,}700$ lb

Density: $\rho = 0.959\rho_0 e^{-2.96 \times 10^{-5}z}$

where $\rho_0 = 0.002378$ slug/ft^3 is taken as air density at sea level.

As before t_1 denotes the time of application of control, this item being left blank in the case of free fall. The amount of control is indicated as a certain per cent of the total available; it is continued until a new control is applied at a new value of t_1, or until impact. The amount of control is also given in terms of the value of the lift coefficient C_L, where this quantity is positive for sail, negative for dive.

The runs of RDA No. 130, Editions No. 1 and 2 given in this index differ from all previous trajectories in that any changes of control are applied continuously by the analyzer, the rate of change being determined by the time, 1.7 seconds, during which the control surface passes from the neutral position of no control to that of maximum control. This is, when sail is first applied at $t_1 = 8$ seconds, C_L is given by

$$C_L = \frac{0.65}{1.7}(t - S)$$

up until the instant when C_L reaches the specified value, after which it is held constant until a new time t_1, at which time a challenge is initiated in a similar way.

C_D is also varied continuously with C_L according to the formula

$$C_D = 0.209 + 0.310 C_L.$$

In all the runs of this index 30 per cent of the brakes were applied, corresponding to a value of C_D of 0.209, when there is no control.

As before t_f is the time-of-flight in seconds, $x\ (t_f)$ the range-at-impact in feet. Trail angle at impact is given in mils, using as the definition of a mil 6,400 mils = 360 degrees.

RDA No. 130 Editions No. 1 and 2 00-1000-V Roc

Run No.	Initial Velocity (mph)	Control	C_L	C_D	t_1(sec)	t_f(sec)	$x\ (t_f)$ (feet)	Max. Roc Velocity (mph)	Trail (mils)
Initial Altitude 5,000 feet									
7	200	None	0	0.209	—	18.417	4827	378	116.7
4	250	None	0	0.209	—	18.507	5961	391	166.6
8	300	None	0	0.209	—	18.601	7061	404	225.0
Initial Altitude 12,500 feet									
44	150	None	0	0.209	—	30.034	5606	500	81.4
55	150	60%	0.390	0.256	8				
		60%	—0.390	0.256	15.4	30.687	4667	480	168.3
30	200	None	0	0.209	—	30.168	7371	505	119.8
56	200	80%	0.520	0.293	8				
		80%	—0.520	0.293	15.4	31.467	6159	467	245.3
29	250	None	0	0.209	—	30.311	9085	508	164.0
35	250	75%	0.488	0.283	8				
		75%	—0.488	0.283	15	31.488	7747	475	300.5
38	250	75%	0.488	0.283	8				
		75%	—0.488	0.283	12.5	30.779	6706	477	357.7
48	250	100%	0.650	0.340	8				
		100%	—0.650	0.340	15.4	32.576	7541	449	345.0
52	250	95%	0.618	0.327	8				
		95%	—0.618	0.327	15.4	32.351	7613	455	333.7
31	300	None	0	0.209	—	30.468	10734	513	214.5
Initial Altitude 15,000 feet									
43	150	None	0	0.209	—	33.211	6096	528	82.0
42	150	25%	0.163	0.217	8				
		25%	—0.163	0.217	17	33.349	5514	525	123.1
47	150	50%	0.325	0.242	8				
		50%	—0.325	0.242	17	33.874	4997	511	165.3
53	150	45%	0.293	0.236	8				
		45%	—0.293	0.236	17	33.740	5097	515	156.7
5	200	None	0	0.209	—	33.362	8011	531	119.9
13	200	75%	0.488	0.283	8				
		75%	—0.488	0.283	20	35.661	7883	490	173.2
15	200	75%	0.488	0.283	8				
		75%	—0.488	0.283	21	35.909	8393	489	144.3
18	200	50%	0.325	0.242	8				
		50%	—0.325	0.242	20	34.563	7987	513	145.0
19	200	71%	0.462	0.276	8				
		71%	—0.462	0.276	17	34.827	6471	496	249.2
20	200	71%	0.462	0.276	8				
		71%	—0.462	0.276	17.2	34.856	6568	496	243.5
22	200	71%	0.462	0.276	8				
		71%	—0.462	0.276	20.6	35.606	8201	494	151.2
26	200	71%	0.462	0.276	8				
		71%	—0.462	0.276	22.4	36.061	9084	492	101.0
33	200	65%	0.423	0.264	8				
		65%	—0.423	0.264	20.6	35.311	8213	499	144.6
41	200	50%	0.325	0.242	8				
		50%	—0.325	0.242	17	34.059	6914	514	206.1
46	200	60%	0.390	0.256	8				
		60%	—0.390	0.256	17	34.428	6739	506	224.4
50	200	60%	0.390	0.256	8				
		40%	—0.260	0.230	17	34.318	7634	719	163.7
60	200	60%	0.390	0.256	8				
		80%	—0.520	0.293	17	34.779	5860	486	286.9

RDA No. 130 Editions No. 1 and 2 (*Continued*)

Run No.	Initial Velocity (mph)	Control	C_L	C_D	t_1(sec)	t_f(sec)	$x(t_f)$ (feet)	Max Roc Velocity (mph)	Trail (mils)
64	225	None	0	0.209	—	33.431	8927	533	142.0
62	225	67%	0.436	0.268	8				
		67%	—0.436	0.268	17	34.816	7463	500	267.1
63	225	67.5%	0.439	0.269	8				
		67.5%	—0.439	0.269	17	34.831	7455	500	268.0
2	250	None	0	0.209	—	33.523	9870	534	163.1
3	250	75%	0.488	0.283	8				
		75%	—0.488	0.283	20	36.206	9732	492	236.3
11	250	75%	—0.488	0.283	8				
		83.5%	—0.543	0.300	21	36.600	9966	485	230.6
12B	250	75%	0.488	0.283	8				
		75%	—0.488	0.283	17	35.344	8205	494	312.7
40	250	75.5%	0.491	0.284	8				
		75%	—0.488	0.283	18.5	35.752	8974	493	274.0
6	300	None	0	0.209	—	33.683	11672	538	210.7
21	300	79%	0.514	0.291	8				
		79%	—0.514	0.291	17	35.909	9875	492	383.2
23	300	79%	0.514	0.291	8				
		79%	—0.514	0.291	14.6	35.065	8630	487	433.5
24	300	79%	0.514	0.291	8				
		79%	—0.514	0.291	13.4	34.661	8032	494	456.9
25	300	79%	0.514	0.291	8				
		79%	—0.514	0.291	12	34.194	7347	494	483.0
36A	300	100%	—0.650	0.340	8				
		100%	—0.650	0.340	13.5	35.896	7155	462	532.2
37	300	85%	0.553	0.304	8				
		85%	—0.553	0.304	14	35.156	8093	485	465.5

Initial Altitude 17,500 feet

Run No.	Initial Velocity (mph)	Control	C_L	C_D	t_1(sec)	t_f(sec)	$x(t_f)$ (feet)	Max Roc Velocity (mph)	Trail (mils)
45	150	None	0	0.209	—	36.194	6543	552	82.4
59	150	38%	0.247	0.228	8				
		38%	—0.247	0.228	18.5	36.416	5552	550	142.3
61	150	40%	0.260	0.230	8				
		40%	—0.260	0.230	18.5	36.750	5376	539	156.4
27	200	None	0	0.209	—	36.342	8602	554	119.1
58	200	50%	0.325	0.242	8				
		50%	—0.325	0.242	18.5	37.292	7162	533	216.5
28	250	None	0	0.209	—	36.503	10603	556	160.6
34	250	75%	0.488	0.283	8				
		75%	—0.488	0.283	19	39.046	8650	507	319.0
39	250	75.5%	0.491	0.284	8				
		75%	—0.488	0.283	21.5	39.726	10092	507	255.0
49	250	75%	0.488	0.283	8				
		75%	—0.488	0.283	18.5	38.903	8345	507	332.3
51	250	50%	0.325	0.242	8				
		50%	—0.325	0.242	18.5	37.566	9152	536	263.0
54	250	65%	0.423	0.264	8				
		65%	—0.423	0.264	18.5	38.290	8692	520	302.1
57	250	63%	0.410	0.261	8				
		63%	—0.410	0.261	18.5	38.179	8757	523	296.4
32	300	None	0	0.209	—	36.675	12539	559	206.5

Initial Altitude 30,000 feet

Run No.	Initial Velocity (mph)	Control	C_L	C_D	t_1(sec)	t_f(sec)	$x(t_f)$ (feet)	Max Roc Velocity (mph)	Trail (mils)
9	250	None	0	0.209	—	49.288	13704	636	147.3
14	250	75%	0.488	0.283	8				
		75%	—0.488	0.283	28	55.973	9616	573	355.2
16	250	75%	0.488	0.283	8				
		75%	—0.488	0.283	31	56.382	12522	570	270.2
17	250	50%	0.325	0.242	8				
		50%	—0.325	0.242	31	52.805	13831	604	185.7
10	300	None	0	0.209	—	49.481	16241	637	185.7

GLOSSARY

AIBR. Acceleration-integrator bombing release. A name for toss-bombing component of Bureau of Standards origin.

AZON. Stands for AZimuth ONly. A guided bomb for linear ground targets. Radio-controlled and flare-observed.

BARB. British-type angular rate bombsight. An American name for certain instruments based on the British-invented principle of absolute angular-rate bombing criteria for low altitudes.

BUG. An apocryphal name for target used in bombing trainers. Crawls on the floor. Certain species crawl in circles, or are unpredictable to the trainee.

CART. Constant-angular-rate turntable. Testing apparatus for angular rate meters. Typically, suspended torsion pendula for accurate studies.

CARP. Chronometric, automatic, ROC (or RAZON) predictor. Alternatively, CRAB actually reaches perfection. A sight for guided bombs, involving a program leading to terminal collinearity over an interval.

CLIP. Computation of lead with an inertia-pendulum. A discarded, but often recalled, project of questionable origin.

CRAB. Chronometric range-anticipator bombsight. Other names have also been applied and are appropriate. A sight for aiding the guider of RAZON in range, principally, in which the terminal point is predicted in terms of the time of flight. Attaches to the bombsight Mark 15.

DBS. Dive bombsight. A sight developed by M. Alkan and Specialties, Inc. for the Navy. Toss principle with freed gyro.

DOVE. Origin unknown to writer. A high-angle guided or homing bomb with "spoilers" or "deflectors" in front for steering. Polaroid, Navy project; Division 5 cognizance.

GCB. Ground-controlled bombing. As with RAZON; ground stations steer bomb to target on known bearings.

GOAD. Giving only angular depression. Local name for rudimentary "triangular-solution" bombsights (visual).

GRASP. Gyroscopic, rocketeers, automatic, sighting predictor; or gyroscopic RASP. A rocket sight and computer for attachment to the gyro gunsight, as Mark 23.

JAG. Just another gadget. A time-of-flight corrector, for use with guided bombs and CRAB.

MIMO. Miniature IMage Orthocon. Small television camera and transmitter for ROC.

ORTHOPENTAX. Linkage having five perpendicular axes, of many and diverse applications.

PACT. Pilot's automatically computing toss bombsight. A linkage computer, with (pneumatic) integrator of incremental acceleration, and automatic release.

PARS. Pilot's automatic rocket sight. A miniature computer as aiming control for forward-fired rockets.

PRESET. Name given to computers for bombers or bombardiers. For auxiliary function to bombsight Mark 15, giving rate-knob adjustment, etc., in advance.

PUSS. Pilot's universal sighting systems. Computer for guns, rockets, and bombs, to be employed in fighter airplanes, etc.

RASP. Rocketeer's automatic sighting predictor. An early project which resulted in an experimental pilot's rocket sight.

RAZON. Stands ostensibly for RAnge and AZimuth ONly. Very misleading. Should have been AZAR, for AZimuth And Range, for example. Two-coordinate AZON, intended for point targets.

ROC. Douglas high-angle guided bomb. Named for the roc or rokh, a giant bird of Arabian mythology.

SNIFFER. Low-altitude blind bombsight, involving f-m radar. Employs range, range rate, and altitude in automatic computation of release condition.

VASS. Vosseller antisubmarine sight. Hand-held bombsight based on Vosseller's method of extrapolation. Chronometric.

VERB. Prehistoric name used for RASP. Rocket bombsight, preceded by two words, which are now unknown; perhaps, very exceptional.

BIBLIOGRAPHY

Numbers such as Div. 7-112.11-M1 indicate that the document listed has been microfilmed and that its title appears in the microfilm index printed in a separate volume. For access to the index volume and to the microfilm, consult the Army or Navy agency listed on the reverse of the half-title page.

PART I

Chapter 1

1. *Corrections Necessary in Aiming a Machine Gun, Mounted on an Airplane, at a Moving Target*, J. R. Moore, K8064695, General Electric Company, June 1941.

   A good exercise in vectorial methods, and some fundamental ballistics for aerial gunnery, by one of the most active workers in this field.

2. *Gyroscopic Lead Computing Sights*, Report to the Services 13, NDRC Section D-2, August 1941. Div. 7-112.11-M1

   An exposition, on fundamental grounds, of the technique of lead computing, principally with regard to a single "disturbed" gyro of the eddy current dome variety. Derivations of the familiar formulas are included, as also is a compete physical interpretation of the theoretical steps.

3. *An Introduction to the Analytical Principles of Lead Computing Sights*, Saunders MacLane, OEMsr-1007, AMP Memo 55.1, AMG-Columbia, March 1944. AMP-503.6-M21

   Mathematically rigorous and accomplished treatment of the classical lead computing dynamics. Fundamental concepts are explained and developed to show the current stage of the theory. A glossary of notation (with translations) and a good bibliography are included.

4. *Pursuit Courses*, Walter Leighton, OEMsr-1007, AMP Memo 57.4, AMG-Columbia, Mar. 29, 1944. AMP-503.7-M1

   A study in detail of such courses, in the air and with respect to the target, with thoroughgoing rigor. Both pure pursuit and that with lead are treated, although "mushing" is not considered. Tables are included for several standard approaches.

5. *The Extrapolation, Interpolation and Smoothing of Stationary Time Series*, Norbert Wiener, OSRD 370, Research Project DIC-6037, Report to the Services 19, NDRC Section D-2, MIT, Feb. 1, 1942. Div. 7-313.1-M2

   An extension of some Russian work, using the methods of communications engineering, statistical theory, and Fourier analysis. The author's earlier work is relevant hereto. Not so difficult to read as is commonly supposed.

6. *Statistical Method of Prediction in Fire Control*, Norbert Wiener and Weaver, NDCrc-83, Report to the Services 59, Research Project 6, Dec. 1, 1942. Div. 7-112.2-M2

   Contains a discussion of some of the results of reference 5 above, and of their probable importance in the prediction of straight flight and of flight which is accelerated in various ways. Comparisons among known methods of prediction are included, together with certain historical material.

7. *An Exposition of Wiener's Theory of Prediction*, N. Levinson, OEMsr-1384, AMP Note 20, AMG-Harvard, June 1945. AMP-13-M21

   Develops the autocorrelation function more gradually, showing that the linear prediction problem reduces to that of the solution of an integral equation. Practical and mathematical difficulties in application are discussed, and errors in prediction are interpreted in mathematical form.

8. *Collision Courses by Method of Overcorrecting Changes in True Bearing*, (Capt.) V. A. Kimberly, USN, Letter to Chief BuOrd from Special Board on Naval Ordnance, September 1927.

   This letter gives an elementary explanation, in terms of surface vessels, of the method named, and shows how and why the method leads to a straight interception as the range closes up. There are not many such explanations available in the literature.

Chapter 2[a]

1. *Electrical Simulation of the Human Operator in Tracking Mechanisms as an Aid in the Study of Sight Dynamics*, R. H. Randall and F. A. Russell, OEMsr-1237, TR T-13, NDRC-Section 7.2, Columbia University Division of War Research, June 21, 1945.

   A report on work originally urged by the present writer. The attempt is described to employ a proposed representation of the operator ϕ in lieu of the real thing for laboratory tracking experiments.

2. *Investigation of the Operator's Response in Manual Control of a Power Driven Gun*, A. Tustin, C. S. Memorandum 169 (British).

   A review of this memorandum is given in the report last mentioned.

3. *Tracking Aircraft with Heavy Turrets*, Merz and McLellan, British Liaison Code WA-1711-1.

   Sets up and studies the hystero-differential equations which result for the ensemble when a direct time delay is attributed to the human operator. Conditions for stability and learning are considered somewhat qualitatively.

4. *Some Characteristics of Human Operators in Control Systems*, K. J. W. Craik (Cambridge University), Ministry of Supply Informal Panel on Servomechanisms, British Liaison Code WA-1641-6, Great Britain, Feb. 4, 1944.

   Apparatus and techniques are described whereby the human operator was studied when performing with direct tracking. Curves of pursuit agree substantially with those obtained in our laboratories. The effects of the excellence of the display and other psychological aspects of the problem are discussed. Also, the transient and harmonic methods of investigation are compared.

5. *The Conduction of the Nervous Impulse*, Keith Lucas, Longmans Green and Company, revised, 1917.

   Contains evidence that nerve conduction, in the large, is a lineaphenomenon, the time relations therein being apparently inder pendent of the strength of stimulus. (This is contrary to the popular conception.) Reflex phenomena are discussed as being a local affair connected with the nerves themselves.

6. *The Mechanism of Nervous Action*, E. D. Adrian, Eldridge Reeves Johnson Foundation, 1931 Lectures,

[a] The reader's attention is called to the related bibliography included in the list for Chapter 1. The writings of H. Whitney on tracking, which have been promulgated by AMG-Columbia of the Applied Mathematics Panel, should also be referred to. Whitney's views, which are arrived at through a minimum of quantitative experiment and a maximum of personal intuition, are nevertheless worthy of attention. Although most of his work has been in connection with turrets, see also his AMG-Columbia Working Paper 329, of Dec. 13, 1944, *Notes on the Tracking Problem for Fighter Planes*, which advances explanations for some of the anomalous results observed in this kind of tracking. For quantitative material, with extensive collation and analysis, relating to the man-machine interactions with standard types of tracking with lead-computing sights, see the rich body of literature which has accumulated under the program sponsored by Section 7.2 at The Franklin Institute. This work will be reported upon fully by S. H. Caldwell, J. B. Russell, and H. C. Wolfe of that section. For the contractor, the psychologists Preston and Irwin, who have been protégés of S. W. Fernberger at the University of Pennsylvania, were in control of the experimental and analytic procedures involved, and their reports are models of exhaustive disclosure and of zealous adherence to the dictates of the data.

Press of the University of Pennsylvania, 1932.

In this book evidence is given, and is referred to there as *proof*, that the workings of the nervous system are not beyond mechanical description.

7. *Pursuitmeters: Alcohol and Human Efficiency*, W. R. Miles, Carnegie Institute Publication.

Apparatus involves complex harmonic generation using wattmeter, error indicator, and manually operated slide-rheostat. Very modern in spirit and objective. Continuous records are shown.

8. "A Pursuit Device for Obtaining both Quantitative and Qualitative Records," R. M. Collier, *Journal of Psychology*, Vol. 2, 1936, pp. 295-300.

Description of curve-drawing apparatus similar to that constructed at The Franklin Institute in the earliest work on the human tracking response. In contrast to the pursuitmeters of this reference and reference 7, most such equipment is of the discontinuous, or averaging, variety.

Chapter 3

1. *Further Application of the Strain Gauge to Gyroscopic Measurements of Angular Rates*, J. D. Eisler, U. C. S. Dilks, and W. W. Felton, Report 152, Franklin Institute, Nov. 9, 1943.

Title is self-explanatory. The principal application in mind was to lead-computing sights. Methods and apparatus are described for compensation required for gyro speed changes, temperature, etc. The noise problem is explained, together with its possible solution. Circuits and photographs are shown.

2. *Symmetrically Constrained Gyros as Angular Rate Indicators*, M. Golomb, Report 218, Franklin Institute, July 27, 1945.

A theoretical treatment of the dynamics involved in the gyro suspensions proposed for the simultaneous measurement of angular rate about two axes with a single gyro. Several types of constraint are considered, and the stability conditions expressed. The effects are described of static friction in the gimbals.

3. *Servo-Controlled Gyroscope*, U. C. S. Dilks and M. Golomb, Report 238, Franklin Institute, Oct. 7, 1945.

Theory and construction of the two-dimensional, electric captive gyro under development for PUSS. Its effectiveness, in speed and accuracy for large and small absolute angular rates, is discussed along with its limitations. Circuits are given, as well as methods of testing, and the numerical data are included on the basic gyro component involved.

Chapter 4[b]

1. *Control Circuits for Radio Controlled Units*, J. R. Ragazzini and L. Julie, Diary of Columbia Project, OEMsr-1237, Dec. 8, 1943.

Contains a brief exposition of the simulative philosophy as applied to control problems. Electronic details are omitted but are implicit. Practical exercises, for sight-simulation, guided bombs, etc., should be consulted for circuits.

2. *Diary Concerning a Conference on November 29 on Subject of Columbia Project*, J. R. Ragazzini, OEMsr-1237, Dec. 14, 1945.

Proposals for simulation of control deflections having boundary conditions, as under human operation. Use of feedback amplifiers, as integrators, etc., outlined, preparatory to the simulative project on guided missiles.

3. *Simulation of PUSS Rocket Sight*, J. R. Ragazzini, OEMsr-1237, NO-265, Report M-35, Columbia University, June 11, 1945.

Shows the then proposed PUSS formulas, and details for their simulation electronically.

4. *Aerodynamic Constants for a Simulated Airplane*, F. A. Russell, OEMsr-1237, NO-265, Report M-38, Columbia University, June 13, 1945.

Arrangement of degrees of freedom, and their connecting relations, for the simulative project.

5. *Axis Conversion*, R. H. Randall, OEMsr-1237, Report M-36, Columbia University, June 21, 1945.

Application of rigid-body dynamics to the problem of airplane simulation. Coordinate systems are worked out, after much deliberation, which appear manageable for incorporation in the electronic model components.

6. *Airplane Simulator (for Small Angles)*, F. A. Russell and R. H. Randall, OEMsr-1237, NO-265, Report M-32, Columbia University, June 21, 1945.

Summary of the first working phase of the simulative development for PUSS. References to the aerodynamic literature are included, as are the schematic circuits employed in simulation.

7. *On the Study of Cyclic Dynamical Systems by Means of Equivalent Networks*, L. Jafek, Cossor Industries, Aug. 5, 1942.

[b] See also Chapter 8.

Chapter 5[c]

1. *Theory of Gyroscope Suspended in a GAP Linkage*, M. Golomb, Memo to G. A. Philbrick, Franklin Institute, Dec. 14, 1944.

An analysis of the orthopentax employed as a gyro suspension. Its possibility as forming a lead-computing system for PUSS was then being explored. Certain practical advantages were expected, but no practical application has yet been made.

Chapter 6

1. *Torpedo Director, Mark 32*, R. W. Pitman, OSRD 5079, OEMsr-330, NO-106, Report 172, Franklin Institute, Jan. 26, 1944. Div. 7-141-M1

Describes the final model of the torpedo director which resulted from Project NO-106. Foreword by A. L. Ruiz. Figures and rewriting by G. A. Philbrick. This document is in the form of an instruction manual, but is roughly definitive of the whole development prior to 1944.

2. *Torpedo Director, Model Number One*, R. W. Pitman, R. K. Marshall, F. W. Schlesinger, and others, NO-106, Report 120, Franklin Institute, Aug. 15, 1942.

Covers phases of work on this project during the first half-year of development. Describes vector theory and first mechanizations thereof. Discusses the problem of stabilization in azimuth. Appendices treat related problems, such as the optical ones connected with the pilot's sight head. References are included to even earlier documents.

[c] For documents on the computing linkages such as are being prepared for the PUSS project at The Franklin Institute, this contractor's final report, not received at the present writing, should be consulted. It is now expected that this work is to be extended under Navy auspices, so that another future source of information will be Section Re8c of the Bureau of Ordnance.

The final report of The Bristol Company under their contract (OEMsr-1387) with NDRC Section 7.2, may also be consulted, in connection particularly with manipulated linkage developed within the PUSS project.

See also Radiation Laboratory (MIT), and Librascope Corporation reports, for other computing linkages.

3. *The Electrical Solution for the Torpedo Director (NO-106)*, J. D. Eisler, Report 135, Franklin Institute.

Describes the a-c vector embodiment of the same theory as in mechanical directors. Certain advantages in flexibility of installation were thus sought, and a technique was experimentally developed for further applications. Many circuits and photographs are shown.

4. *Stabilization of the Mark 31 Torpedo Director for Motor Torpedo Boats*, J. D. Eisler, NO-134, Report 148, Franklin Institute, June 15, 1945.

Mainly a servo development which derived stabilization for the target velocity, as set in this director, by means of the Flux-gate compass. Details of the construction. Test data are not included, but good operation was experienced in field tests at the MTBSTC at Melville, Rhode Island.

Chapter 7

1. *U. S. Navy Bombsight Mark 20*, R. W. Pitman and others, Report 200, Franklin Institute, August 1943. Div. 7-122.2-M2

A descriptive manual of the theory and instrumental principles, and of the installation and maintenance of this bombsight. Detailed instructions for operation, with pictorial aids, are included.

2. *Auxiliary Ground Speed Computer*, John A. Bevan and others, Report 167, Franklin Institute, Dec. 12, 1943.

Description of, with instructions for use of, a miniature mechanical computer for use with hand-held bombsights. Of wider applicability, however.

3. *Automatic Altitude Adjustment for Bombsight Mark 20*, U. C. S. Dilks, OSRD 5080, OEMsr-330, NO-129, Report 161, Franklin Institute, Jan. 24, 1944. Div. 7-122.2-M1

Account of an experimental project wherein the altitude determined by the f-m radio altimeter is servoed into the bombsight continuously. Very small equipment resulted, attachable without interference to a hand-held instrument. A d-c resistive feedback principle is applied.

4. *Bombsight V, Model 3*, John A. Bevan, Report 168, Franklin Institute, Mar. 6, 1944.

This is one of the many adaptations of the basic principle of the Mark 20, although without chronometric extrapolation. Each modification led to diminishment in size and complexity. Very satisfactory results were obtained in flight tests at NAS Banana River.

5. *Bombsight VI, LTA*, John A. Bevan, Report 178, Franklin Institute, May 6, 1944. Div. 7-122.4-M2

An adaptation of bombsight V, so-called, to the bombing problem of blimps. This sight became bombsight Mark 24 and enabled a surprising accuracy of dropping.

6. *Slant Range Computer*, R. W. Pitman, Report 156, Franklin Institute, Nov. 5, 1943. Div. 7-123-M1

Description of a small, flat, manually operated computer, giving the slant range at which bomb release should occur, in horizontal flight at low altitude, in terms of altitude and closing speed. One-hand operation. Model submitted to BuAer.

7. *The Preset Computer Model 3*, John A. Bevan, OSRD 5088, OEMsr-330, NO-191, Report 189, Franklin Institute, July 7, 1944. Div. 7-123-M2

A mechanical computer, the final model of several developed, supplying preliminary information on how synchronizing run to the high-altitude bombsight operates. Inputs are manually applied, except azimuth stabilization, which is automatically provided.

8. *Report on the Strain Gauge BARB*, J. D. Eisler, Report 155, Franklin Institute, June 1943.

Experimental angular-rate measurement from precessing torque of a restrained gyro, strain gauges being applied to the restraining members. Also its tentative application to a low-altitude bombsight.

9. *Notes on Low Altitude Bombing, I—VII*, G. A. Philbrick and R. M. Peters, Research Project 33, NDRC Section 7.2, Franklin Institute, May 1943—March 1944. Div. 7-122.1-M3-9

A series of studies, concerned principally with the theoretical evaluation of bombing errors, for the various known methods (including BARB), in terms of the several variables of the problem. An account of the BARB theory is included, with predictions, later upheld, of the comparative results obtainable. Offset bombing and glide bombing are considered by various methods. A "hybrid" BARB is shown to be substantially unworkable, so that no time was wasted on it. The writing of L. Goldberg, under the auspices of Section 7.3, should also be referred to in these connections. BARB eventually became bombsights Mark 23 and 27. These are, more precisely, BARB and SuperBARB.

10. *Gyroscopic Lead Computing Sights*, Report to the Services 13, August 1941. Div. 7-122.11-M1

Chapter 8

1. *Norden Bombsight Attachment for Guiding AZON/RAZON Bombs*, John A. Bevan, OSRD 5740, OEMsr-330, AC-36, Report 194, Report to the Services 97, Franklin Institute, Oct. 10, 1944. Div. 7-122.4-M1

A description, briefly, of the theory and nature of the CRAB sight, with instructions for its installation, calibration, and employment operationally. Adapted by NDRC Division 5 to *Preliminary Technical Manual, CRAB, An Attachment to the Norden-Type Bombsight for Aid in Guiding Flare-Equipped Bombs*, issued in June 1945.

2. *Notes on Calibration, Installation, and Operation of CARP*, John A. Bevan, Report 199, Franklin Institute, Oct. 12, 1944.

Descriptive material on CARP, with an appendix giving theory, especially of the *artificial disk speed* technique. Includes functional diagrams, circuits, and photographs. No clues as to why CARP ROC combination failed as a collinear flare-guiding system. For related matters on ROC, and the other projectiles and vehicle-controls, the reader is referred to the report of NDRC Division 5 and its contractors, particularly Gulf Research and Development, and Douglas Aircraft. Note in particular the writings of W. B. Klemperer of the latter organization.

3. *Study of AZON Control and Regulation*, L. Julie, OEMsr-1237, Report 7-1, Columbia University, May 10, 1944.

Describing the initial simulative system whereby the dynamics of guiding were electronically reproduced in the laboratory. Records of the RDA No. 2 at MIT should be consulted for more precise data on trajectory shapes.

4. *Completion of Trainer Project*, L. Julie, OEMsr-1237, Report M-23, Columbia University, Oct. 21, 1944.

Exposition of the final experimental form of the universal guided-bomb simulator, for AZON, RAZON, and ROC. This device was later produced as a field trainer for guiders by Division 5 agencies. Circuits and alignment techniques are given.

5. *The AZON-RAZON Bombing Trainer*, (Models X-1010 and X-1020), NDRC Division 5, Aug. 15, 1945. Div. 5-233-M1

Based on writings and developments of Section 7.2, applied to guided bombing equipment. Attachment of electronic simulative equipment to Army A-6 and A-5 bombing trainers. Similar combinations, with the Navy 7A3 bombing trainer, were also carried out, in which a moving spot was projected on the synthetic landscape of the photographic trainer.

6. *Study of the Guiding Characteristics of the Television Bomb; including a Regulator for Guiding*, L. Julie, OEMsr-1237, Report 7-14, Columbia University, June 29, 1945.

Employment of electronic simulative techniques to study the stability of a proposed control system for attaining interception under remote manual guiding, even with target motion in the air

mass. Cooperation of simulator and human operator was exploited. Criticalness of control parameters was exhaustively studied and later fully corroborated elsewhere.

Chapter 9

1. *Development of Rocket-Gunsights; Summary Report on NO-216*, Eugene P. Cooper, Report 233, Franklin Institute, Sept. 30, 1945.

A review in some detail of the development on rocket sights at The Franklin Institute, with attention to historical sequence. An attempt is made to supply the logical steps which were followed. A brief account of the various field-testing programs is included, together with connected references to other local reports on rocketry developments. This is essentially a final statement by the author, who was given considerable responsibility in this field.

2. *Sighting of Rocket Projectiles from Aircraft*, C. W. Gilbert, British Liaison Code, WA-3190-3, Gunnery Research Unit, RAF, Oct. 19, 1944.

The tactics and aiming problem are developed, with a breakdown of the sighting problem in terms of the allowances for relative speed, trail (attack angle and skid), and gravity. Proposals are made for future development, and a general vector theory included.

3. *On the Use of the Lead-Computing Characteristics of the Mark 18 to Solve the Super-ElevationProblem for Rocketry*, Eugene P. Cooper, Memo to G. A. Philbrick, Franklin Institute, July 18, 1944.

Cooper gives here the theory later embodied in the so-called British method of rocket aiming. Shows reasons for pessimism as to result. Note that this method ultimately generalized to PUSS.

4. *Gravity Drop Formulas for Airborne Rockets*, Harry Pollard, [OEMsr-1007], AMG-C Working Paper 347, AMG-Columbia, Jan. 4, 1945. AMP-603-M2

Fitting of firing-table data to second-order space paths. See also various CIT rocket publications and firing tables.

5. *The RASP Rocket Sight—Model I*, U. C. S. Dilks, E. C. Lewis, and others, OEMsr-330, NO-216, Report 188, Franklin Institute, Aug. 17, 1944. Div. 7-132-M1

A descriptive account of the RASP project in its primary phase. Includes the derivation of equations and their mechanization. A study of the theoretical accuracy is included, together with firing data from tests—with final reductions thereof, photographs, and circuits. Foreword by G. A. Philbrick of Section 7.2.

6. *The RASP Rocket Sight—Model III*, U. C. S. Dilks, E. C. Lewis, and W. C. Sheppard, OSRD 5091, OEMsr-330, NO-265, Report 206, Franklin Institute, Feb. 1, 1945 Div. 7-132-M2

Comparisons with Model I are included throughout. The computational technique is newly explained and an appendix is devoted to a more theoretical treatment of this technique, with general applications. Many photographs are included, and circuits and servo components are described in detail. Analyses of test data, with statistical reduction, are included. A foreword is attached, prepared by G. A. Philbrick.

7. [The] *GRASP Sight for Forward Firing Aircraft Rockets, Model I*, Eugene P. Cooper and Marjorie C. Cooper, OSRD 4991, OEMsr-330, NO-216, Report 211, Franklin Institute, Apr. 9, 1945. Div. 7-131-M1

The method is described, and a derivation is given of the equations to be employed. The details of the physical computation and of the instrumental components are elaborated upon, with a study of errors in firing. Very complete testing data are given. A foreword by G. A. Philbrick is attached.

8. *The GRASP Rocket Sight, Model II*, Eugene P. Cooper and Marjorie C. Cooper, OSRD 6040, OEMsr-330, NO-216, Report 213, Franklin Institute, Sept. 30, 1945. Div. 7-131-M2

Report 211 is here brought up to date for the newer model. (The Aircraft Rocket Sight Mark 2. For ARS Mark 3, the writings of H. Whitney, *et al*, of AMG-C should be referred to by the reader, where corresponding details of the jointly pursued PARS project are more fully covered.) In this report an exhaustive account of laboratory calibration is included, and, in the final edition, the results of firing tests at NOTS, Inyokern. A foreword by the present writer is attached.

9. *Measurement of Angle of Attack and Skid in Rocket Fire Problems*, H. L. Garabedian, OEMsr-1379, AMG-N Working Paper 61 (revised), AMG-Northwestern, Sept. 12, 1945. AMP-502.14-M13

A study of relations preparatory to a test program for skid. Camera techniques and coordinates. Bibliography of related writings.

Chapter 10

1. *Analysis of Optical Systems for PUSS*, I. M. Levitt, Report 241, Franklin Institute, Oct. 31, 1945.

Containing a review of the problems and proposals in connection with the development of universal sight heads for the pilot. Collimating lenses and mirror linkages are discussed, as are the possible modifications to this end of the interesting Fly's Eye design. Illumination problems are explored. Some tentative overall conclusions are arrived at. The final report of the Bristol Company contract—with Section 7.2—should also be consulted.

2. *Interim Report on PUSS*, John A. Bevan, Report 224, Franklin Institute, Aug. 24, 1945.

A compilation of recent documents, arranged as seven independent appendices, and with an introduction on the current status of the project, including theory and design. Instrumental techniques are discussed, with a review of the input components, computers (including PACT for toss bombing), and sight head. The electric and pneumatic versions of PUSS are described and compared.

The appendices of this report, which are worthy bibliographic items in their own right, include memoranda and theoretical papers on gyro systems, computing dynamics (with competing theories), roll stabilization, etc., and were prepared chiefly by M. Golomb and R. O. Yavne.

3. *Servo-Controlled Gyroscope*, U. C. S. Dilks and M. Golomb, Report 238, Franklin Institute, Oct. 2, 1945.

Theory and construction of the two-dimensional, electrically captured gyro in development for PUSS. Effectiveness for small angular rates is discussed, with limitations included. Electronic circuits are given in detail, together with the electrical and other properties of the basic gyro unit which was adapted for this purpose.

4. *Development of Pilot-Operated Fire Control Equipment; Outline of the General Project*, G. A. Philbrick, Apr. 25, 1944.

Preview of the PUSS Project. Aims and hopes for the development. Gives scope and instrumental specifications. Discusses aiming methods, including those without explicit range measurement. Proposed electronic tracking simulator for pilot-operated fire controls.

5. *Notes on Pneumatic PUSS; I*, L. Charles Hutchinson, [OEMsr-1007], AMG-C Working Paper 461, AMG-Columbia, July 17, 1945. AMP-502.1-M28

Considers pneumatic embodiment of computing dynamics. Gives theory and brief experimental data.

6. *Notes on Pneumatic PUSS; II*, L. Charles Hutchinson, [OEMsr-1007], AMG-C Working Paper 478, AMG-Columbia, Aug. 10, 1945. AMP-502.1-M30

Considers pneumatic component for the input giving rate of change of altitude. Circuits, theory, and numerical data. Includes nonlinearities.

7. *PUSS, Target, Sight, Horizon Presentation*, J. R. Ragazzini and R. H. Randall, OEMsr-1237, Report M-34, Columbia University, May 29, 1945.

Simulative computational arrangements for pilot's tracking simulator. Includes dynamics, coordinates to be employed, and the optical or oscilloscopic display.

8. *A Theory of Toss-Bombing*, Harry Pollard, OEMsr-1007, AMP Report 146.1R, AMG-Columbia, September 1945. AMP-803.5-M12

Principles of toss bombing on a revised basis. Measurables of the problem, including target motion. Mechanization of formulas, employing integral of normal acceleration. This is the definitive theory for PACT.

9. *The Azimuth Problem in Toss-Bombing*, Harry Pollard, AMG-C Working Paper 495, AMG-Columbia, Sept. 18, 1945. AMP-803.5-M13

Shows an exact solution for the target motion in azimuth. The effect of roll and bank is included, and the appropriate sighting dynamics, for use with PUSS and PACT, are specified.

PART II

The following reports were all issued by The Franklin Institute under Contract OEMsr-330.

1. *Reflecting Sight for Torpedo Director*, Report 330-1706-102, Apr. 17, 1942.
2. *Torpedo Director NO-106—Tests on Sight Containing a Miniature Ship Model*, Report 330-1706-103, May 6, 1942.
3. *Use of Radar Data in Torpedo Director NO-106*, Report 330-1706-105, May 12, 1942.
4. *Altitude Speed Range Slide Rule for Simplified Torpedo Director NO-106*, Report 330-1706-106, May 25, 1942.
5. *Torpedo Director NO-106: Principle and Operation of Experimental Director*, Report 330-1706-107, May 26, 1942.
6. *Torpedo Director NO-106—Stabilizing Systems Using a Turn Gyro*, Report 330-1706-108, June 1, 1942.
7. *Proposed Continuous Electrical Solution for Torpedo Director*, Report 330-1706-109, July 25, 1942.
8. *A Stabilizing Method which Might be Applied to the Torpedo Director*, Report 330-1706-110, July 25, 1942.
9. *Explicit Relation Between Present Range and Torpedo Run in the Torpedo Director NO-106*, Report 330-1706-111, Aug. 6, 1942.
10. *A Two-Phase Stabilizer for Torpedo Director NO-106*, Report 330-1706-112, Aug. 15, 1942.
11. *An AC Current Controller for Stabilizing the Torpedo Director NO-106*, Report 330-1706-113, Aug. 15, 1942.
12. *Reflecting Sight for Torpedo Director NO-106*, Report 330-1706-114, Aug. 19, 1942.
13. *Torpedo Director NO-106—Further Notes on Two-Man Operation*, Report 330-1706-115, Aug. 20, 1942.
14. *Wind Effects Relative to Torpedo Director*, Report 330-1706-116, Aug. 15, 1942.
15. *Effects of Interchange of Torpedo Run and Range on the Performance of Torpedo Director in Attacks on Bow of Target*, Report 330-1706-117, Aug. 17, 1942.
16. *Torpedo Director NO-106 Model Number One: Principles, Construction, Operation, Accuracy and Specifications*, Report 330-1706-120, Aug. 15, 1942.
17. *Calibration of Torpedo Director for Use by Army Air Forces*, Report 330-1706-121, Aug. 8, 1942.
18. *Torpedo Director NO-106, Flight Tests at Norfolk, Virginia*, Report 330-1706-123, Sept. 30, 1942.
19. *Preliminary Studies Leading to the Development of a Photoelectric Stabilizer for the NO-106 Torpedo Director*, Report 330-1706-124, Oct. 15, 1942.
20. *A Photoelectric Stabilizer for the NO-106 Torpedo Director*, Report 330-1706-125, Oct. 25, 1942.
21. *Conditions of Torpedo Running Time Affecting the Use of Torpedo Director NO-106*, Report 330-1706-128, Oct. 12, 1942.
22. *Tables and Graphs for Calibration of Torpedo Director NO-106*, Report 330-1706-129, Oct. 10, 1942.
23. *Calibration Tests, Torpedo Director NO-106, Type "D"*, Report 330-1706-130, Oct. 26, 1942.
24. *Preliminary Study of Japanese Torpedo Director*, Report 330-1706-132, Nov. 10, 1942.
25. *Use of Torpedo Director—Errors in Lead Angle to be Expected from Errors in Estimation of V_s, Target Speed*, Report 330-1706-133, Dec. 7, 1942.
26. *Two-Autosyn Controller for Stabilizing the NO-106 Torpedo Director*, Report 330-1706-134, Dec. 21, 1942.
27. *The Electrical Solution of NO-106 Torpedo Director*, Report 330-1706-135, Jan. 12, 1943.
28. *Mark 32 Torpedo Director—Inaccuracies in Theory and Design*, Report 330-1706-137, Mar. 14, 1943.
29. *General Solution of the Determination of Minimum Range, Time of Travel to This Point and Angle Between Course of M.T.B. and Minimum Range*, Report 330-1706-140, Mar. 24, 1943.
30. *Adaptation of the General Electric Automatic Pilot System to the Stabilization of the NO-106 Torpedo Director*, Report 330-1706-142, Apr. 19, 1943.
31. *Use of a Rule-of-Thumb Method of Torpedo Direction*, Report 330-1706-145, May 4, 1943.
32. *M.T.B. Director: Preliminary Study of Applicability of Radar Data to Use of*, Report 330-1706-146, May 1, 1943.
33. *Stabilization of Mark 31 Torpedo Director for M.T.B.*, Report 330-1706-148, June 15, 1943.
34. *Field Tests of Stabilized Mark 31 Torpedo Director NO-134*, Report 330-1706-149, June 16, 1943.
35. *Helmsman Direction Indicator for M.T.B. Mk 31 Torpedo Director*, Report 330-1706-154, Oct. 5, 1943.
36. *Stabilizer for Torpedo Director Mark 30*, OSRD 5086, Report 330-1706-162, February 1944.
37. *Study of Apparent Length Method for Aiming Air-Borne Torpedoes*, Report 330-1706-169, July 8, 1944.
38. *Analysis of Films from Project 7-43, A.A.U., U.S. Naval Air Station, Norfolk, Virginia*, Report 330-1706-170, Mar. 29, 1944.
39. *Torpedo Director Mark 32*, R. W. Pitman, OSRD 5079, OEMsr-330, NO-106, Report 330-1706-172, Jan. 26, 1944. Div. 7-141-M1
40. *Linear Target Track Errors Caused by Accidental Deviation from Horizontal of the Flight Path of a Torpedo Plane*, Report 330-1706-175, May 3, 1944.
41. *Torpedo Director Type B-3*, OSRD 6223, Report 330-1706-180, May 15, 1944.
42. *Combination Torpedo Director, Fixed Gun Sight and Bomb Sight (NO-241)*, OSRD 6576, Report 330-1706-195, Nov. 13, 1944.

43. *Range Type Torpedo Director*, Report 330-1706-196, Oct. 19, 1945.

44. *Apparent Length Method for Aiming Tossed Torpedoes*, Report 330-1706-201, Nov. 27, 1944.

45. *Torpedo Director for Maneuvering Targets*, Report 330-1706-226, Nov. 16, 1945.

46. *Torpedo Trainer Computer*, Report 330-1706-227, October 1945.

The following reports were issued by the Statistical Research Group, Columbia University:

47. *Lead Angles for Aerial Torpedo Attacks Against Turning Ships*, OEMsr-618, AMP Report 8.1R, SRG Report 190, July 1944. AMP-405.1-M6

48. *Tables of Aircraft Torpedo Lead Angles*, OEMsr-618, AMP Report 8.2R, SRG Report 453, May 1945. AMP-405.1-M9

PART III

1. *Military Airborne Radar Systems*, NDRC Summary Technical Report, Division 14, Vol. 2, Radiation Laboratory, Massachusetts Institute of Technology.

2. *Analytical Studies in Aerial Warfare*, NDRC Summary Technical Report, Applied Mathematics Panel, Vol. 2.

3. *Aerial Gunnery Problems*, Saunders MacLane, AMG-C 491, AMP, Aug. 31, 1945.

4. *Bibliography of Papers Written at the Applied Mathematic Group, Division of War Research, Columbia University*, L. LaSala and M. Reiner, OEMsr-1007, AMG-C 496, October 1945. AMP-905.2-M2

5. *Pursuit Courses*, Walter Leighton, [OEMsr-1007], AMG-C 141, AMP Memo 57.4, AMG-Columbia, Mar. 29, 1944. AMP-503.7-M1

6. *Aerodynamic Lead Pursuit Curves*, Daniel Zelinsky, [OEMsr-1007], AMG-C 273, AMG-Columbia, Sept. 29, 1944. AMP-503.7-M8

7. *The Aerodynamic Pursuit Curve*, M. M. Day and W. Prager, [OEMsr-1066], AMG-B Memo 31M, AMP Memo 106.1M, AMG-Brown, July 12, 1944. AMP-503.7-M4

8. *Aerodynamic Lead Pursuit Curves for Overhead Attacks*, G. H. Handelman and W. R. Heller, OEMsr-1066, AMG-B 79, AMP Report 106.2R, AMG-Brown, Oct. 31, 1945. AMP-503.7-M15

9. *Equations for Aerodynamic Lead Pursuit Courses*, Leon W. Cohen, OEMsr-1007, AMG-C 316, AMP 153.1R, AMG-Columbia, July 1945. AMP-503.7-M12

10. *Aerodynamic Lead Pursuit Courses*, Leon W. Cohen, OEMsr-1007, AMG-C 443, AMP 153.2R, AMG-Columbia, July 1945. AMP-503.7-M11

11. *Interception and Escape Techniques at High Speed and High Altitudes*, W. B. Klemperer, Report SM-3263 (revised), Douglas Aircraft Company, Inc., Oct. 23, 1941. AMP-504.6-M1

12. *Graphs of Pursuit Curve Characteristics*, N. V. Mayall, OEMsr-101, Carnegie Institute of Washington and Mt. Wilson Observatory, Apr. 5, 1944. AMP-503.7-M2

13. *Pursuit-Bomber Attack Calculations*, Report D.A. 71353, General Electric Company, Dec. 9, 1943.

14. *The Bank of an Airplane and Load Factor Under Conditions of General Flight*, William M. Borgman, Jam Handy Organization, Inc., June 30, 1944. AMP-504.6-M2

15. *Experimental Determination of the Path of a Fighter Plane in Attacking a Bomber*, Navy Contract N166s-2052, Jam Handy Organization, Inc., Mar. 13, 1944.

16. *The Current Status of the Simplest Attackability Problem*, John W. Tukey, [OEMsr-1365], AMG-P Memo 12, AMG-Princeton, June 7, 1945. AMP-504.4-M14

17. *Firing Sidewise from an Airplane—I. Theoretical Considerations*, H. P. Hitchcock, Report 116, Ballistic Research Laboratory, Aberdeen Proving Ground, Aug. 12, 1938.

18. *The Effect of Yaw Upon Aircraft Gunfire Trajectories*, Theodore E. Sterne, Report 345, Ballistic Research Laboratory, Aberdeen Proving Ground.

19. *Analytical Trajectories for Type 5 Projectiles*, Theodore E. Sterne, Report 346, Ballistic Research Laboratory, Aberdeen Proving Ground.

20. *On the Motion of a Projectile with Small or Slowly Changing Yaw*, Report 446, Ballistic Research Laboratory, Jan. 29, 1944.

21. *Analysis and Computation Procedures for 50 Cal. Machine Gun Ballistic Corrections*, Report DF-71342, General Electric Company, Aug. 14, 1942.

22. *Simple Formulas to Fit the Values Tabulated in the Firing Tables FT 0.50 AC-M-1*, George Piranian, OEMsr-1007, AMG-C 130, AMP Memo 104.1, AMG-Columbia, Apr. 6, 1944. AMP-503.1-M2

23. *Ballistic and Deflection Formulas for Aerial Gunnery*, Alex E. S. Green, Laredo Army Air Field Research Bulletin 106, AMP Memo 104.2M, June 27, 1944. AMP-503.1-M6

24. *On Direct Firing Tables for Flexible Aircraft Gunnery, with Particular Reference to Caliber 0.50 A.P. M2 Ammunition*, Theodore E. Sterne, Report 396, Ballistic Research Laboratory, Aberdeen Proving Ground, Sept. 2, 1943.

25. *Deflection Formulas for Airborne Fire Control*, Magnus R. Hestenes, OEMsr-1007, AMG-C 247R, AMP 104.2R, AMG-Columbia, October 1945. AMP-503.3-M8

26. *The Problem of True Lead Under Evasive Action (Two-Dimensional Case)*, [OEMsr-1007], AMG-C 235, AMP Study 104, AMG-Columbia, July 25, 1944. AMP-503.6-M29

27. *Tables of True Leads for Two Pure Pursuit Courses*, Irving Kaplansky, [OEMsr-1007], AMG-C 222, AMP Study 104, AMG-Columbia, July 8, 1944. AMP-503.6-M26

28. *Tables Giving True Lead for Three Pure Pursuit Courses*, Gustav A. Hedlund, [OEMsr-1007], AMG-C 231, AMP Study 104, AMG-Columbia, July 20, 1944. AMP-503.6-M28

29. *Some Uses of Variable Speed Mechanisms In Fire Control*, Magnus R. Hestenes, OEMsr-1007, AMG-C 149, AMP Memo 103.1, AMG-Columbia, Apr. 14, 1944. AMP-503.1-M3

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